Quantum States (Chs. 38, 39)

Today:

1. **Review: photons and electrons**
2. **Trapped electron waves**
3. **Quantized energies**

Photons

- Quantization
  - Quantize energy and momentum of wave.
- Photon energy: $E = hf$
- Photon momentum: $p = h/\lambda$

$E = pc$

Planck's constant: $h = 6.6 \times 10^{-34}\ Js$

Photon has zero mass and always moves with speed c!

Light waves and photons

- **Light is an electromagnetic wave.**

  $f\lambda = c \qquad c = 3.0 \times 10^{8}\ m/s$
- **Energy and momentum carried by photons.**

  $E = hf \qquad p = E/c \qquad h = 6.6 \times 10^{-34}\ Js$
- **Energetic photons have short wavelengths.**

  $E = hf = hc/\lambda \qquad hc = 1240\ eV \cdot nm$

For example a photon with energy **3 eV** has $\lambda = 414\ nm$ **(violet)** while a photon with energy **2 eV** has $\lambda = 622\ nm$ **(red)**.

Example

A beam of orange light has wavelength 600*nm* in air. What is the energy of one photon?

Solution:

We know $E = hf \quad and \quad h = 6.6 \times 10^{-34}\ Js$

Therefore:

$$f = \frac{c}{\lambda} = \frac{3 \times 10^{8} m/s}{600 \times 10^{-9}\ m} = 5.0 \times 10^{14}\ Hz$$

$$E = hf = (6.6 \times 10^{-34}\ J \cdot s) \times (5.0 \times 10^{14}\ s^{-1}) = 3.3 \times 10^{-19}\ J$$

$$= \frac{3.3 \times 10^{-19}\ J}{1.6 \times 10^{-19}\ J/eV} = 2.1\ eV$$

Example: Better solution

A beam of orange light has wavelength 600*nm* in air. What is the energy of one photon?

Forget the SI units and use $hc = 1240\ eV \cdot nm$

$$E = hf = \frac{hc}{\lambda} = \frac{1240\,eV \cdot nm}{600\,nm} = 2.1\ eV$$

Q.38-1

- **If a photon of blue light (400 nm) has energy 3 eV, what's the energy of a UV photon of wavelength 200 nm?**

(1) 0.75 eV (2) 1.5 eV (3) 6 eV (4) 12 eV

Q.38-1

- If a photon of blue light (400 nm) has energy 3 eV, what's the energy of a UV photon of wavelength 200 nm?

$$E = hf = hc/\lambda$$

$$E \propto 1/\lambda$$

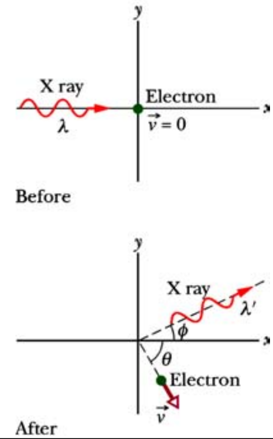
Half the wavelength: twice the energy!

(1) 0.75 eV (2) 1.5 eV (3) 6 eV (4) 12 eV

Compton scattering

In photon collision with electron, use conservation of energy and momentum to solve for change in f,λ for given recoil angle θ.

Calculate exactly like collision of two billiard balls in Ch. 9.



Q.38-2

•A photon collides with a stationary electron.
•After the collision, the electron has a recoil velocity.
•How does the photon's wavelength change?

1. The wavelength increases.
2. The wavelength decreases.
3. The wavelength stays the same.

Q.38-2

•A photon collides with a stationary electron.
•After the collision, the electron has a recoil velocity.
•How does the photon's wavelength change?

$$\lambda = h/p$$

Energy loss ⇒ lower p ⇒ larger λ

1. The wavelength increases.
2. The wavelength decreases.
3. The wavelength stays the same.

Trapped Electron waves

We have seen that an electron wave determines the probability of detecting an electron.

So far we have talked about *running waves*, which represent free electrons, such as the electron beam inside a cathode ray tube.

$$p = h/\lambda$$

But we can also have *standing waves*, which represent *bound* electrons, such as electrons in an atom or in a nanostructure.

Free Electrons

An electron wave determines the probability of detecting an electron.

