Chapter 8: Potential Energy and Conservation of Energy

Work and kinetic energy are energies of motion.

$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_{net} = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$

Consider a vertical spring oscillating with mass $m$ attached to one end. At the extreme ends of travel the kinetic energy is zero, but something caused it to accelerate back to the equilibrium point.

We need to introduce an energy that depends on location or position. This energy is called potential energy.
Conservative forces

\[ W_{ab,1} + W_{ba,2} = 0 \]
\[ W_{ab,2} + W_{ba,2} = 0 \]
therefore: \( W_{ab,1} = W_{ab,2} \)

A force is a \textbf{conservative} force if the net work it does on a particle moving around every closed path is zero.

So, ... choose the easiest path!!
Elastic Potential Energy

• Spring force is also a conservative force

\[ F = -kx \]

\[ \Delta U = -W = \int_{x_i}^{x_f} (-kx) \, dx \]

\[ U_f - U_i = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2 \]

Choose the free end of the relaxed spring as the reference point:
that is: \[ U_i = 0 \text{ at } x_i = 0 \]

\[ U = \frac{1}{2} kx^2 \quad \text{Elastic potential energy} \]
Conservation of Mechanical Energy

• Mechanical energy

\[ E_{\text{mec}} = K + U \]

• For an isolated system (no external forces), if there are only conservative forces causing energy transfer within the system….

We know: \[ \Delta K = W \] (work-kinetic energy theorem)
Also: \[ \Delta U = -W \] (definition of potential energy)
Therefore: \[ \Delta K + \Delta U = 0 \]
\[ (K_f - K_i) + (U_f - U_i) = 0 \]
therefore \[ K_1 + U_1 = K_2 + U_2 \]
\[ E_{\text{mec,}1} = E_{\text{mec,}2} \] the mechanical energy is conserved
The figure shows four situations: one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. Which situation will the block have the greatest kinetic energy at B?

5) all have the same kinetic energy at B
Conservation of mechanical energy.

Fig. 8-7
Conservation of mechanical energy

For an isolated system with only conservative forces \(( F = mg, F = -kx )\) act inside

\[ E_{\text{mec,1}} = E_{\text{mec,2}} \]

\[ K_1 + U_1 = K_2 + U_2 \]

\[ \frac{1}{2} mv_1^2 + mgy_1 + \frac{1}{2} kx_1^2 = \frac{1}{2} mv_2^2 + mgy_2 + \frac{1}{2} kx_2^2 \]
A 61.0 kg bungee-cord jumper is on a bridge 45.0 m above a river. The elastic bungee cord has a relaxed length of $L = 25.0$ m. Assume that the cord obeys Hooke’s law, with a spring constant of 160 N/m. If the jumper stops before reaching the water, what is the height $h$ of her feet above the water at her lowest point?

The person gains $\Delta U = mgL$ before the cord starts stretching. It all goes into kinetic energy.

The person still gains $\Delta U = mgd$ from gravity, but some of it is being stored in the cord and being converted into kinetic energy.

Now all the energy is stored in the cord.
Potential Energy Curve

- We know
\[ \Delta U(x) = -W = -F(x) \Delta x \]
Therefore \[ F(x) = -\frac{dU(x)}{dx} \]
Work done by external force

- When no friction acts within the system, the net work done by the external force equals to the change in mechanical energy
  \[ W = \Delta E_{\text{mec}} = \Delta K + \Delta U \]

- Friction is a non-conservative force
- When a kinetic friction force acts within the system, then the thermal energy of the system changes:
  \[ \Delta E_{\text{th}} = f_k d \]
  Therefore \[ W = \Delta E_{\text{mec}} + \Delta E_{\text{th}} \]
Work done by external force

When there are non-conservative forces (like friction) acting on the system, the net work done by them equals to the change in mechanical energy

\[ W_{\text{net}} = \Delta E_{\text{mec}} = \Delta K + \Delta U \]

\[ W_{\text{friction}} = -f_k d \]
Conservation of Energy

• The total energy $E$ of a system can change only by amounts of energy that are transferred to or from the system

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}$$

If there are no internal energy change, but there is friction acts within the system:

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}$$

If there are only conservative forces act within the system:

$$W = \Delta E_{\text{mec}}$$

If we take only the single object as the system

$$W = \Delta K$$
Law of Conservation of Energy

• For an isolated system (\( W = 0 \)), the total energy \( E \) of the system cannot change
\[
\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0
\]

• For an isolated system with only conservative forces, \( \Delta E_{\text{th}} \) and \( \Delta E_{\text{int}} \) are both zero. Therefore:
\[
\Delta E_{\text{mec}} = 0
\]
In the figure, a 2.0kg package slides along a floor with speed \( v_1 = 4.0 \text{m/s} \). It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic friction force from the floor, of magnitude 15 N, act on it. The spring constant is 10,000 N/m. By what distance \( d \) is the spring compressed when the package stops?