Exam 03 Tomorrow!!

Will cover Chapters 06, 07, and 08 (through conservative forces)
Conservation of mechanical energy

For an isolated system with only conservative forces (e.g., \( F = mg \) and \( F = -kx \)) acting on the system:

\[
E_{\text{mec,1}} = E_{\text{mec,2}} = E_{\text{total}}
\]

\[
K_1 + U_1 = K_2 + U_2 = E_{\text{total}}
\]

\[
\frac{1}{2} mv_1^2 + mgx_1 + \frac{1}{2} kx_1^2 = \frac{1}{2} mv_2^2 + mgx_2 + \frac{1}{2} kx_2^2 = E_{\text{total}}
\]

Initial mechanical energy  Final mechanical energy
**Work done by external force**

- When no friction acts within the system, the net work done by the external force equals to the change in mechanical energy
  \[ W = \Delta E_{\text{mec}} = \Delta K + \Delta U \]

- Friction is a non-conservative force

- When a kinetic friction force acts within the system, then the thermal energy of the system changes:
  \[ \Delta E_{\text{thermal}} = f_k d \]

  Therefore
  \[ W = \Delta E_{\text{mec}} + \Delta E_{\text{thermal}} \]
Work done by external force

• When there are non-conservative forces (like friction) acting on the system, the net work done by them equals to the change in mechanical energy

\[ W_{\text{net}} = \Delta E_{\text{mec}} = \Delta K + \Delta U \]

\[ W_{\text{friction}} = f_k d \]

Work done by friction takes mechanical energy *out of* the system by converting it to thermal energy.
Conservation of Energy

• The total energy $E$ of a system can change only by amounts of energy that are transferred to or from the system

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

If there are no internal energy change, but there is friction acts within the system:

$$W = \Delta E_{mec} + \Delta E_{th}$$

If there are only conservative forces act within the system:

$$W = \Delta E_{mec}$$

If we take only the single object as the system

$$W = \Delta K$$
Law of Conservation of Energy

• For an isolated system (\( W = 0 \)), the total energy \( E \) of the system cannot change
  \[
  \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0
  \]

• For an isolated system with only conservative forces, \( \Delta E_{\text{th}} \) and \( \Delta E_{\text{int}} \) are both zero. Therefore:
  \[
  \Delta E_{\text{mec}} = 0
  \]
Sample Problem 8-6

A wooden crate of \( m = 14\text{kg} \) is pushed along a horizontal floor with a constant force of \( |F| = 40\text{N} \) for a total distance of \( d = 0.5\text{m} \), during which the crate’s speed decreased from \( v_o = 0.60\text{ m/s} \) to \( v = 0.20\text{m/s} \).

A) Find the work done by \( F \).

\[
W = Fd \cos \phi = (40\text{N})(0.50\text{m})\cos 0^\circ = 20\text{J}
\]

B) Find the increase in thermal energy.

\[
W = \Delta E_{\text{mec}} + \Delta E_{\text{thermal}} = 20\text{J} = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 + \Delta E_{\text{thermal}}
\]

\[
\Rightarrow \Delta E_{\text{thermal}} = W - \Delta E_{\text{mec}} = 20\text{J} - (\frac{1}{2}mv^2 - \frac{1}{2}mv_o^2) = 22.2\text{J}
\]
Law of Conservation of Energy

Count up the initial energy in all of its forms.

\[ E_i = K_i + U_i + E_{\text{thermal}_i} + E_{\text{internal}_i} \]

Count up the final energy in all of its forms.

\[ E_f = K_f + U_f + E_{\text{thermal}_f} + E_{\text{internal}_f} \]

These two must be equal.

\[ E_i = E_f \]

\[ \Rightarrow E_i = K_i + U_i + E_{\text{thermal}_i} + E_{\text{internal}_i} = E_f = K_f + U_f + E_{\text{thermal}_f} + E_{\text{internal}_f} \]
Law of Conservation of Energy

\[ E_i = K_i + U_i + E_{\text{thermal}_i} + E_{\text{internal}_i} \]
\[ = E_f = K_f + U_f + E_{\text{thermal}_f} + E_{\text{internal}_f} \]

Rearrange terms.

\[ (E_f - E_i) = \]
\[ (K_f - K_i) + (U_f - U_i) + (E_{\text{thermal}_f} - E_{\text{thermal}_i}) + (E_{\text{internal}_f} - E_{\text{internal}_i}) = 0 \]

\[ \Delta E = (\Delta K + \Delta U) + \Delta E_{\text{thermal}} + \Delta E_{\text{internal}} \]
\[ = \Delta E_{\text{mechanical}} + \Delta E_{\text{thermal}} + \Delta E_{\text{internal}} = 0 \]
Sample Problem 8-7

In the figure, a 2.0kg package slides along a floor with speed $v_1 = 4.0 \text{m/s}$. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic friction force from the floor, of magnitude 15 N , act on it. The spring constant is 10,000 N/m. By what distance $d$ is the spring compressed when the package stops?
The change in mechanical energy must equal the energy converted to thermal energy.

\[
\Delta E_{\text{mechanical}} + \Delta E_{\text{thermal}} = 0
\]

\[
\Rightarrow (E_{\text{mec,2}} - E_{\text{mec,1}}) + \Delta E_{\text{thermal}} = 0
\]

\[
\Rightarrow E_{\text{mec,2}} = E_{\text{mec,1}} - \Delta E_{\text{thermal}} = 0
\]

Initial mechanical energy,

\[
E_{\text{mec,1}} = K_1 + U_1 = \frac{1}{2} mv_1^2 + U_1 = \frac{1}{2} mv_1^2 + 0
\]

