Waves and Particles

Today:

1. **Photon: the elementary particle of light.**
2. **Electron waves**
3. **Wave-particle duality**

Photons

- **Light is Quantized**
  - **Einstein, 1905**
  - **Energy and momentum is carried by photons.**
- **Photon energy:** $E = hf$
- **Photon momentum:** $p = h/\lambda$

Planck's constant: $h = 6.6 \times 10^{-34}\ Js$

Note: Because h is so small in SI units, we don't notice photons in the light we see around us.

Photons

- Photon energy: $E = hf$
- Photon momentum: $p = h/\lambda$

Energy-momentum relation for an ordinary particle (ch.37): $E = \sqrt{(pc)^2 + (mc^2)^2}$

But for a photon we find: $E = hf = hc/\lambda = pc$

Equations agree if m=0.

The photon has zero rest mass and always moves with speed c.

Summary

- **Light is an electromagnetic wave.**

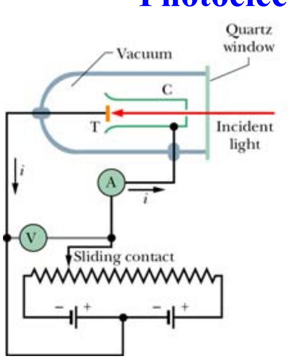
  $f\lambda = c \qquad c = 3.0 \times 10^8\ m/s$
- **Energy and momentum carried by photons.**

  $E = hf \qquad p = E/c \qquad h = 6.6 \times 10^{-34}\ Js$
- **Energetic photons have short wavelengths.**

  $E = hf = hc/\lambda \qquad hc = 1240\ eV \cdot nm$

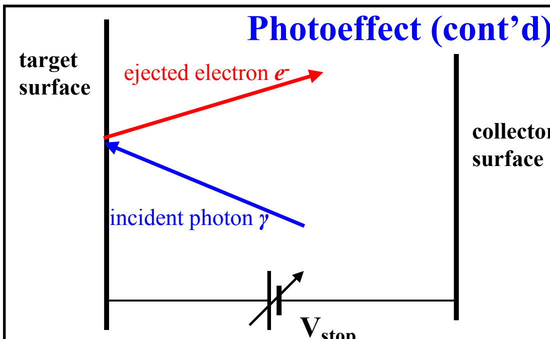
For example a photon with energy **3 eV** has $\lambda = 414\ nm$ **(violet)** while a photon with energy **2 eV** has $\lambda = 622\ nm$ **(red)**.

Photoelectric Effect



- **Light striking a solid target causes electrons to be emitted.**
- **Collect electrons to form a current.**
- **Measure current as function of voltage applied between target and collector.**

Photoeffect (cont'd)



Measure maximum electron energy by applying opposing voltage until current stops.

Photoeffect (cont'd)

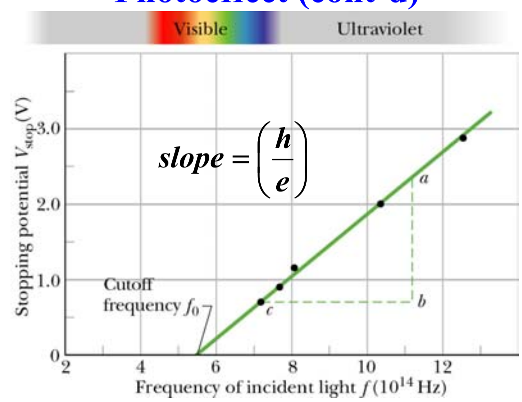
- **Result: The maximum electron kinetic energy *K* is determined by the frequency *f* of the light used, *not by its brightness*.**
- **There is a direct proportionality between energy and frequency. Shows that electron gets its energy from a *photon* of energy *E = hf*.**

Experiment:

$$K_{\max} = hf - \Phi = eV_{stop}$$

$$V_{stop} = \left(\frac{h}{e}\right) f - V_0$$

Photoeffect (cont'd)



Interaction of light with matter

- **Detection of light:** A photon is ***absorbed*** by an electron, atom, or molecule.
  - Photocell
  - Photographic film
  - Digital camera
  - Retina of the eye
- **Production of light:** A photon is ***emitted*** by an electron, atom, or molecule.
- **Scattering of light:** A photon can collide with an electron, exchanging energy and momentum, just like two billiard balls.

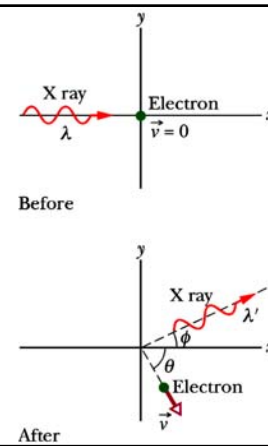
E = hf

$$p = h / \lambda$$

Compton scattering

In photon collision with electron, use conservation of energy and momentum to solve for change in f,λ for given recoil angle θ.

Calculate exactly like collision of two billiard balls in Ch. 9.



Light waves and photons

We detect light by observing the photon.

So what is the *wave* doing?

The electromagnetic wave *intensity* determines the *probability* that we will detect a photon.

