Solution to Problem 4 in Problem Set 2

“In your solutions to the following problems, show your reasoning clearly” means that you need to provide enough narrative to show understanding. A sequence of formulas is often not sufficient, especially if a similar problem has already been worked in class. The following model provides enough explanation to receive full credit. Don’t follow such a model too slavishly, though. There are several acceptable ways to explain this solution.

4. For an inferior planet, derive the relationship between the sidereal and synodic periods.

Solution:

Relative to the stars, the angular speed of the Earth’s revolution is \( \frac{2\pi}{P_\odot} \).
Relative to the stars, the angular speed of the planet’s revolution is \( \frac{2\pi}{P} \).
Relative to the Earth, the angular speed of the planet’s revolution is \( \frac{2\pi}{S} \).

According to the rules of relative motion (for example, imagine that the stars are the river bank, the Earth is the river, and the planet is the boat), the velocities add as follows.

\[
\frac{2\pi}{P} = \frac{2\pi}{P_\odot} + \frac{2\pi}{S}
\]

Therefore,

\[
\frac{1}{P} = \frac{1}{P_\odot} + \frac{1}{S}
\]