Linear Momentum

- The linear momentum of a particle is a vector defined as
  \[ \vec{p} = m\vec{v} \]

- Newton’s second law in terms of momentum
  \[ \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} = m\vec{a} = \vec{F}_{net} \]

Most of the time the mass doesn’t change, so this term is zero. Exceptions are rockets (Tuesday)

Linear Momentum is conserved

\[ \vec{P}_{f} = \vec{P}_{i} \]

Let the mass \( M \) change \((M + \text{d}M)\), which in turn makes the velocity change. Rocket exhausts \(-\text{d}M\) in time \( \text{d}t \) at a velocity \( U \) relative to our inertial reference frame.

\[ M\vec{v} = -\text{d}M U + (M + \text{d}M)(v + \text{d}v) \]

\[ \begin{align*}
  \text{Velocity of rocket relative to ref. frame} &= \text{Velocity of rocket relative to exhaust} + \text{Velocity of exhaust relative to ref. frame} \\
  (v + \text{d}v) &= v_{\text{rel}} + U
\end{align*} \]

\[ v_{\text{rel}} \quad U \quad v + \text{d}v \]
Linear Momentum is conserved
\[ \mathbf{P}_f = \mathbf{P}_i \]

\[ U = (v + dv) - v_{\text{rel}} \]
Substitute and divide by dt

\[ Mv = -dM U + (M + dM)(v + dv) \]

\[ \frac{\text{d}M}{\text{d}t} v_{\text{rel}} = M \frac{dv}{dt} \]

\[ U \quad v_{\text{rel}} \quad v + dv \]

Linear Momentum is conserved
\[ \mathbf{P}_f = \mathbf{P}_i \]

\[ U = (v + dv) - v_{\text{rel}} \]
Substitute and divide by dt

\[ Mv = -dM U + (M + dM)(v + dv) \]

\[ dv = -\frac{dM}{M} v_{\text{rel}} \Rightarrow \int_{v_i}^{v_f} dv = -v_{\text{rel}} \int_{M_i}^{M_f} \frac{dM}{M} \]

\[ v_f - v_i = v_{\text{rel}} \ln \left( \frac{M_i}{M_f} \right) \]

\[ v_{\text{rel}} \quad v + dv \]
Collisions

In absence of external forces, 

**Linear momentum is conserved.**

Mechanical energy may or may not be conserved.

Elastic collisions: Mechanical energy is conserved.

Inelastic collisions: Mechanical energy is NOT conserved.

But, **Linear momentum is always conserved.**

Conservation of Linear Momentum

\[
\begin{align*}
\vec{p}_i &= \vec{p}_f \quad \Rightarrow \\
\vec{p}_{1i} + \vec{p}_{2i} &= \vec{p}_{1f} + \vec{p}_{2f} \\

m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} &= m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}
\end{align*}
\]
Center of Mass motion is constant

\[ \vec{p} = \vec{p}_{1i} + \vec{p}_{2i} = M \vec{v}_{c.m.} = (m_1 + m_2) \vec{v}_{c.m.} \]

\[ \Rightarrow \quad \vec{v}_{c.m.} = \frac{\vec{p}}{m_1 + m_2} \]
Perfectly inelastic collision: The two masses stick together

\[ m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = m_1 \vec{V}_{1f} + m_2 \vec{V}_{2f} = (m_1 + m_2) \vec{V}_f \]

\[ \vec{V}_f = \frac{m_1}{m_1 + m_2} \vec{V}_{1i} \quad (= \vec{V}_{c.m.}) \]
Inelastic Collisions

Was the mechanical energy:
- conserved \((E_i = E_f)\);
- lost \((E_i > E_f)\); or
- gained \((E_i < E_f)\);
in the collision?

Inelastic Collisions

How much mechanical energy was lost in the collision?

\[
\frac{1}{2} (m_1 + m_2) v_i^2 = \frac{1}{2} (m_1 + m_2) \left( \frac{m_1}{m_1 + m_2} \right)^2 v_{i1}^2 = \frac{1}{2} \left( \frac{m_1}{m_1 + m_2} \right) v_{i1}^2 \Rightarrow
\]

\[
E_{\text{lost}} = E_i - E_f = \frac{1}{2} m_i v_{i1}^2 - \frac{1}{2} \left( \frac{m_i^2}{m_1 + m_2} \right) v_{i1}^2 = \frac{1}{2} \left( \frac{m_i m_2}{m_1 + m_2} \right) v_{i1}^2
\]

\[
E_{\text{lost}} = \frac{1}{2} \left( \frac{m_1 m_2}{m_1 + m_2} \right) v_{i1}^2
\]
Perfectly elastic collision: Mechanical energy is conserved

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (= M \vec{v}_{c.m.}) \]

\[ E_i = \frac{1}{2} m_1 \vec{v}_{1i}^2 + \frac{1}{2} m_2 \vec{v}_{2i}^2 = \frac{1}{2} m_1 \vec{v}_{1f}^2 + \frac{1}{2} m_2 \vec{v}_{2f}^2 = E_f \]

---

Perfectly elastic collision: Mechanical energy is conserved

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (= M \vec{v}_{c.m.}) \]

\[ \vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} \vec{v}_{1i} + \frac{2 m_2}{m_1 + m_2} \vec{v}_{2i} \]

\[ \vec{v}_{2f} = \frac{2 m_1}{m_1 + m_2} \vec{v}_{1i} + \frac{m_2 - m_1}{m_1 + m_2} \vec{v}_{2i} \]
What about two (or more) dimensions?

Simply break the momenta and velocities into their x-, y-, and z-components.

Opinion Survey
(i.e., Daily Quiz, October 05, 2004)

Which part of the course works best for you?

– I learn most from:

(1) The lectures.    (2) The Thursday discussion groups.
(3) The labs.        (4) Doing homework problems myself.
(5) The exams.       (6) Reading the textbook.
Opinion Survey

What about the remote-response quizzes during the lectures? How do you like them on a scale of 1-5?
Give a numerical response with 1 the worst and 5 the best.

1 5
Ughh, bleethe  Super, they rock!

Opinion Survey

What about the remote-response quizzes during the lectures? Same question with some thoughts to influence your rating!

(1) Terrible. Better to just have homework graded.
(2) They’re a drag. They’re too easy to be fun.
(3) I’m neutral. Nothing makes any difference.
(4) OK. At least it breaks up the (boring) lecture.
(5) Great. Helps me keep up with the main ideas.