Problem 07-50

A 0.25 kg block is dropped on a relaxed spring that has a spring constant of $k = 250.0 \text{ N/m} \ (2.5 \text{ N/cm})$. The block becomes attached to the spring and compresses it 0.12 m before momentarily stopping.

While being compressed,
A) What is the work done on it by gravity?
B) What is the work done on it by the spring force?

C) What is the speed of the block just prior to hitting the spring?
A 0.25 kg block is dropped on a relaxed spring that has a spring constant of $k = 250.0$ N/m (2.5 N/cm). The block becomes attached to the spring and compresses it 0.12 m before momentarily stopping.

While being compressed,
A) What is the work done on it by gravity?
B) What is the work done on it by the spring force?

50. (a) The compression of the spring is $d = 0.12$ m. The work done by the force of gravity (acting on the block) is, by Eq. 7-12,

$$W_1 = mgd = (0.25 \text{ kg}) \left(9.8 \text{ m} / \text{s}^2\right) (0.12 \text{ m}) = 0.29 \text{ J}.$$

(b) The work done by the spring is, by Eq. 7-26,

$$W_2 = -\frac{1}{2} kd^2 = -\frac{1}{2} \left(250 \text{ N} / \text{m}\right) (0.12 \text{ m})^2 = -1.8 \text{ J}.$$
Problem 07-50

A 0.25 kg block is dropped on a relaxed spring that has a spring constant of $k = 250.0 \text{ N/m}$ (2.5 N/cm). The block becomes attached to the spring and compresses it 0.12 m before momentarily stopping.

C) What is the speed of the block just prior to hitting the spring?

(c) The speed $v_i$ of the block just before it hits the spring is found from the work-kinetic energy theorem (Eq. 7-15).

$$\Delta K = 0 - \frac{1}{2}mv_i^2 = W_1 + W_2$$

which yields

$$v_i = \sqrt{\frac{-2(W_1 + W_2)}{m}} = \sqrt{\frac{-2(0.29 - 1.8)}{0.25}} = 3.5 \text{ m/s.}$$
Problem 07-60

Force \( \vec{F} = (3.0\hat{i} + 7.0\hat{j} + 7.0\hat{k}) \) in \( \Delta t = 4.0\text{s} \)

Initial Displacement \( \vec{d}_i = (3.0\hat{i} - 2.0\hat{j} + 5.0\hat{k}) \)

Final Displacement \( \vec{d}_f = (-5.0\hat{i} + 4.0\hat{j} + 7.0\hat{k}) \)

A) Work done on the particle

B) Average power

C) Angle between \( \vec{d}_i \) and \( \vec{d}_f \)
Problem 07-60

Force \( \vec{F} = (3.0\hat{i} + 7.0\hat{j} + 7.0\hat{k}) \) in \( \Delta t = 4.0\) s

Initial Displacement \begin{align*} \vec{d}_i &= (3.0\hat{i} - 2.0\hat{j} + 5.0\hat{k}) \end{align*} Final Displacement \begin{align*} \vec{d}_f &= (-5.0\hat{i} + 4.0\hat{j} + 7.0\hat{k}) \end{align*}

A) Work done on the particle

\( \vec{F} \) is constant in this problem, so

\[
\begin{align*}
W &= \vec{F} \cdot (\vec{d}_f - \vec{d}_i) \\
&= (3.0\hat{i} + 7.0\hat{j} + 7.0\hat{k}) \cdot ((-5.0 - 3.0)\hat{i} + ((4.0 + 2.0)\hat{j} + (7.0 - 5.0)\hat{k}) \\
&= (3.0\hat{i} + 7.0\hat{j} + 7.0\hat{k}) \cdot (-8.0\hat{i} + 6.0\hat{j} + 2.0\hat{k}) \\
&= -24.0 + 42.0 + 14.0 = 32.0 \text{J}
\end{align*}
\]
Problem 07-60

Force \( \vec{F} = (3.0\hat{i} + 7.0\hat{j} + 7.0\hat{k}) \) in \( \Delta t = 4.0s \)

Initial Displacement \( \vec{d}_i = (3.0\hat{i} - 2.0\hat{j} + 5.0\hat{k}) \)

Final Displacement \( \vec{d}_f = (-5.0\hat{i} + 4.0\hat{j} + 7.0\hat{k}) \)

B) Average power

\[
P_{\text{ave}} = \frac{W}{\Delta t} = \frac{32.0J}{4s} = 8.0W
\]
### Problem 07-60

<table>
<thead>
<tr>
<th>Force</th>
<th>( \vec{F} = (3.0\hat{i} + 7.0\hat{j} + 7.0\hat{k}) )</th>
<th>in ( \Delta t = 4.0) s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Displacement</td>
<td>( \vec{d}_i = (3.0\hat{i} - 2.0\hat{j} + 5.0\hat{k}) )</td>
<td>Final Displacement</td>
</tr>
<tr>
<td>(</td>
<td>\vec{d}_i</td>
<td>= \sqrt{38} )</td>
</tr>
</tbody>
</table>

