

## Physics 4210/5210 Theoretical Mechanics Fall 2024 - Exam 2

- 1.) A particle of mass  $m$  is constrained to move only on a sphere of radius  $b$  that is centered at the origin. The acceleration due to gravity has magnitude  $g$  and acts in the negative  $Z$  direction. (a) Choose an appropriate set of generalized coordinates and write the Lagrangian for the particle. [**2 points**] (b) Derive the Lagrange's equations of motion. [**2 points**] (c) By integrating one of these equations obtain a constant of motion. [**1 point**] (d) What is the physical meaning of this constant? [**1 point**]
  
- 2.) A particle with mass  $m$  and angular momentum,  $\ell$ , moves in a potential  $U(r) = -k/r^3$ , where  $k > 0$  and  $r$  is the radial coordinate. (a) Use the effective potential to find the radius  $r_0$  of a circular orbit. [**2 points**] (b) Is the orbit stable? [**2 points**]
  
- 3.) A particle of mass  $m$  moves only along the  $X$  axis under the influence of a force  $f(x) = -F_0 \sinh(cx)$ , where  $F_0$  and  $c$  are positive constants. (a) Find the potential energy  $U(x)$ . [**2 points**] (b) Find the approximation to  $U(x)$  for simple harmonic oscillations to occur. [**2 points**] (c) Find the frequency of such oscillations. [**1 point**]

Some relevant information:  $e^y = 1 + y + y^2/2 + y^3/6 + y^4/24 + \dots$