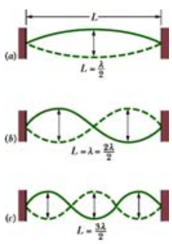
A free electron traveling with momentum *p* has a simple wavefunction and wavelength:

$$\psi = \psi_0 \sin(kx - \omega t) \quad with \quad \lambda = h/p$$

But what about an electron that is bound inside an atom or a solid? It is not moving in a straight line with a constant momentum. How do we determine its wavefunction and its energy?

Bound Electrons

Simplest case: electron trapped between two walls.
Just like vibrating string.
Standing wave instead of running wave.



Ch. 16: modes of vibration of a string. Allowed wavelengths, fundamental frequency and harmonics:

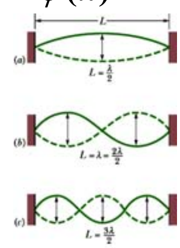
$$L = n\left(\frac{\lambda}{2}\right) \qquad \lambda = \frac{2L}{n}$$

$$f = \frac{v}{\lambda} = n\left(\frac{v}{2L}\right) = nf_0$$

Bound Electrons

The deBroglie wave for a trapped electron is exactly the same, and gives the same allowed wavelengths.

$\psi(x)$



But now the energy calculation is different:

wavelength ⇒ momentum

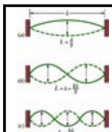
momentum ⇒ energy

$$\lambda = \frac{2L}{n} \qquad p = \frac{h}{\lambda} = \frac{nh}{2L}$$

$$E = \frac{p^2}{2m} = \frac{n^2h^2}{8mL^2} = n^2E_0$$

Quantization!

Quantized energy levels

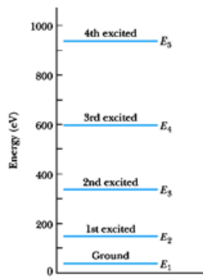


Case L = 100 pm = 0.1 nm = 1 Å = 10^{-10} m.

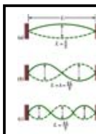
$$E_n = \frac{n^2h^2}{8mL^2} = n^2E_0$$

$$E_0 = \frac{h^2}{8mL^2} = \frac{(hc)^2}{8mc^2L^2}$$

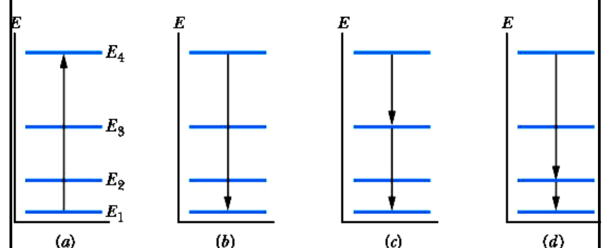
$$E_0 = \frac{(1240\ eV\cdot nm)^2}{8\times .51\times10^6\ eV\times(0.1\ nm)^2} = 37.7\ eV$$



Quantum jumps



System can gain or lose only specific quanta of energy. For example, absorb or emit a photon of energy hf = $E_m - E_n$.

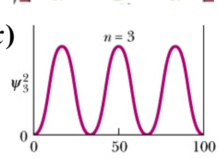


Position Probability Density

The wave function ψ alternates between positive and negative values like any wave. But the probability of finding the electron at a particular point is given by ψ^2. So it is always positive as a probability must be.

$\psi(x)$

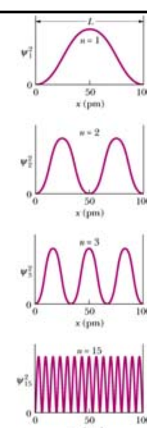
$\psi^2(x)$



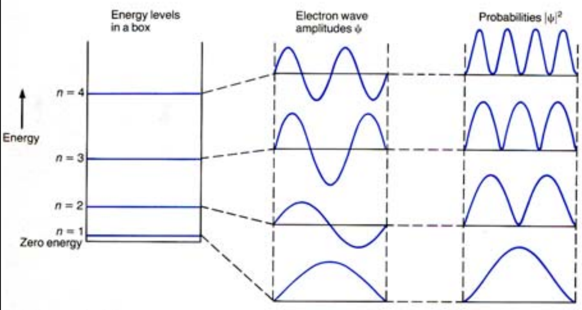
Four states of a trapped electron

The "quantum number" n gives the number of peaks in the probability density.

"Antinodes" for the vibrating string in chapter 16.