**C) Angle between \( \vec{d}_i \) and \( \vec{d}_f \)**

\( \vec{d}_i \) and \( \vec{d}_f \) are two vectors, so take the dot product

\[
\vec{d}_f \cdot \vec{d}_i = d_f d_i \cos \phi \quad \Rightarrow
\]

\[
\cos \phi = \frac{(-5.0\hat{i} + 4.0\hat{j} + 7.0\hat{k}) \cdot (3.0\hat{i} - 2.0\hat{j} + 5.0\hat{k})}{(|d_f||d_i|)}
\]

\[
(-15.0 - 8.0 + 35.0\hat{k}) / ((9.49)(6.16)) = 12.0 / 58.5 = 0.205
\]

\[
\Rightarrow \quad \phi = \cos^{-1} 0.205 = 78.2^\circ
\]
Problem 07-70

m = 230kg, L = 12.0m
variable horizontal force F
horizontal distance d = 4.0m

A) magnitude of F in final position
B) Total Work done on crate
C) Gravitational Work
D) Work done by rope
E) Work done by force F
Problem 07-70

\[ m = 230 \text{kg}, \quad L = 12.0 \text{m} \]

variable horizontal force \( F \)

horizontal distance \( d = 4.0 \text{m} \)

70. (a) To hold the crate at equilibrium in the final situation, \( \vec{F} \) must have the same magnitude as the horizontal component of the rope’s tension \( T \sin \theta \), where \( \theta \) is the angle between the rope (in the final position) and vertical:

\[
\theta = \sin^{-1} \left( \frac{4.00}{12.0} \right) = 19.5^\circ.
\]

But the vertical component of the tension supports against the weight: \( T \cos \theta = mg \). Thus, the tension is

\[
T = (230)(9.80)/\cos 19.5^\circ = 2391 \text{ N}
\]

and \( F = (2391) \sin 19.5^\circ = 797 \text{ N} \).

An alternative approach based on drawing a vector triangle (of forces) in the final situation provides a quick solution.
Problem 07-70

m = 230kg, L = 12.0m
variable horizontal force F
horizontal distance d = 4.0m

(b) Since there is no change in kinetic energy, the net work on it is zero.

(c) The work done by gravity is \( W_g = \vec{F}_g \cdot \vec{d} = -mgh \), where \( h = L(1 - \cos \theta) \) is the vertical component of the displacement. With \( L = 12.0 \) m, we obtain \( W_g = -1547 \) J which should be rounded to three figures: \(-1.55 \) kJ.

(d) The tension vector is everywhere perpendicular to the direction of motion, so its work is zero (since \( \cos 90^\circ = 0 \)).

(e) The implication of the previous three parts is that the work due to \( \vec{F} \) is \(-W_g \) (so the net work turns out to be zero). Thus, \( W_F = -W_g = 1.55 \) kJ.

(f) Since \( \vec{F} \) does not have constant magnitude, we cannot expect Eq. 7-8 to apply.

\[
F = T \sin \theta = mg \tan \theta
\]
Daily Quiz, September 21, 2004

Which content do you prefer for the Tuesday lectures without the scheduled exams?

1) Lecture over material as we’ve done so far.
2) Concentrate on working problems and examples.
3) No Lecture, just hold informal office hours.
Directions are Important in Space!

Vector (direction important) Quantities:
- displacement
- velocity
- acceleration

Scalar (direction not important) Quantities:
- distance
- speed
- acceleration (same word, but it really is a vector)
Components of Vectors

\[ a_x = a \cos \theta \]
\[ a_y = a \sin \theta \]

- Component notation vs magnitude-angle notation

\[ a = \sqrt{a_x^2 + a_y^2} \]
\[ \tan \theta = \frac{a_y}{a_x} \]
Add vectors by components

\[ \mathbf{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \]
\[ \mathbf{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k} \]
\[ \mathbf{r} = \mathbf{a} + \mathbf{b} = (a_x + b_x) \hat{i} + (a_y + b_y) \hat{j} + (a_z + b_z) \hat{k} \]
Scalar product

- Scalar product of two vectors \( \mathbf{a} \) and \( \mathbf{b} \)

\[
\mathbf{a} \cdot \mathbf{b} = a \cdot b \cos \phi
\]

\( a, b \): magnitude of \( a, b \)

\( \phi \): angle between the directions of \( a \) and \( b \)

\[
\begin{align*}
\mathbf{a} &= a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k} \\
\mathbf{b} &= b_x \mathbf{i} + b_y \mathbf{j} + b_z \mathbf{k} \\
\mathbf{a} \cdot \mathbf{b} &= (a_x b_x) + (a_y b_y) + (a_z b_z)
\end{align*}
\]
Displacement

• Displacement is the change in position (or location)
  \[ \Delta x = x_2 - x_1 \]

• Displacement is a vector with both magnitude and direction
Instantaneous velocity

• Velocity at a given instant
  \[ V = \lim_{\Delta t \to 0} \left( \frac{\Delta x}{\Delta t} \right) = \frac{dx}{dt} \]
  – the slope of \( x(t) \) curve at time \( t \)
  – the derivative of \( x(t) \) with respect to \( t \)
  – VECTOR! -- direction and magnitude

