Newton’s Law of Gravitation

The force that makes an apple fall is the same force that holds moon in orbit.

**Newton’s law of gravitation:** Every particle attracts any other particle with a gravitation force given by:

\[
\vec{F} = G \frac{m_1 m_2}{r^2} \left( -\hat{r} \right)
\]

G: gravitation constant, \( G = 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2 \)

The minus sign means this force is always **attractive**.
Gravitational Attraction

\[ \vec{F} = G \frac{m_1 m_2}{r^2} (-\hat{r}) \]

“Resistance” to Acceleration

\[ \sum_{i} \vec{F}_i = m\vec{a} \]

Einstein Equivalence Theorem:
Inertial mass = Gravitational mass
**Shell Theorem**

**Shell theorem:** a uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell’s mass were concentrated at its center.

![Shell Diagram]

is the same as

![Force Diagram]

**Problem 13-20**

Two concentric spheres $M_1$ and $M_2$. Find $F$ at radii $a$, $b$, and $c$.

\[ F = G \frac{Mm}{r^2} \]

20. (a) What contributes to the $GmM/r^2$ force on $m$ is the (spherically distributed) mass $M$ contained within $r$ (where $r$ is measured from the center of $M$). At point $A$ we see that $M_1 + M_2$ is at a smaller radius than $r = a$ and thus contributes to the force:

\[ |F_{\text{total}}| = G \frac{(M_1 + M_2)m}{a^2}. \]

(b) In the case $r = b$, only $M_1$ is contained within that radius, so the force on $m$ becomes $GM_1m/b^2$.

(c) If the particle is at $C$, then no other mass is at smaller radius and the gravitational force on it is zero.
Problem 13-31

Three masses. Move B from near A to near C. Find work done by a) you, b) by gravity.

31. (a) The work done by you in moving the sphere of mass \(m_B\) equals the change in the potential energy of the three-sphere system. The initial potential energy is

\[
U_i = -\frac{G m_A m_B}{d} - \frac{G m_B m_C}{L} - \frac{G m_A m_C}{L - d}
\]

and the final potential energy is

\[
U_f = -\frac{G m_A m_B}{L - d} - \frac{G m_A m_C}{L} - \frac{G m_B m_C}{d}.
\]

The work done is

\[
W = U_f - U_i = G m_B \left( m_A \left( \frac{1}{d} - \frac{1}{L - d} \right) + m_C \left( \frac{1}{L - d} - \frac{1}{d} \right) \right)
\]

\[
= (6.67 \times 10^{-11} \, \text{m}^3/\text{s}^2\cdot\text{kg})(0.010 \, \text{kg}) \left[ (0.080 \, \text{kg}) \left( \frac{1}{0.040 \, \text{m}} - \frac{1}{0.080 \, \text{m}} \right) \right. \\
\left. + (0.020 \, \text{kg}) \left( \frac{1}{0.080 \, \text{m}} - \frac{1}{0.040 \, \text{m}} \right) \right]
\]

\[
= +5.0 \times 10^{-13} \, \text{J}.
\]

(b) The work done by the force of gravity is \((-U_f - U_i) = -5.0 \times 10^{-13} \, \text{J}.

Problem 13-44

Find distance between the foci of the Earth’s orbit.

44. (a) The distance from the center of an ellipse to a focus is $ae$ where $a$ is the semimajor axis and $e$ is the eccentricity. Thus, the separation of the foci (in the case of Earth’s orbit) is

$$2ae = 2\left(1.50 \times 10^1 \text{ m}\right) (0.0167) = 5.01 \times 10^0 \text{ m}.$$  

(b) To express this in terms of solar radii (see Appendix C), we set up a ratio:

$$\frac{5.01 \times 10^0 \text{ m}}{6.96 \times 10^8 \text{ m}} = 7.20.$$

Problem 13-46

Find distance for geosynchronous orbit.

http://science.nasa.gov/Realtime/Jtrack/3d/JTrack3D.html
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Find distance for geosynchronous orbit.

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46. To “hover” above Earth \((M_E = 5.98 \times 10^{24} \text{ kg})\) means that it has a period of 24 hours \((86400 \text{ s})\). By Kepler’s law of periods,

\[
(86400)^2 = \left( \frac{4\pi^2}{GM_E} \right) r^3 \Rightarrow r = 4.225 \times 10^7 \text{ m}.
\]

Its altitude is therefore \(r - R_E\) (where \(R_E = 6.37 \times 10^6 \text{ m}\)) which yields \(3.58 \times 10^7 \text{ m}\).

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Satellites and Orbits

Potential energy \(U(r) = -G \frac{Mm}{r}\)

Centripetal force \(F_c = m \frac{v^2}{r} = G \frac{Mm}{r^2}\)

Kinetic energy \(K = \frac{1}{2} mv^2 = G \frac{Mm}{2r} = -\frac{1}{2} U\)

Total energy \(E = K + U = \left(-\frac{1}{2} U\right) + U = -G \frac{Mm}{2r}\)
Elliptical Orbits

Total energy \[ E = -\frac{G M m}{2a} \]

Daily Quiz, October 25, 2004

Total Energy \[ E = -\frac{G M m}{2a} \]

\( a_1 < a_3 < a_2 \Rightarrow E_2 \) is least negative.

All three orbits intersect at P. Which path has the greater total energy?

1) 1 2) 2 3) 3
4) all have the same total energy
Planets and Satellites: Kepler’s laws

The law of orbits: All planets move in elliptical orbits, with the Sun at one focus.

\[ \text{Angular momentum is conserved} \quad \frac{dA}{dt} = \frac{L}{2m} \]
**Kepler’s Laws**

**The law of periods:** the square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

Circular orbit  e = 0

\[ T^2 = \left( \frac{4\pi^2}{GM} \right) r^3 \]

Elliptical orbit  e > 0

\[ T^2 = \left( \frac{4\pi^2}{GM} \right) a^3 \]

a is the major axis