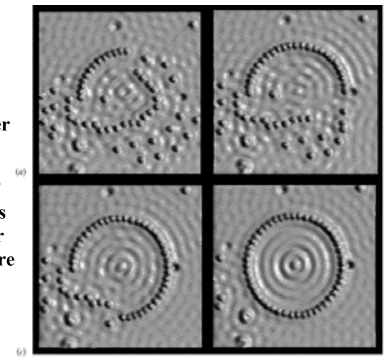
Quantum states of trapped electron



Do such electron traps exist?

Textbook Chapter 39 Section 6 describes several kinds of “nanostructures” which can confine an electron to a nanometer sized region.

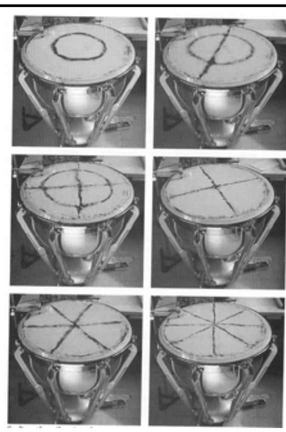
The “quantum corral” made by IBM arranges iron atoms on a copper surface. The ripples are the standing electron waves.



Oscillations in 2 and 3 dimensions

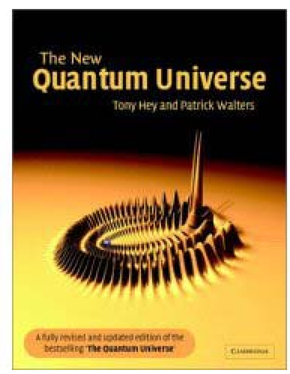
For this vibrating drum, need 2 “quantum numbers”, one for the radial and one for the angular oscillations.

For trap in 3 dimensions, need 3 quantum numbers.



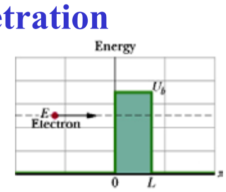
Hey and Walters

Very good paperback, $24.41 at Amazon.com

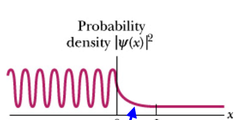


Barrier Penetration

An electron (or any other particle) can “tunnel” through a potential energy barrier that’s too high for it to go over. (Chapter 38 Section 9)



In the “classically forbidden” region, ψ is ***exponentially decreasing*** instead of oscillating.



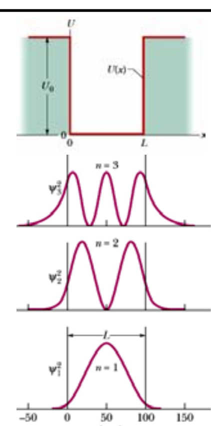
$$\psi(x) = \psi_0 e^{-r/a}$$

Potential Well

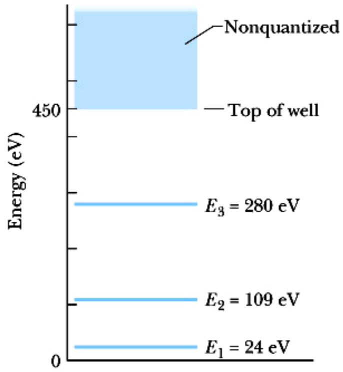
Ground state and first two excited states. Note penetration into classically forbidden region outside the well.

$$L = 100\ pm$$
$$U_0 = 450\ eV$$

But what if E > U_0?



Bound and Unbound States

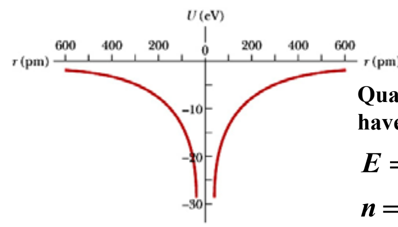


Any E > U_0 is allowed.

Note ground state has kinetic energy. "Zero point energy."

The Hydrogen Atom

General idea: Electron trap in 3 dimensions. Allowed states are labeled with 3 quantum numbers, described by energy levels and wave functions. The trap is provided by the Coulomb potential.



Quantum states have energies

$$E = -E_0 / n^2$$

$$n = 1,2,3,\cdots$$

Energy levels of hydrogen

Arrows show some of the observed quantum jumps. These give rise to photon emission and absorption.