Veteran’s Day, November 11

No Recitation Classes, but Chapter 17 problems are still due on Friday!

Waves: Quantum Physics

Types of waves

– Mechanical waves

exist only within a material medium. e.g. water waves, sound waves, etc.

– Electromagnetic waves

require no material medium to exist. e.g. light, radio, microwaves, etc.

– Matter waves

waves associated with electrons, protons, etc.
**Particle – Wave Duality**

Our language fails us in the description of the microscopic world.

**Example:** How would you describe an electron or proton?

This is *not* accurate.

Elementary particles have both wave and particle characteristics. -- and the world is in constant motion.

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**Start with a cosine wave.**
\[ y = \frac{1}{2} \cos(28\pi x) + \cos(30\pi x) + \frac{1}{2} \cos(32\pi x) \]

\[ y = \frac{1}{5} \cos(22\pi x) + \frac{1}{4} \cos(24\pi x) + \frac{1}{3} \cos(26\pi x) + \frac{1}{2} \cos(28\pi x) + \cos(30\pi x) + \frac{1}{3} \cos(32\pi x) + \frac{1}{4} \cos(34\pi x) + \frac{1}{5} \cos(36\pi x) + \frac{1}{2} \cos(38\pi x) \]
\[ y = \sum_{\kappa=6}^{24} \frac{1}{|5 - \kappa| + 1} \cos(2\pi \kappa x) \]

What’s happened?

As we’ve added more cosine waves, the “disturbance” is becoming localized (i.e., *particle*-like).

Add an infinite number to accurately represent free electrons, protons, etc.
As we’ve added more cosine waves, the “disturbance” is becoming localized (i.e., particle-like).

Position of a particle is given by \( x \).

Linear momentum of a particle is related to \( \kappa \).

known momentum
(one \( \kappa \)).

\[ \Delta p = 0 \]

position unknown
(spread out through all space).

\[ \Delta x = \infty \]
Linear momentum of a particle is related to $\kappa$.

Position of a particle is given by $x$.

range of momenta (19 $\kappa$’s). $\Delta p > 0$

position known within range (localized in space) $\Delta x < \infty$

$\Delta x \Delta p > $ constant

$\Delta x \Delta p \geq \frac{\hbar}{2}$

Heisenberg Uncertainty Principle

$\Delta x \Delta p \geq \frac{\hbar}{2}$
Interference

For distant points,
\[ \Delta L = d \sin \theta \]

\[ \Delta \phi / 2\pi = \Delta L / \lambda \]

Constructive interference: \( \Delta L = d \sin \theta = m\lambda \)
Destructive interference: \( \Delta L = d \sin \theta = (m + \frac{1}{2})\lambda \)

\( S_1 \) and \( S_2 \) are now atoms in a crystal. The waves are particles.

\[ y/D = \tan \theta \]

\[ \Delta L = d \sin \theta = m\lambda \]

\[ y/D = \tan \theta \]

If \( \Delta \phi = m \, (2\pi) \) or \( \Delta L / \lambda = m \) \( (m = 0, 1, 2 \ldots) \)
We have fully constructive interference \( \Delta L = d \sin \theta = m\lambda \)

If \( \Delta \phi = (2m + 1)\pi \) or \( \Delta L / \lambda = m + \frac{1}{2} \) \( (m = 0, 1, 2 \ldots) \)
We have fully destructive interference \( d \sin \theta = (m + \frac{1}{2})\lambda \)
Daily Quiz, November 10, 2004

S₁ and S₂ are emitting identical waves in phase. A minimum is occurring at P. What happens when S₂ is turned off?

1) nothing changes
2) interference pattern disappears
3) a maximum occurs at P
4) constant intensity all along y
5) none of the above

Interference pattern requires at least two waves.
Photons, Electrons, and Neutrons

x-rays (photons) on aluminum \( \lambda = 0.071 \text{ nm} \)
electrons on aluminum \( \lambda = 0.05 \text{ nm} \)
neutrons on copper \( \lambda = 0.12 \text{ nm} \)

Reference: Tipler, *Foundations of Modern Physics*

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Quantum World -- Nuclei

Protons and neutrons experience a potential energy function mathematically similar to clamped string. -- Standing waves inside the nucleus

energy

- ground state
- 1\(^{\text{st}}\) excited state
- 2\(^{\text{nd}}\) excited state
Quantum World -- atoms

Particles are waves and waves are particles!

Only discrete values (quantized) of energy, momentum, etc. are allowed.

Consider an atom
(hydrogen)

electron probability functions

Quantum World -- atoms

Particles are waves and waves are particles!

Only discrete values (quantized) of energy, momentum, etc. are allowed.

Consider an atom
(sodium)

electron energy levels
Quantum World -- molecules

Particless are waves and waves are particles!

Only discrete values (quantized) of energy, momentum, etc. are allowed.

Consider a molecule, angular momentum is quantized.

\[ L = \hat{\hbar} n \quad n = 0, 1, 2, 3, \ldots \]

Rotational energy is then quantized.

\[ K_{\text{rotation}} = \frac{1}{2} I \omega^2 = \frac{1}{2I} L^2 = \frac{1}{2I} \hat{\hbar}^2 n^2 \]
Quantum World -- molecules

Particles are waves and waves are particles!

Only discrete values (quantized) of energy, momentum, etc. are allowed.

Consider a molecule, vibrational energy is also quantized.

\[ K_{\text{vibration}} = \left( n + \frac{1}{2} \right) \hbar \omega \]

Quantum World -- molecules

Consider a molecule

Only discrete values (quantized) of energy, momentum, etc. are allowed.

\[ K_{\text{vibration}} = \left( n + \frac{1}{2} \right) \hbar \omega \]
\[ K_{\text{rotation}} = \frac{1}{2I} \hbar^2 n^2 \]
**Microscopic View of Material**

How do we know matter is made of atoms?

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**Heat Capacity**

Interested in how well material can absorb energy as we increase the temperature (Chapters 18-19).

Classical theory predicts this to be a constant $C_v / R = 1/2$ times the number of degrees of freedom. See Section 19-9, pages 523-525.

Ok for human scale, but wrong for microscopic low temperature scale.
Speed of Sound  (Chapter 17)

Sound (longitudinal wave) results in compression and/or rarification of the atoms in the material.

\[ v_{\text{sound}} = \sqrt{\frac{B}{\rho}} \quad B = \text{bulk modulus}, \rho = \text{density} \]

Measure \( v_{\text{sound}} \) to obtain the Bulk modulus.

Use \( B \) to obtain the force \( F \). Use Hook’s Law to obtain vibrational frequencies.

Quantum Theory

Vibrational frequencies are quantized.

Einstein Theory: Atoms vibrate independently.

Debye Theory: Neighboring atoms affect vibrations (Coupled oscillations).
**Einstein and Debye Theories**

Vibrational frequencies are quantized.

Einstein Theory: Atoms vibrate independently.

\[ C_V = 3R \left( \frac{\Theta_E}{T} \right)^2 e^{-\Theta_E/T} \quad \Theta_T = \text{Einstein temperature (characteric constant)} \]

Debye Theory: Neighboring atoms affect vibrations (Coupled oscillations).

So, matter is made of atoms continually vibrating!!