Chapter 07: Kinetic Energy and Work

Like other fundamental concepts, “energy” is harder to define in words than in equations. It is closely linked to the concept of “force”.

Conservation of Energy is one of Nature’s fundamental laws that is not violated.

Energy can take on different forms in a given system.
Forms of energy

**Kinetic energy**: the energy associated with motion.

**Potential energy**: the energy associated with position or state. *(Chapter 08)*

**Heat**: thermodynamic quantity related to entropy.

**Conservation of Energy** is one of Nature’s fundamental laws that is not violated.

That means the grand total of all forms of energy in a given system is (and *was*, and *will be*) a constant.
Different forms of energy

**Kinetic Energy:**
- Linear motion
- Rotational motion

**Potential Energy:**
- Gravitational
- Spring compression/tension
- Electrostatic/magnetostatic
- Chemical, nuclear, etc....

**Mechanical Energy** is the sum of **Kinetic energy** + **Potential energy.** (reversible process)

**Friction** will convert **mechanical energy** to **heat.** Basically, this (conversion of mechanical energy to heat energy) is a **non-reversible** process.
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• Kinetic Energy is the energy associated with the motion of an object

\[ K = \frac{1}{2} mv^2 \]

m: mass and v: speed

This form of K.E. holds only for speeds \( v \ll c \).

• SI unit of energy:

1 joule = 1 J = 1 kg\cdot m^2/s^2

• Other useful unit of energy is the electron volt (eV)

1 eV = 1.602 \times 10^{-19} J
Work

- Work is energy transferred to or from an object by means of a force acting on the object

- Work done by a constant force:
  \[ W = (F \cos \phi) \, d = F \, d \cos \phi \]

In general,

\[ W = \int \vec{F} \cdot d\vec{s} \]
• Work done by a constant force:

\[ W = F \cdot d \cos \phi = \vec{F} \cdot \vec{d} \]

– when \( \phi < 90^\circ \), \( W \) positive
– when \( \phi > 90^\circ \), \( W \) negative
– when \( \phi = 90^\circ \), \( W = 0 \)
– When \( F \) or \( d \) is zero, \( W = 0 \)

– Work done on the object by the force
– **Positive work**: object receives energy
– **Negative work**: object loses energy
• \( W = F \cdot d \cos \phi = \vec{F} \cdot \vec{d} \)

• SI Unit for Work
  
  \[ 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m/s}^2 \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1 \text{ J} \]

• Net work done by several forces
  
  \[ \Sigma W = \vec{F}_{\text{net}} \cdot \vec{d} = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3) \cdot \vec{d} \]
  
  \[ = \vec{F}_1 \cdot \vec{d} + \vec{F}_2 \cdot \vec{d} + \vec{F}_3 \cdot \vec{d} = W_1 + W_2 + W_3 \]
Work-Kinetic Energy Theorem

The change in the kinetic energy of a particle is equal the net work done on the particle

\[ \Delta K = K_f - K_i = W_{\text{net}} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]

.... or in other words,

final kinetic energy = initial kinetic energy + net work

\[ K_f = \frac{1}{2} m v_f^2 = K_i + W_{\text{net}} = \frac{1}{2} m v_i^2 + W_{\text{net}} \]
A particle moves along the x-axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle’s velocity changes:

(a) from $-3 \text{ m/s}$ to $-2 \text{ m/s}$?

(b) from $-2 \text{ m/s}$ to $2 \text{ m/s}$?

(c) In each situation, is the work done positive, negative or zero?
Work done by Gravitation Force

\[ W_g = mg \cdot d \cdot \cos \phi \]

throw a ball upwards:

- during the rise,
  \[ W_g = mg \cdot d \cdot \cos 180^\circ = -mgd \]
  negative work, \( K \), \( v \) decrease

- during the fall,
  \[ W_g = mgd \cdot \cos 0^\circ = mgd \]
  positive work, \( K \), \( v \) increase
• Work done by an applied force in lifting and lowering an object:

In the special case that the object is stationary both before and after the lift

\[ v_i = v_f = 0 \quad \text{then} \quad K_i = K_f = 0 \]

\[ K_f - K_i = W_{\text{net}} = W_a + W_g \]

so \[ W_a + W_g = 0 \]

\[ \Rightarrow \quad W_a = - W_g = -mg \, d \, \cos \phi \]
Sample problem 7-5: An initially stationary 15.0 kg crate is pulled a distance \(L = 5.70\) m up a frictionless ramp, to a height \(H\) of 2.5 m, where it stops.

(a) How much work \(W_g\) is done on the crate by the gravitational force \(F_g\) during the lift?
How much work $W_t$ is done on the crate by the force $T$ from the cable during the lift?
Work done by a variable force

\[ \Delta W_j = F_j, \text{avg} \Delta x \]

\[ W = \Sigma \Delta W_j = \Sigma F_j, \text{avg} \Delta x = \lim_{\Delta x \to 0} \Sigma F_j, \text{avg} \Delta x \]

\[ W = \int_{x_i}^{x_f} F(x)dx \]

- Three dimensional analysis

\[ W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz \]
Work done by a spring force

- The spring force is given by $\vec{F} = -k\vec{x}$ (Hooke’s law)
  - $k$: spring (or force) constant
  - $k$ relates to stiffness of spring;
  - unit for $k$: N/m.

Spring force is a variable force
- $x = 0$ at the free end of the relaxed spring.
Work done by a spring force

The spring force tries to restore the system to its equilibrium state (position).

Examples:
- springs
- molecules
- pendulum

Motion results in Simple Harmonic Motion
Work done by Spring Force

• Work done by the spring force

\[ W_s = \int_{x_i}^{x_f} Fdx = \int_{x_i}^{x_f} (-kx)dx = \frac{1}{2} kx_i^2 - \frac{1}{2} kx_f^2 \]

• If \( |x_f| > |x_i| \) (further away from \( x = 0 \)); \( W < 0 \)
  If \( |x_f| < |x_i| \) (closer to \( x = 0 \)); \( W > 0 \)

• If \( x_i = 0, x_f = x \)
  then \( W_s = -\frac{1}{2} k x^2 \)
• The work done by an applied force.
• If the block is stationary both before and after the displacement:

\[ v_i = v_f = 0 \quad \text{then} \quad K_i = K_f = 0 \]

work-kinetic energy theorem: \( W_a + W_s = \Delta K = 0 \)

therefore: \( W_a = -W_s \)
Power

• The time rate at which work is done by a force
• Average power
  \[ P_{\text{avg}} = \frac{W}{\Delta t} \]
• Instantaneous power
  \[ P = \frac{dW}{dt} = \frac{(F \cos \phi \, dx)}{dt} = F \, v \cos \phi = F \, v \]
  Unit: watt
  1 watt = 1 W = 1 J / s
  1 horsepower = 1 hp = 550 ft lb/s = 746 W
• kilowatt-hour is a unit for energy or work:
  1 kW h = 3.6 M J