• (Instantaneous) speed is the magnitude of the (instantaneous) velocity
Acceleration

• Acceleration is the change in velocity
• The average acceleration
  \[ \ddot{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} \]
• (Instantaneous) acceleration
  \[ \ddot{a} = \frac{d\vec{v}}{dt} \quad \Rightarrow \quad \ddot{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} \left( \frac{d\vec{x}}{dt} \right) = \frac{d^2 \vec{x}}{dt^2} \]
• Unit: m/s²
• It is correctly a vector!
Projectile Motion

Acceleration is constant and only acts in the \(-y\) direction

- The horizontal motion and the vertical motion are independent of each other

- **Horizontal motion**: (Motion with constant velocity)
  
  \[
  v_x = v_{0x} = v_0 \cos \theta_0 \\
  x - x_0 = (v_0 \cos \theta_0) t
  \]

- **Vertical motion**: (Motion of free-falling object)
  
  \[
  v_y = v_{0y} + a_y t = (v_0 \sin \theta_0) - g t \\
  y - y_0 = v_{0y} t + \frac{1}{2} a t^2 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2
  \]

  assume the upward direction is positive
Chapter 5  Force and Motion

• **Force** $\mathbf{F}$
  – is the interaction between objects
  – is a vector
  – causes acceleration
  – Net force: **vector** sum of all the forces on an object.

$$\vec{F}_{\text{total}} \equiv \vec{F}_{\text{net}} = \sum_{i=1}^{N} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + ... = m\ddot{a}$$
Newton’s Laws of Motion

**Newton’s first law**: If no force acts on a body, then the body’s velocity cannot change, that is, the body can not accelerate

– rest, still rest

– moving, continue moving with same velocity

**Newton’s second law**: The net force on a body is equal to the product of the body’s mass and the acceleration of the body: $\Sigma \vec{F} = m \vec{a}$

**Newton’s third law**: When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction: $\vec{F}_{AB} = - \vec{F}_{BA}$
Newton’s Second Law

- **Newton’s second law:** The net force on a body is equal to the product of the body’s mass and the acceleration of the body

  \[ \Sigma \vec{F} = m \vec{a} \]

  \( \Sigma \vec{F} \): vector sum of all the forces that act on that body

  \[ \Sigma F_x = m a_x \quad \Sigma F_y = m a_y \quad \Sigma F_z = m a_z \]

- **Unit:** \( 1 \text{ N} = (1\text{kg}) \cdot (1\text{m/s}^2) = 1 \text{ kg.m/s}^2 \)
**Frictional Forces**

A block lies on a horizontal floor.

a) What is the magnitude of the friction force \( f \) on it from the floor? \( f = 0 \text{N}, \) but note \( f_{S,\text{max}} = \mu_s F_N \)

b) If a horizontal force of 5 N is now applied to it, but it does not move, what is \( f \) now? \( f_s = 5\text{N} \)

c) If \( f_{s,\text{max}} = 10 \text{N}, \) will the block move if the horizontal applied force is 8 N? no, because \( F < f_s \)

d) How about 12 N? yes, because \( F > f_s \)
Uniform circular motion

- Period of revolution (period)
  \[ T = \frac{2\pi r}{v} \]

- Centripetal acceleration
  magnitude: \( a = \frac{v^2}{r} \)
  direction: radially inward

- \( v \) and \( a \): constant magnitude
  but vary continuously in direction

Centripetal force: \( \vec{F} = m\vec{a} = \frac{mv^2}{r} \ (\vec{-r}) \)
Work-Kinetic Energy Theorem

The change in the kinetic energy of a particle is equal the net work done on the particle

$$\Delta K = K_f - K_i = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = W_{\text{net}} = \int_{r_i}^{r_f} \mathbf{F} \cdot d\mathbf{r}$$

.... or in other words,

final kinetic energy = initial kinetic energy + net work

$$K_f = \frac{1}{2} mv_f^2 = K_i + W_{\text{net}} = \frac{1}{2} mv_i^2 + W_{\text{net}}$$
Work done by a variable force

\[ \Delta W_j = F_{j, \text{avg}} \Delta x \]

\[ W = \Sigma \Delta W_j = \Sigma F_{j, \text{avg}} \Delta x = \lim_{\Delta x \to 0} \Sigma F_{j, \text{avg}} \Delta x \]

\[ W = \int_{x_i}^{x_f} F(x) \, dx \]

- Three dimensional analysis

\[ W = \int_{x_i}^{x_f} F_x \, dx + \int_{y_i}^{y_f} F_y \, dy + \int_{z_i}^{z_f} F_z \, dz \]

\[ = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r} \]
Power

• The time rate at which work is done by a force
• Average power
  \[ P_{\text{avg}} = \frac{W}{\Delta t} \]  (energy per time)
• Instantaneous power
  \[ P = \frac{dW}{dt} = (F \cos \phi \, dx)/dt = F \cdot v \cdot \cos \phi = F \cdot v \]

Unit: watt
1 watt = 1 W = 1 J / s
1 horsepower = 1 hp = 550 ft lb/s = 746 W

• kilowatt-hour is a unit for energy or work:
  1 kW h = 3.6 M J