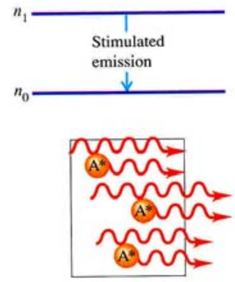
Electrons in Solids

Today:

1. **Review Chapters 40, 41**
   - **Fermions and bosons**
   - **Boltzmann factor**
   - **Energy bands in solids**
   - **Fermi-Dirac distribution function**
2. **Some solid-state applications**

Bosons

Lasers work because photons are bosons.



Atom emits a new photon into *exactly the same quantum state* as the original photon. This keeps happening until there is a strong beam of many photons *all in the same quantum state.*

Produce intense, strongly collimated laser beam.

Probability: Boltzmann Factor

Given any system in thermal equilibrium at temperature T.

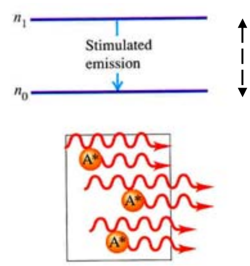
The relative probability of any two states is

$$\frac{P(E_2)}{P(E_1)} = e^{-(E_2 - E_1)/kT} = e^{-\Delta E / kT}$$

So if $\Delta E > kT$ **then** $P(E_2) << P(E_1)$

For example in a gas the number of atoms in an excited state is smaller than the number in the ground state.

Stimulated Emission



$$\Delta E = hf$$

Without pumping, at temperature T:

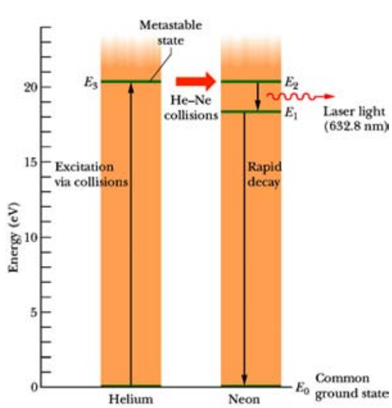
$$\frac{n_1}{n_0} = e^{-\Delta E / kT} < 1$$

Need pumping of a metastable level to produce population inversion ($n_1 > n_0$).

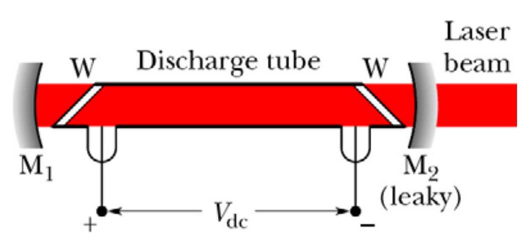
He-Ne Laser

Key to any laser is finding a metastable state which can be pumped (populated) somehow.

Get population inversion (cheat the Boltzmann factor).



Laser Construction



Fermions

- Pauli exclusion principle: No two electrons in the same state.
- That is, one electron for each set of quantum numbers: (n, l, m_l, m_s).
- This gives the *periodic table!*

Q.40-3

5	6	7	8	9	10
B	C	N	O	F	Ne
10.81	12.01	14.01	16.00	19.00	20.18

Carbon ground state: $1s^2\,2s^2\,2p^2$

Which of the following describes the ground state of the *nitrogen* atom (Z=7)?

(1) $1s^2\,2s^2\,3s^3$ (2) $1s^2\,2s^2\,3s^2\,3p$ (3) $1s^2\,2s^3\,2p^2$

(4) $1s^2\,2s^2\,2p^3$ (5) $1s^2\,2s^2\,3s\,3p\,3d$ (6) $1s^2\,2p^5$

Q.40-3

5	6	7	8	9	10
B	C	N	O	F	Ne
10.81	12.01	14.01	16.00	19.00	20.18

Carbon ground state: $1s^2\,2s^2\,2p^2$

Which of the following describes the ground state of the nitrogen atom (Z=7)?

Begin with carbon and place one more electron in the lowest available subshell, which is 2p. Note that p orbitals can hold 6 electrons.

(1) $1s^2\,2s^2\,3s^3$ (2) $1s^2\,2s^2\,3s^2\,3p$ (3) $1s^2\,2s^3\,2p^2$

(4) $1s^2\,2s^2\,2p^3$ (5) $1s^2\,2s^2\,3s\,3p\,3d$ (6) $1s^2\,2p^5$

Ch.40 Sections 5, 6, 10

(Read these sections but don't worry about the details.)

40-5: Stern-Gerlach Experiment

- History
- Showed quantization of angular momentum

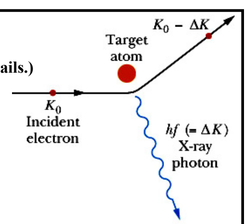
40-6: Magnetic Resonance

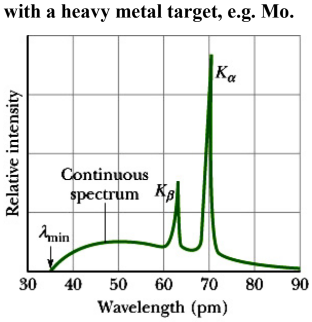
- Proton spins are flipped by oscillating magnetic field.
- Energy of flip is different for different molecules: NMR.
- MRI gives maps of different molecules inside the body.

