Ch 13: Newton’s Law of Gravitation

The force that makes an apple fall is the same force that holds moon in orbit.

Newton’s law of gravitation: Every particle attracts any other particle with a gravitation force given by:

\[ \vec{F} = G \frac{m_1 m_2}{r^2} \left( -\hat{r} \right) \]

G: gravitation constant, \( G = 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2 \)

The minus sign means this force is always attractive.

This force depends on the masses *and* the distance squared between them.

Between the earth and a 60 kg person standing on the earth’s surface: \( F = 588 \text{ N} \)
If the person moved to twice the earth’s radius, the force will now be divided by \( 2^2 \) (or 4). \( F_{2R} = \frac{588}{4} = 147 \text{ N} \)

Gravitational force between two 60 kg persons standing 1 m apart: \( F = 2.4 \times 10^{-7} \text{ N} \)
• The principle of superposition: net effect is the sum of the individual effects

\[ \vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n} = \sum_{i=2}^{n} \vec{F}_{1i} \]

Sample 13-1: What is the net gravitational force \( F_1 \) that act on particle 1 due to the other two particles?

\[ \vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} \]

Shell Theorem

Shell theorem: a uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell’s mass were concentrated at its center

is the same as
Gravitation near Earth’s Surface

- Assume the Earth is a uniform sphere of mass \( M \), the gravitation force on a particle of mass \( m \) located outside earth a distance \( r \) from Earth’s center is

\[
F_g = G \frac{Mm}{r^2}
\]

since \( F = ma \)

\[
a_g = G \frac{M}{r^2}
\]

- \( a_g \) varies with attitude,

Near earth’s surface: \( a_g = 9.801 \text{ m/s}^2 \)

at an altitude = 35700 km, \( a_g = 0.225 \text{ m/s}^2 \)

Gravitation inside earth

Consider a uniform sphere of matter. We want to find the force on \( m \) a radial distance \( a \) from the center

Opposing forces cancel for masses \( r > a \). Net force on \( m \) comes from the mass that is inside \( r < a \).

\[
|F| = G \left( \frac{M_{\text{inside}} m}{r^2} \right)
\]

where \( M_{\text{inside}} = \int_{0}^{a} \rho \int_{0}^{\pi} \sin \theta \ d\theta \int_{0}^{2\pi} \ d\phi \)

For a uniform density \( \rho \), the integral reduces to \( r \) times the volume inside \( a \).

\[
M_{\text{inside}} = \rho \int_{0}^{a} r^2 \ dr (2\pi) = \frac{4}{3} \rho \pi a^3 = M_{\text{total}} \left( \frac{a^3}{R^3} \right)
\]
Gravitational Potential Energy

- Gravitational potential energy of a system of two particles M and m:

\[
U(r) = -W = - \int F(r) \, dr = - \int F(r) dr \cos 180°
\]

\[
= \int F(r) dr = \int \frac{GMm}{r^2} dr = - \frac{GMm}{r} + \text{constant}
\]

Let \( U(r) = 0 \) when \( r = \infty \) then \( \text{constant} = 0 \)

so we have \( U(r) = -\frac{GMm}{r} \)

for any finite value of \( r \), \( U \) is negative
From energy conservation:

$$K_i + U_i = \frac{1}{2} mv^2 + (-GMm/R) = K_f + U_f = E_{tot} \geq 0$$

This yields: $v = \sqrt{\frac{2GM}{R}}$

Earth: $M = 5.98 \times 10^{24}$ kg, $R = 6.37 \times 10^6$ m, $v = 11.2$ km/s

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**Planets and Satellites: Kepler’s laws**

**The law of orbits**: All planets move in elliptical orbits, with the Sun at one focus.
**The law of areas:** A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet’s orbit in equal times; that is, the rate $\frac{dA}{dt}$ at which it sweeps out area $A$ is constant. 

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega$$

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow$$

$$L = r p_\perp = (r)(mv_\perp) = mr^2 \omega$$

Angular momentum is conserved

$$\frac{dA}{dt} = \frac{L}{2m}$$

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**Kepler’s Laws**

**The law of periods:** the square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

Circular orbit $e = 0$

$$T^2 = \left( \frac{4\pi^2}{GM} \right) r^3$$
**Kepler’s Laws**

**The law of periods:** the square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

\[ T^2 = \left( \frac{4\pi^2}{GM} \right) a^3 \]

Elliptical orbit, \( e > 0 \)

**Daily Quiz, October 22, 2004**

At which point is \( m \) moving the fastest?

1) 1  
2) 2  
3) 3  
4) 4  
5) always moves at the same speed  
6) some other point on the orbit
Daily Quiz, October 22, 2004

Reason: m sweeps equal areas in equal times.

Another way of looking at it: U(r) is most negative at 1, so K must be greatest there to keep E constant.

At which point is m moving the fastest?

1) 1  2) 2  3) 3  4) 4
5) always moves at the same speed
6) some other point on the orbit

http://science.nasa.gov/Realtime/Jtrack/3d/JTrack3D.html