Chapter 8: Potential Energy and Conservation of Energy

*Work* and *kinetic energy* are energies of *motion*.

\[ \Delta K = K_f - K_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = W_{\text{net}} = \int_{r_i}^{r_f} \mathbf{F} \cdot d\mathbf{r} \]

Consider a vertical spring oscillating with mass \( m \) attached to one end. At the extreme ends of travel the kinetic energy is zero, but something caused it to accelerate back to the equilibrium point.

We need to introduce an energy that depends on *location* or *position*. This energy is called *potential energy*. 
Chapter 8: Potential Energy and Conservation of Energy

- Work done by gravitation for a ball thrown upward then fell down
  \[ W_{ab} + W_{ba} = -mgd + mgd = 0 \]
  The gravitational force is said to be a conservative force.

- A force is a conservative force if the net work it does on a particle moving around every closed path is zero.
Conservative forces

\[ W_{ab,1} + W_{ba,2} = 0 \]
\[ W_{ab,2} + W_{ba,2} = 0 \]
therefore: \[ W_{ab,1} = W_{ab,2} \]

i.e. The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle

So, ... choose the easiest path!!
Conservative & Non-conservative forces

**Conservative Forces:** (path independent)
- gravitational
- spring
- electrostatic

**Non-conservative forces:** (dependent on path)
- friction
- air resistance
- electrical resistance

These forces will convert mechanical energy into heat and/or deformation.
Gravitation Potential Energy

• **Potential energy** is associated with the configuration of a system in which a conservative force acts: \( \Delta U = -W \)

• For a general conservative force \( F = F(x) \)
  \[
  \Delta U = -W = -\int_{x_i}^{x_f} F(x) \cdot dx
  \]

• **Gravitational potential energy**:
  \[
  \Delta U = -\int_{y_i}^{y_f} (-mg)dy = mg(y_f - y_i)
  \]
  assume \( U_i = 0 \) at \( y_i = 0 \) (reference point)

\[ U(y) = mgy \]  (gravitational potential energy)

only depends on vertical position
Nature only considers changes in Potential energy important. Thus, we will always measure $\Delta U$.

So, . . . We need to set a reference point!

The potential at that point could be defined to be zero (i.e., $U_{\text{ref. point}} = 0$) in which case we drop the “$\Delta$” for convenience.

BUT, it is always understood to be there!
- **Potential energy and reference point**

  A 0.5 kg physics book falls from a table 1.0 m to the ground. What is \( U \) and \( \Delta U \) if we take reference point \( y = 0 \) (assume \( U = 0 \) at \( y = 0 \)) at the ground?

\[
\Delta U = U_f - U_i = -(mg\Delta y)
\]

The book lost potential energy.

Actually, it was converted to kinetic energy.
• **Potential energy and reference point**

A 0.5 kg physics book falls from a table 1.0 m to the ground. What is $U$ and $\Delta U$ if we take reference point $y = 0$ (assume $U = 0$ at $y = 0$) at the table?

\[
\Delta U = U_f - U_i = mg(-h)
\]

The book still the same lost potential energy.
Sample problem 8-1: A block of cheese, \( m = 2 \) kg, slides along frictionless track from a to b, the cheese traveled a total distance of 2 m along the track, and a net vertical distance of 0.8 m. How much is \( W_g \)?

\[
W_{\text{net}} = \int_{a}^{b} \vec{F}_g \cdot d\vec{r}
\]

But, ... We don’t know the angle between \( F \) and \( dr \)
Sample problem 8-1: A block of cheese, \( m = 2 \text{ kg} \), slides along frictionless track from \( a \) to \( b \), the cheese traveled a total distance of 2 m along the track, and a net vertical distance of 0.8 m. How much is \( W_g \)?

Easier (or another way)?

Split the problem into two parts since gravity is conservative

\[
W_{\text{net}} = \int_{a}^{c} \mathbf{F}_g \cdot d\mathbf{x} + \int_{c}^{b} \mathbf{F}_g \cdot d\mathbf{y}
\]

\[
0 = mg(b-c) = mgd
\]
Elastic Potential Energy

• Spring force is also a conservative force
  \[ F = -kx \]

  \[ \Delta U = -W = \int_{x_i}^{x_f} (-kx)dx \]

  \[ U_f - U_i = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2 \]

Choose the free end of the relaxed spring as the reference point:
that is: \( U_i = 0 \) at \( x_i = 0 \)

\[ U = \frac{1}{2} kx^2 \quad \text{Elastic potential energy} \]
Conservation of Mechanical Energy

- Mechanical energy
  \[ E_{\text{mec}} = K + U \]
- For an isolated system (no external forces), if there are only conservative forces causing energy transfer within the system....
  
  We know: \[ \Delta K = W \] (work-kinetic energy theorem)
  
  Also: \[ \Delta U = -W \] (definition of potential energy)
  
  Therefore: \[ \Delta K + \Delta U = 0 \] \[ (K_f - K_i) + (U_f - U_i) = 0 \]
  
  therefore \[ K_1 + U_1 = K_2 + U_2 \]
  
  \[ E_{\text{mec},1} = E_{\text{mec},2} \] the mechanical energy is conserved
Daily Quiz Sept 22, 2004

- The figure shows four situations: one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. Which situation will the block have the greatest kinetic energy at B?

5) all have the same kinetic energy at B
Daily Quiz Sept 22, 2004

5) all have the same kinetic energy at B

Greatest Kinetic Energy

(1) (2) (3) (4)

5) all have the same kinetic energy at B
Daily Quiz Sept 22, 2004

• The figure shows four situations: one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. Which situation will the block have the greatest kinetic energy at B?

5) all have the same kinetic energy at B
The bowling ball pendulum demonstration

Mechanical energy is conserved if there are only conservative forces acting on the system.