Review: nuclear terminology

- \( Z = \) atomic number = number of protons
- \( N = \) number of neutrons
- \( A = \) mass number = \( Z + N \) = number of nucleons
- Radioactive nuclide: spontaneous radioactive decay with an exponential decay curve and “half-life”.
- \( \text{Alpha (}\alpha\text{)}: \) Emission of a helium nucleus (\( \alpha \) particle).
- \( \text{Beta (}\beta\text{)}: \) Emission of an electron (\( \beta^- \)) or positron (\( \beta^+ \)).
- \( \text{Gamma (\( \gamma \))}: \) Emission of a photon (\( \gamma \)-ray).
- Neutrino: Like electron but zero charge and mass.
- Antiparticles: positrons and antineutrinos.

Leptons: \((e^-, \nu)\)\ Antileptons: \((e^+, \bar{\nu})\)

Review: nuclear masses

The mass of the nucleus is also approximately proportional to \( A \), the number of nucleons.

But not exactly, thanks to \( E = mc^2 \).

The strong force pulls the nucleons together. Work would be needed to pull apart the nucleons. This amount of work is called the binding energy \( \Delta E \).

So the nucleus has less energy than \( A \) separate nucleons. So the nucleus has less mass than \( A \) separate nucleons.

\[
M c^2 = Z \, m_p \, c^2 + N \, m_n \, c^2 - \Delta E
\]

Example: Problem 42-7

If the sun became a neutron star without losing mass, what would be its radius?

A neutron star is basically a huge nucleus, containing only neutrons.

Number of neutrons = “mass number” = \( A \)

\[
A = \frac{M_{\text{sun}}}{M_{\text{neutron}}} = \frac{2 \times 10^{30} \, \text{kg}}{1.7 \times 10^{-27} \, \text{kg}} = 1.2 \times 10^{57}
\]

\[
r = (1.2 \, \text{fm}) A^{1/3} = (1.2 \times 10^{-15} \, \text{m})(1.2 \times 10^{57})^{1/3} = (1.2 \times 10^{-18} \, \text{km})(1.06 \times 10^{19}) = 13 \, \text{km} = 8 \, \text{miles}
\]
Q.42-1

Suppose a certain nucleus has a radius about twice as great as the radius of a helium nucleus.
(Remember helium has 2 protons and 2 neutrons.)
Approximately what is the mass number $A$ of this larger nucleus?

(1) 2  (2) 4  (3) 8  (4) 16  (5) 32  (6) 64  (7) 128

---

**Announcement**

- Turning Technologies, who makes our RF student response units (“clickers”) will be in the Student Union, outside the bookstore, trading new improved units for old ones.
- **MAY 1, 2: 11AM – 6PM**
- Bookstore will continue to buy back old units as promised, but if you were planning on keeping yours for future use, then definitely “trade up” for an “improved model.”

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Q.42-2

Without looking up the numbers, what do you know about the mass of the neutron $m_n$ compared to the proton $m_p$ and electron $m_e$?

(1) $m_n < m_p - m_e$  (2) $m_n = m_p - m_e$  (3) $m_n = m_p$
(4) $m_n = m_p + m_e$  (5) $m_n > m_p + m_e$  (6) $m_n = m_e$

---

Beta decay

$$n \rightarrow p + e^- + \bar{\nu}$$

The fundamental beta-decay process changes a neutron into a proton, with the emission of an electron.
But there is also emission of an antineutrino.
Electrons ($e^-$) and neutrinos ($\nu$) are leptons ($L=1$)
Positrons ($e^+$) and antineutrinos ($\bar{\nu}$) are antileptons ($L=-1$)
Neutrinos have zero mass, zero charge, but they have spin $\frac{1}{2}$ like electrons, and are fermions.

**Conservation Laws:**

- **Charge:** ($\Delta Z = 0$) $0 = 1 - 1 + 0$
- **Nucleons:** ($\Delta A = 0$) $1 = 1 + 0 + 0$
- **Energy:** ($\Delta E = 0$) $M_n c^2 = M_p c^2 + M_e c^2 + KE$
- **Leptons:** ($\Delta L = 0$) $0 = 0 + 1 - 1$

---

Q.42-2

Without looking up the numbers, what do you know about the mass of the neutron $m_n$ compared to the proton $m_p$ and electron $m_e$?

(1) $m_n < m_p - m_e$  (2) $m_n = m_p - m_e$  (3) $m_n = m_p$
(4) $m_n = m_p + m_e$  (5) $m_n > m_p + m_e$  (6) $m_n = m_e$

Because of the basic beta decay reaction, it **must** be true that the neutron has more rest energy than the proton plus electron, so that some energy is left over for the kinetic energy of the antineutrino.
Energy in beta decay \[ n \rightarrow p + e^- + \bar{\nu} \]

\[
M_n c^2 = M_p c^2 + M_e c^2 + KE
\]

\[
M_e c^2 = 0.511 \text{ MeV}
\]

\[
M_p c^2 = 938.63 \text{ MeV}
\]

\[
K_{tot} = M_e c^2 - M_p c^2 - M_e c^2 = 0.62 \text{ MeV} = 620 \text{ keV}
\]

So electron and antineutrino share this kinetic energy.

\[ 0 < K_e < 620 \text{ keV} \]