Final mechanical energy,

\[
E_{\text{mec,2}} = K_2 + U_2 = \frac{1}{2} mv_2^2 + U_2 = 0 + \frac{1}{2} kd_2^2
\]
The change in mechanical energy must equal the energy converted to thermal energy.

\[ \Delta E_{\text{thermal}} = f_k d \]

\[ \Rightarrow (E_{\text{mec,2}} - E_{\text{mec,1}}) + \Delta E_{\text{thermal}} = \left( \frac{1}{2} kd^2 - \frac{1}{2} mv_1^2 \right) + f_k d = 0 \]

\[ \Rightarrow \left( \frac{1}{2} kd^2 - \frac{1}{2} mv_1^2 \right) + f_k d = 0 \]

\[ \Rightarrow \frac{1}{2} kd^2 + f_k d - \frac{1}{2} mv_1^2 = 0 \quad \Rightarrow d = 0.055\text{m} \]

Initial mechanical energy,

\[ E_{\text{mec,1}} = K_1 + U_1 = \frac{1}{2} mv_1^2 + U_1 = \frac{1}{2} mv_1^2 + 0 \]

Final mechanical energy,

\[ E_{\text{mec,2}} = K_2 + U_2 = \frac{1}{2} mv_2^2 + U_2 = 0 + \frac{1}{2} kd_2^2 \]
A hydrogen atom with kinetic energy of 4 eV is approaching another hydrogen atom in its ground state. The potential energy is shown to the right.
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Will this H atom be captured and thereby become a H₂ molecule?
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1) yes    2) no    3) maybe
A hydrogen atom with kinetic energy of 4 eV is approaching another hydrogen atom in its ground state. The potential energy is shown to the right.

\[ E_{\text{initial}} = K_{\text{initial}} + U_{\text{initial}} \]

\[ 4.0 \text{ eV} + 0 \text{ eV} = 4.0 \text{ eV} \]
The H-atom hits the repulsive potential energy wall at about 0.04 nm and is reflected back to infinity. Another object is needed to absorb the excess kinetic energy. Note that the speed (kinetic energy) increases as the potential well becomes more negative, but the total energy is constant.

Will this H atom be captured and thereby become a H$_2$ molecule?

1) yes  2) no  3) maybe
Review: Chapters 6, 7, 8
Chapter 06: Friction

A block lies on a horizontal floor.

a) What is the magnitude of the friction force \( f \) on it from the floor? \( f = 0 \text{N}, \) but note \( f_{\text{s,max}} = \mu_s F_N \)

b) If a horizontal force of 5 N is now applied to it, but it does not move, what is \( f \) now? \( f_s = 5 \text{N} \)

c) If \( f_{\text{s,max}} = 10 \text{N} \), will the block move if the horizontal applied force is 8 N? \( \text{no, because } F < f_s \)

d) How about 12 N? \( \text{yes, because } F > f_s \)
Chapter 6: Uniform circular motion

- For uniform circular motion, the centripetal acceleration
  \[ a = \frac{v^2}{R} \]
  which is caused by a force called centripetal force \( F \)
  \[ F = ma = m\left(\frac{v^2}{R}\right) \]
  - direction of \( F \): points radially inward
  - Centripedal force is not a new kind of force
Car rounding a banked curve

Friction and \( mg \sin \theta \) (the Normal component) are causing the car to move in a circle.

For the example in the book:

\[
\text{fr} + F_N = m\ddot{a}_C = m \frac{v^2}{R}
\]

\[
F_{Nr} = mg \sin \theta = m \frac{v^2}{R}
\]

\[
F_{Ny} = mg \cos \theta
\]

\[
tan \theta = \frac{v^2}{gR}
\]
Example: (Sample problem 6-8 in the book) In the figure, a person is riding the Rotor.
Suppose that the coefficient of static friction $\mu_s$ between the rider’s clothing and the canvas is 0.4 and the cylinder’s radius $R$ is 2.1 m.

(A) What minimum speed $v$ must the cylinder and rider have if the rider is not to fall when the floor drops.

\[
\ddot{N} = m\ddot{a}_C = m \frac{v^2}{R}
\]

\[
\ddot{a}_C = \frac{v^2}{R}
\]

\[
\mu_s m \frac{v^2}{R} = mg
\]

\[
v = \sqrt{\frac{Rg}{\mu_s}} = 7.17 \text{ m/s}
\]

\[
f_S = \mu_s N = mg
\]
Consider 1-D motion.

\[ W = \int_{x_i}^{x_f} F \, dx = \int_{x_i}^{x_f} (ma) \, dx = \int_{x_i}^{x_f} m \left( \frac{dv}{dt} \right) \, dx \]

\[ = \int_{x_i}^{x_f} m \left( \frac{dv}{dx} \frac{dx}{dt} \right) \, dx = \int_{x_i}^{x_f} mv \left( \frac{dv}{dx} \right) \, dx \]

\[ = \int_{v_i}^{v_f} mv \, dv = \frac{1}{2} mv^2 \bigg|_{v_i}^{v_f} = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 \]

So, kinetic energy is mathematically connected to work!!
Work done by Spring Force

- Work done by the spring force:
  \[
  W_s = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} (-kx)dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2
  \]

- If \( |x_f| > |x_i| \) (further away from \( x = 0 \)); \( W < 0 \)
- If \( |x_f| < |x_i| \) (closer to \( x = 0 \)); \( W > 0 \)

- If \( x_i = 0, x_f = x \) then \( W_s = -\frac{1}{2}kx^2 \)

This work went into potential energy, since the speeds are zero before and after.