40-10: X-rays

(Read this section but don't worry about the details.)

Electron beam in an X ray tube collides with a heavy metal target, e.g. Mo.





Resulting x-ray photon energies form "continuous" and "characteristic" spectra.

$$e + A \rightarrow e + A^*$$

$$A^* \rightarrow A + hf$$

X-rays (continued)

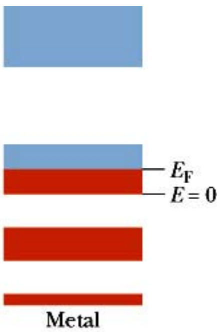
(Read this section but don't worry about the details.)

Characteristic X rays are from collisions in which a *vacancy* is created in an inner shell of a heavy atom.

For example in cobalt (Z=27) if there is a vacancy in the n=1 shell (*K shell*), then an electron from the n=2 shell (L shell) can jump down to fill the vacancy. The energy of the emitted photon in that case is about 7 keV (wavelength about 0.18 nm).

The jump from n=2 to n=1 is called K_α; from n=3 to n=1 is K_β etc. Transitions down to an n=2 vacancy are called L_α L_β etc.

Ch. 41: Electrons in a metal



Atomic core states are full.

Conduction band is half full.

Fermi energy E_F shows highest filled state.

Easy for electron to jump to slightly higher energy state, and then move through solid.

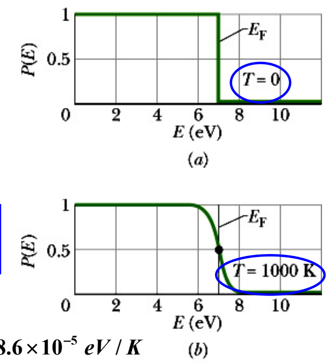
Good conductor.

Fermi-Dirac Distribution Function

At T=0, states are filled just up to Fermi energy.

At T>0, some electrons have gotten a thermal boost.

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$



Boltzmann's constant: $k = 8.6 \times 10^{-5}\ eV/K$

Example: (Based on Problem 41-5)

Consider a state in the conduction band of a metal, with energy 0.05 eV above the Fermi energy.

What is the probability that this state is occupied at room temperature?

$$kT = (8.6 \times 10^{-5}\ eV/K)(300\ K) = .0258\ eV \approx \frac{1}{40}\ eV$$

Solution 1:

$$P(E) \approx e^{-\Delta E/kT} = e^{-\frac{.05}{1/40}} = e^{-2} = \frac{1}{e^2} = \frac{1}{7.39} = .135$$

Solution 2:

$$P(E) = \frac{1}{e^{\Delta E/kT} + 1} = \frac{1}{e^2 + 1} = \frac{1}{8.39} = .119$$

Q.41-1

Consider a metal at temperature T = 2500 K. Find the probability for an electron to occupy a state in the conduction band with energy 0.20 eV above the Fermi energy: $E - E_F = 0.20\ eV$.

1. 3.5 %
2. 7 %
3. 21 %
4. 28 %
5. 35 %

Q.41-1

Consider a metal at temperature T = 2500 K. Find the probability for an electron to occupy a state in the conduction band with energy 0.20 eV above the Fermi energy: $E - E_F = 0.20\ eV$.

Solution: use the Fermi-Dirac distribution function:

1. 3.5 %
2. 7 %
3. 21 %
4. 28 %
5. 35 %

$$kT = (8.6 \times 10^{-5}\ eV/K)(2500\ K) = 0.215\ eV$$

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$= \frac{1}{e^{0.20/0.215} + 1} = \frac{1}{2.535 + 1} = 0.28 = 28\%$$

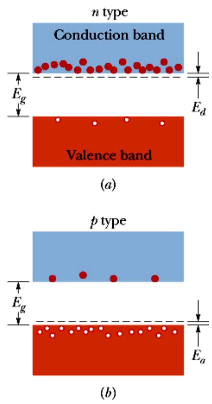
Additional topics

- Doped semiconductors
- p-n junctions
- LEDs
- Photocells

Doped semiconductors

n-type: add atom with one more valence electron. e.g. add P (Z=15) to Si (Z=14) Extra e's lie in "donor" levels.

p-type: add atom with one less valence electron. e.g. add Al (Z=13) to Si (Z=14) Extra empty "acceptor" levels.

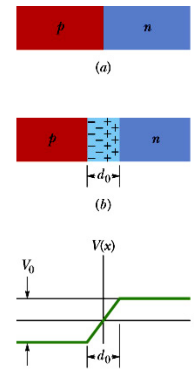


p-n Junction

Junction between p-type and n-type materials.