Where the wave is strong, the light is bright, the eye will detect many photons. Where the wave is weak, the light is dim, there are very few photons.

The double slit revisited

Suppose we decrease the intensity of the light in a double-slit experiment until we are only detecting one photon per second.

Then each photon hits *somewhere* on our screen, and we can record that hit.

We cannot predict where the next photon will hit.

But if we keep this up for a long time, the predicted interference pattern will *gradually* emerge!

Wave-Particle Duality

So is it weird that light is both a particle and a wave?

Not really. Everything is both a particle and a wave.

A photon is a quantum of a light wave.

An electron is a quantum of an electron wave.

… etc …

Electron waves

We have seen: A light wave determines the probability of detecting a photon.

Now we find: An electron wave determines the probability of detecting an electron!

And the equation relating the wave and the particle is the same:

$$p = h/\lambda$$

For historical reasons this wave is called a deBroglie wave and λ is the deBroglie wavelength.

Particle Energies

Note that $\lambda = h/p$

applies to both photons and electrons (and all other particles also). But the relation between momentum and energy is different, as we already know, depending on the mass of the particle:

Photons: $E = pc$

Slow particles: $K = \dfrac{p^2}{2m}$

Fast particles: $E = \sqrt{(pc)^2 + (mc^2)^2}$

Example

Electron microscope: Problem 38-50

For studying very small structures (molecules, etc.) we need a wave with a very short wavelength.

To get a wavelength the size of an atom we need λ = 0.1 nm, which means pc = hc/λ = (1240 eV-nm)/(0.1 nm) = 12 keV.

For light this means a photon with energy E = pc = 12 keV. But this is an Xray which has so much energy it will likely destroy the structure to be studied.

But for an electron this means only 1% the kinetic energy:

$$K \cong \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{(pc)^2}{2mc^2} = \frac{(12\times10^3\,eV)^2}{2\times0.51\times10^6\,eV} \cong 140\,eV$$

Electron Interference

Electron waves interfere just like any other wave.

A *double-slit experiment with electrons* works just like the Young's double slit with light.

Each electron hits at a random spot, but as more electrons pass through the two-slit pattern emerges. See text figure 38-8 on page 1069.

The electron wave intensity determines the probability of finding an electron.

All other particles show the same probability-wave behavior (protons, neutrons, even atoms).

Double-slit Simulation

Photons strike screen *randomly.*

Next hit is more *probable* where light wave is more intense.

Double-slit interference pattern gradually emerges.

This simulations is exactly the same for electrons, or any other particle!

Schrödinger's Equation

Just as a light wave is governed by Maxwell's Equations, there is a differential equation that governs the propagation of the deBroglie wave.

The function that describes the wave, and satisfies Schrödinger's Equation, is called the "wave function" (for lack of a better name). It's a scalar function, simpler than the electromagnetic case.

$$\vec{E}(x,t) = \vec{E}_0 \sin(kx - \omega t)$$
$$\vec{B}(x,t) = \vec{B}_0 \sin(kx - \omega t)$$

→ $\psi(x,t) = \psi_0 \sin(kx - \omega t)$

Or in general $\psi(x,y,z,t)$

Schrödinger's Equation

Just as a light wave is governed by Maxwell's Equations, there is a differential equation that governs the propagation of the deBroglie wave.

The function that describes the wave, and satisfies Schrödinger's Equation, is called the "wave function" for lack of a better name.

As with any wave, the intensity is the square of the amplitude, and the probability of finding the electron at point x is given by the square of the wavefunction:

Probability = $P(x) = |\psi(x)|^2$

Wave-Particle Duality

Everything is both a particle and a wave.
A photon is a quantum of a light wave.
An electron is a quantum of an electron wave.
… etc …

Wave gives probability of particle showing up.

Universal relation between wavelength of the wave and momentum of the particle:

$$\lambda = \frac{h}{p}$$

Planck's constant:

$$h = 6.6 \times 10^{-34}\ J \cdot s$$
$$= 4.14 \times 10^{-15} eV \cdot s$$

The Uncertainty Principle

If all particles are represented by waves, then we never know exactly where a particle is at any particular time.

Heisenberg raised this fact to the level of a fundamental postulate of quantum mechanics.

There is a relation between the uncertainty in each position coordinate of a particle, and in the corresponding momentum, and also between the energy and the time.

$$\Delta x \Delta p_x \geq \hbar$$
$$\Delta y \Delta p_y \geq \hbar$$
$$\Delta z \Delta p_z \geq \hbar$$
$$\Delta t \Delta E \geq \hbar$$

$$\hbar = \frac{h}{2\pi} = 9.5 \times 10^{-14}\ eV \cdot s$$

Example

When the structure of the nucleus was not understood, one idea was that it contained protons and electrons.

