Chapter 23: Gauss’s Law

Homework:
• Read Chapter 23
• Questions 2, 5, 10
• Problems 1, 5, 32
• Quiz Thursday on Chapters 23, 24.

Gauss’s Law

• Gauss’s Law is the first of the four Maxwell Equations which summarize all of electromagnetic theory.
• Gauss’s Law gives us an alternative to Coulomb’s Law for calculating the electric field due to a given distribution of charges.

Gauss’s Law: The General Idea

The net number of electric field lines which leave any volume of space is proportional to the net electric charge in that volume.

Flux

The flux \( \Phi \) of the field \( E \) through the surface \( S \) is defined as

\[
\Phi = \int_S \vec{E} \cdot d\vec{A}
\]

The meaning of flux is just the number of field lines passing through the surface.

Best Statement of Gauss’s Law

The outward flux of the electric field through any closed surface equals the net enclosed charge divided by \( \varepsilon_0 \).

Gauss’s Law: The Equation

\[
\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}
\]

• \( S \) is any closed surface.
• \( Q_{\text{enc}} \) is the net charge enclosed within \( S \).
• \( dA \) is an element of area on the surface of \( S \).
• \( \vec{dA} \) is in the direction of the outward normal.

\( \varepsilon_0 = 8.85 \times 10^{-12} \text{ SI units} \)
Flux Examples

Assume two charges, +q and –q. Find fluxes through surfaces. Remember flux is negative if lines are entering closed surface.

\[ \Phi_1 = +\frac{q}{\varepsilon_0} \]
\[ \Phi_2 = -\frac{q}{\varepsilon_0} \]
\[ \Phi_3 = 0 \]
\[ \Phi_4 = \frac{q - q}{\varepsilon_0} = 0 \]

Another Flux Example

Given uniform field \( \mathbf{E} \), find flux through net.

Gauss \( \Rightarrow \) Coulomb

Given a point charge, draw a concentric sphere and apply Gauss's Law:

\[ \vec{E} = E(r) \hat{r} \]
\[ \Phi = \oint \vec{E} \cdot d\vec{A} = E(r) 4\pi r^2 \]

Gauss \( \Rightarrow \)
\[ \Phi = q / \varepsilon_0 \]
\[ \therefore \quad E(r) = q / (4\pi r^2 \varepsilon_0) = \frac{kq}{r^2} ! \]

Q.23-1

What is the name for the product \( \vec{v} \cdot \vec{A} = vA \cos \theta \) ?

1. Curl
2. Energy
3. Flux
4. Gradient

Q.23-1

If a vector field \( \mathbf{v} \) passes through an area \( \mathbf{A} \) at an angle \( \theta \), what is the name for the product \( \mathbf{v} \cdot \mathbf{A} = vA \cos \theta \) ?

(1) curl (2) energy (3) flux (4) gradient
Q.23-1  \[ \Phi = \mathbf{v} \cdot \mathbf{A} = vA \cos \theta = \text{flux} \] from the Latin “to flow”.

Q.23-2  Two charges $Q_1$ and $Q_2$ are inside a closed cubical box of side $a$. What is the net outward flux through the box?

1. $\Phi = 0$
2. $\Phi = \frac{(Q_1 + Q_2)}{\varepsilon_0}$
3. $\Phi = \frac{k(Q_1 + Q_2)}{a^2}$
4. $\Phi = \frac{Q_1 + Q_2}{4\pi \varepsilon_0 a^2}$
5. $\Phi = \frac{Q_1 - Q_2}{4\pi \varepsilon_0 a^2}$

Application of Gauss’s Law

- We want to compute the electric field at the surface of a charged metal object.
- This gives a good example of the application of Gauss’s Law.
- First we establish some facts about good conductors.
- Then we can get a neat useful result:

\[ E = \frac{\sigma}{\varepsilon_0} \]

Fields in Good Conductors

**Fact:** In a steady state the electric field inside a good conductor must be zero.

**Why?** If there were a field, charges would move. Charges will move around until they find the arrangement that makes the electric field zero in the interior.
Charges in Good Conductors

**Fact:** In a steady state, any net charge on a good conductor must be entirely on the surface.

**Why?** If there were a charge in the interior, then by Gauss’s Law there would be a field in the interior, which we know cannot be true.

Field at the Surface of a Conductor

- Construct closed Gaussian surface, sides perpendicular to metal surface, face area = \( A \).
- Flux through left face is zero because \( E = 0 \).
- Field is perpendicular to surface or charges would move, therefore flux through sides is 0.
- So net outward flux is \( EA \).

Large Sheet of Charge

\[
\Phi = \frac{Q}{\varepsilon_0} \quad \text{Gauss’ law}
\]
\[
Q = \sigma \cdot A
\]
\[
\Phi = \int \mathbf{E} \cdot d\mathbf{A} = 2EA
\]
\[
2EA = \frac{\sigma A}{\varepsilon_0} \quad E = \frac{\sigma}{2\varepsilon_0}
\]

Long Line of Charge

\[
\lambda = \frac{\text{charge/length}}{\text{linear charge density}}
\]
\[
\Phi = \frac{Q}{\varepsilon_0} \quad \text{Gauss’ law}
\]
\[
Q = l\lambda
\]
\[
\Phi = \int \mathbf{E} \cdot d\mathbf{A} = EA
\]
\[
A = 2\pi rl
\]
\[
E = \frac{\lambda}{2\pi \varepsilon_0}
\]

Summary for different dimensions

- **Point charge, \( d=0 \)**
  \[
  \Phi = 4\pi r^2E = \frac{Q}{\varepsilon_0} \quad E = \frac{Q}{(4\pi \varepsilon_0 r^2)}
  \]
  \[
  E \propto \frac{1}{r^{2-d}}
  \]
- **Line charge, \( d=1 \)**
  \[
  \Phi = 2\pi rE = \lambda l / \varepsilon_0
  \]
  \[
  E = \lambda / (2\pi \varepsilon_0 r)
  \]
- **Surface charge, \( d=2 \)**
  \[
  \Phi = 2AE = \sigma A / \varepsilon_0
  \]
  \[
  E = \sigma / (2\varepsilon_0)
  \]