Diffusion across the boundary forms a depletion layer, and potential difference V_0.

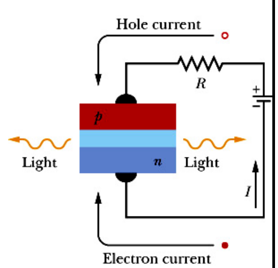
Used to form a diode. Forward voltage allows current to flow LtoR by reducing V_0 and d_0.

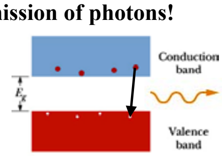


LED Light-emitting diode.

Current passing through p-n junction; electrons and holes meet in the depletion layer; many recombine. This releases an energy approximately equal to the band gap.

Emission of photons!





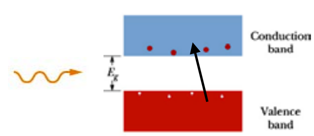
$hf \approx E_g$

Photo-diode

Reverse of LED. Incoming photon creates an electron-hole pair in the depletion layer of a p-n junction.

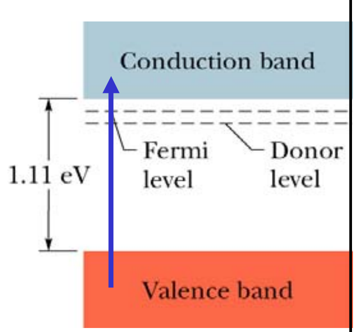
This allows flow of current; light energy is converted into electrical energy.

$hf \approx E_g$

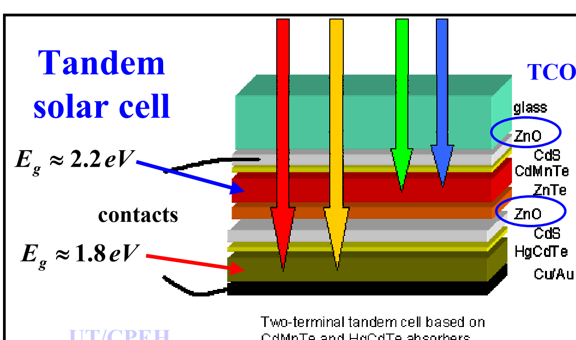


Silicon Photocell

Photon from sun excites electron from valence band to conduction band.



Tandem solar cell



Two thin-film semiconductor junctions.
One band gap tuned to red light, one to blue.

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2. Some solid-state applications

Back to Solids: Chapter 41

Density of states and electrons

A crucial quantity for electronic properties of a solid is the *density of states.* We will not try to learn how to calculate it. Its meaning is straightworward:

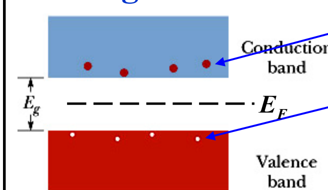
N(E) dE = Number of *quantum states* with energy in the range E to E+dE.

Multiply by P(E) = probability of finding an electron in a state with energy E to get number of *electrons* with energy in range dE.

If e's were not *fermions,* this would be $P(E) = e^{-E/kT}$

But because e's are fermions, they obey the Fermi-Dirac probability distribution function instead.

Charge carriers in semiconductor



Electrons in conduction band

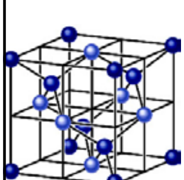
Holes in valence band

For example $E_g = 1\,eV$, $E_F = 0.5\,eV$

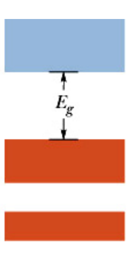
$$P(E_{conduction}) = \frac{1}{e^{(E_g - E_F)/kT} + 1} = \frac{1}{e^{(.5)/.025} + 1} = \frac{1}{e^{20}+1} = e^{-20} = 2\times10^{-9}$$

Multiply by Avagadro's number to get rough estimate of charge carriers per mole: $6\times10^{23}\times2\times10^{-9} = 10^{15}$

Insulators



Diamond lattice



Insulator

- All bands either full or empty.
- Band gap is large.
- Diamond:

$$E_g = 5.5\,eV$$

Excellent insulator

Band gaps

Why is diamond an insulator?
Why is 5.5 eV a "large" band gap?
Electrons in filled bands cannot move because of Pauli exclusion principle.
Must move up to conduction band.
Boltzmann factor doesn't let this happen:

$$E_g >> kT$$

Room temperature kT is about .025 eV.

$$P(E) = e^{-E/kT} = e^{-5.5/.025} = e^{-220} = 3\times10^{-96}$$

Also 5.5eV is larger than energy of photon of visible light.
So diamond is transparent, photons are not absorbed.
Shining light on a diamond doesn't make it a conductor.