Note this means we must use \[ K_e = (\gamma - 1)m_e c^2 \]

instead of \[ K_e = m_e c^2 \]

Another beta decay example

Or a nucleus can emit a positron (anti-electron) while changing a proton into a neutron. For example

\[ ^{64}\text{Cu}_{29} \rightarrow ^{64}\text{Ni}_{28} + e^+ + \nu \]

Conservation Laws:

Charge: \((\Delta Z=0)\) \[ 29 = 28 + 1 + 0 \]

Nucleons: \((\Delta A=0)\) \[ 64 = 64 + 0 + 0 \]

Energy: \((\Delta E=0)\) \[ M_{\text{Cu}} c^2 = M_{\text{Ni}} c^2 + m_e c^2 + KE \]

Leptons: \((\Delta L=0)\) \[ 0 = 0 - 1 + 1 \]

Example: Positron (\(\beta^+\)) emission

\[ ^{64}\text{Cu}_{29} \rightarrow ^{64}\text{Ni}_{28} + e^+ + \nu + K \]

\[ K_{max} = M(^{64}\text{Cu}) c^2 - M(^{64}\text{Ni}) c^2 - m_e c^2 \]

Gamma rays

\[ \gamma \]

Gamma-ray emission during quantum jump between excited states of an aluminum nucleus.

\[ E_\gamma = hf = \frac{hc}{\lambda} \approx 1.2 \text{ MeV} \]

\[ \lambda = \frac{hc}{E} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.2 \text{ MeV}} = 10^{-3} \text{ nm} = 10^3 \text{ fm} \]

Radioactive decay

Spontaneous process. Cannot predict when any particular nucleus will decay.

Result is just like decay of charge on a capacitor in an RC circuit. The number of nuclei remaining decreases exponentially with time. So the decay rate \( R \) (and intensity of the emitted radiation) does also.

\[ R = R_0 e^{-t/\tau} \]

Meanlife is \( \tau \)

Halflife is \( T_{1/2} = (\ln 2) \tau \)

Semilog plot shows straight line. \( \ln R = \ln R_0 - t / \tau \)
Chapter 43: Nuclear energy

Can we get useful energy from nuclear reactions?

Two possibilities:

• **Fission**: Breakup of heavy nuclei.
• **Fusion**: Merging of light nuclei.

In both cases, the final nuclei are *more stable* (greater binding energy per nucleon) than the initial nuclei, so the total mass is less, and the excess energy is released, typically as heat.

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**Typical fission process**

\[ n + ^{235}\text{U} \rightarrow ^{236}\text{U} \rightarrow ^{140}\text{Xe}_{54} + ^{94}\text{Sr}_{38} + 2n + K \]

*Conservation laws:*

- **Charge**: \( \Delta Z = 0 \), \( 92 = 54 + 38 + 0 \)
- **Nucleons**: \( \Delta A = 0 \), \( 1+235 = 140 + 94 + 1+1 \)
- **Energy**: \( \Delta E = 0 \), \( M_1c^2 + M_2c^2 = M_3c^2 + M_4c^2 + 2m_n c^2 + K \)
- **Leptons**: \( \Delta L = 0 \), \( 0 + 0 = 0 + 0 + 0 \)

The two final neutrons lead to the famous *chain reaction!*

Fig. 43-1: U \( \rightarrow \) X+Y where \( Z_X + Z_Y = 92 \) and \( A_X + A_Y \approx 234 \)

---

**Electric power from fission**

- Well-established technology.
- Heat engine drives turbines which drive generators.
- Fission replaces combustion as heat source.
- With proper safeguards a nuclear power plant can be safer and cleaner than a coal-burning plant.
- Environmental factors:
  - Heat is delivered at a lower temperature, so by the second law of thermodynamics there is more waste heat, more thermal pollution of air, rivers, lakes.
  - Less radioactivity dumped into atmosphere than by a coal-burning plant.
  - Used fuel rods are radioactive and must be disposed of very carefully – unsolved political problem.

---

**Fusion**

\[ 2p + 2n \rightarrow \text{He} \ ( +30 \text{ MeV}) \]

How to make this happen in practice?

Can’t get free protons and neutrons to start this reaction.

- Start with heavy isotopes as in the H bomb.
- Use inverse beta decay as in the sun.

Hard to mimic sun on earth: thermonuclear fusion requires \( T \approx 10 \text{ million K} \) to overcome Coulomb repulsion between protons.

---

**Estimate temperature for thermonuclear fusion**

Need to make two protons collide: \( r = 3 \text{ fm} \).

Coulomb potential energy:

\[
V(r) = \frac{kQ}{r} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{3 \times 10^{-15}} = 4.8 \times 10^5 \text{ V}
\]

\[
U(r) = QV = 480 \text{ keV}
\]

For \( T = 100 \text{ million K} \) (10 x center of sun), what fraction of protons will have this much energy?

\[
kT = (8.6 \times 10^{-5} \text{ eV} / K) \times (10^8 \text{ K}) = 8.6 \times 10^5 \text{ eV} = 8.6 \text{ keV}
\]

\[
e^{-U/kT} = e^{-480/8.6} = e^{-56} = 6 \times 10^{-25} = N_A / 100
\]
When will we have fusion power?

Potentially tremendous source.
Still not in view after 50 years of R&D.