The uncertainty principle shows that to be unlikely. Because of the small size of the nucleus, the uncertainty in the electron's momentum would be very large.

$$\Delta x = 10^{-6}\ nm \qquad \Delta p = \hbar / \Delta x$$

$$cp = \frac{hc}{2\pi \Delta x} = \frac{1240\, eV \cdot nm}{2\pi 10^{-6}\ nm} \cong 200\ MeV$$

$$E = \sqrt{(cp)^2 + (mc^2)^2}$$
$$= \sqrt{(200)^2 + (0.51)^2}\ MeV \cong 200\ MeV$$

More about uncertainty

There is sometimes some misunderstanding, even among physicists, about what this means. It is sometimes said that it is just the fact that it is hard to measure small things. But it is more than that.

Quantum mechanics, which has been repeatedly verified by experiments, asserts that these uncertainties are fundamental.

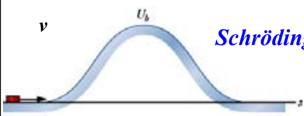
The electron actually does not have a well-determined position. Only the probability of its being at a particular place is determined.

Barrier penetration (tunneling)

One consequence of Schrödinger's Equation is that an electron (or any other particle) can move through a region where it's forbidden to go by energy conservation!

Energy conservation can be violated, but only for a brief time, given by the uncertainty relation. $\Delta t \Delta E \geq \hbar$

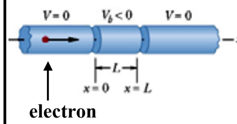
Does puck go over the hill?



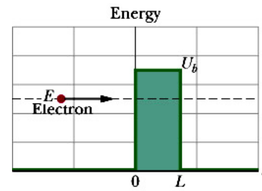
Newton: If ½mv² > U_b, yes. Otherwise, no.

Schrödinger: If ½mv² > U_b, yes. Otherwise, maybe!

Tunneling (barrier penetration)



Very thin wire (nanotube).
Central section given negative potential.
All this on a solid surface (silicon chip).

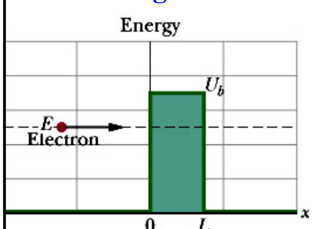


Negative V means positive potential energy U for electron in barrier region.

Suppose E = kinetic energy is less than U = barrier height. What is probability of electron tunneling through?

T = transmission coefficient = ?

Tunneling Result



T = transmission coefficient = ?

Here $b = \sqrt{\frac{2m}{\hbar^2}[U_b - E]}$

So a small change in U_b or L makes a big difference in the current.

For *e* in allowed region (*x*<0 or x>L) we have

$$\psi = \psi_0 \sin(kx)$$

For *e* in forbidden region (0<*x*<L) we have

$$\boxed{\psi = \psi_0 e^{-bx}}$$

This means probability of getting through is

$$T = \frac{\psi(L)^2}{\psi(0)^2} = e^{-2bL}$$

Simplified Schrödinger

Consider simple case, wave function ψ as a function of only *x*:

$$\psi(x) = \psi_0 \sin(kx)$$

Schrödinger's Equation is then a second-order ordinary differential equation, similar to the equations we have studied for simple harmonic motion or oscillations of an LC circuit, except the independent variable is x, rather than t.

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

Just take the second derivative of sin(kx) and see that it's right.

Now we know $k = \frac{2\pi}{\lambda}$ *and* $\lambda = \frac{h}{p}$ *so* $p = \hbar k$

So $\Delta p_x = 0$ *and* $\Delta x = \infty$

"Plane wave"
"Monochromatic"
"Monoenergetic"

Simplified Schrödinger

$$\frac{d^2\psi}{dx^2} = -k^2\psi \qquad \psi(x) = \psi_0 \sin(kx)$$

This works for free particle with fixed momentum $p = \hbar k$

But what if particle is subject to a potential energy function U(x), so that p is not constant?

Kinetic energy is $K = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} = E - U(x)$

So if E>U(x), we have $k^2>0$, and we have a sine wave as before.

But if E<U(x), we have $k^2<0$, and solution is an exponential!

$$\frac{d^2\psi}{dx^2} = b^2\psi \quad \textit{gives} \quad \psi = \psi_0 e^{-bx}$$

$$b^2 = -k^2 = \frac{2m}{\hbar^2}[U(x) - E]$$

Simplified Schrödinger Summary

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

Classically allowed regions, E>U(x):

$$\psi(x) = \psi_0 \sin(kx)$$

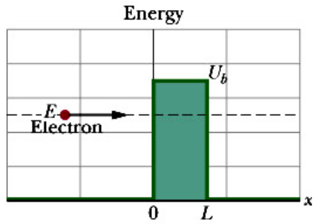
$$p = \hbar k$$

Classically forbidden regions, E<U(x):

$$\psi(x) = \psi_0 e^{-bx}$$

$$b^2 = -k^2 = \frac{2m}{\hbar^2}[U(x) - E]$$

Back to Tunneling



For *e* in forbidden region (0<*x*<L) we have

$$\psi = \psi_0 e^{-bx}$$

This means <u>probability</u> of getting through is

T = transmission coefficient = ?

$$T = \frac{\psi(L)^2}{\psi(0)^2} = e^{-2bL}$$

Here $b = \sqrt{\frac{2m}{\hbar^2}\left[U_b - E\right]}$

So this means a small change in U_b or L makes a big difference in the current.