Relativity

- **Today: Relativistic Mechanics**
  - **Review: Basics of special relativity**
  - **Review: The gamma factor**
  - **Kinetic energy and rest energy**
  - **Examples**

Summary of Special Relativity basics

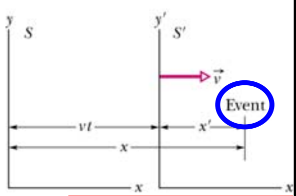
- **The laws of physics are the same for all inertial observers (inertial reference frames).**
- **The speed of light in vacuum is a universal constant, independent of the motion of source and observer.**
- **The space and time intervals between two events are different for different observers.**
- **The equations of Newtonian mechanics (Phys. I) are only "non-relativistic" approximations, valid for speeds small compared to speed of light.**

Lorentz transformation

Einstein found that the old "self-evident" laws for transformations between inertial frames must be replaced by new ones.

Galileo: $x' = x - vt$

$t' = t$



Einstein: use the ***Lorentz transformation:***

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - vx/c^2)$$

where $\boxed{\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}}}$

Everything Follows

- Lorentz transformation equations
- Doppler shift for light
- Addition of velocities
- Length contraction
- Time dilation (twin paradox)
- Correct equations for kinetic energy
- Nothing can move faster than c
- Equivalence of mass and energy $\boxed{E=mc^2}$

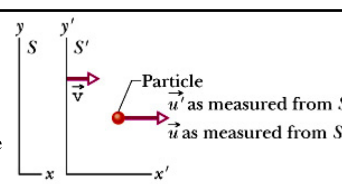
Doppler shift for light

Frequency shift for motion along the line of sight: $$f = f_0\sqrt{\frac{1 \mp v/c}{1 \pm v/c}}$$

Approximation for v << c: $$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

For motion *transverse* to line of sight: $$f = f_0\sqrt{1-(v/c)^2} = f_0/\gamma$$

Adding velocities



If *S'* moves with speed *v* relative to *S*, and a particle moves with speed *u'* relative to *S'*, then what is its speed *u* relative to *S*?

"Obvious" answer: $\cancel{u = u' + v}$

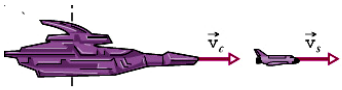
Correct answer:

$$u = (u'+v)/(1+u'v/c^2)$$

This gives desired result that if *u'*=*c*, then *u*=*c* also, independent of the value of *v*!

Example

An enemy spaceship approaches the earth at a speed of 0.5c. It fires a torpedo at us, which has a speed of 0.5c relative to the spaceship. What is the torpedo's speed relative to the earth when it hits us?

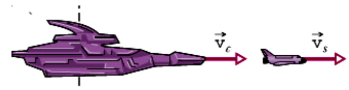


$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{.5c + .5c}{1 + .5 \times .5}$$

$$= \frac{c}{1.25} = \frac{4}{5}c = 0.8c$$

Q.37-1

An enemy spaceship approaches the earth at a speed of 0.5c. It fires an X-ray pulse at us, which has a speed of 1.0c relative to the spaceship. What is the speed of the X-ray pulse relative to the earth when it hits us?



(1) 0 (2) 0.5c (3) 0.8c (4) 1.0c (5) 1.5c

Q.37-1

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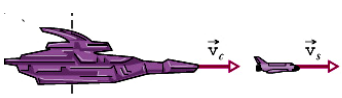


Electromagnetic waves in vacuum always travel at speed *c* independent of motion of source or observer!

(1) 0 (2) 0.5c (3) 0.8c (4) 1.0c (5) 1.5c

Q.37-1 (Alternative solution)

$$u = \frac{v + u'}{1 + u'v/c^2}$$

$$= \frac{.5c + c}{1 + .5 \times 1} = \frac{1.5c}{1.5} = c$$

Time Dilation

A lab observer compares two stationary clocks against a clock moving with speed v, as it passes first one then the other. Lab clocks give Δt, moving clock Δt_0.

$$\Delta t = \gamma \Delta t_0 \geq \Delta t_0$$

Time measured in lab (Δt) is greater than proper time Δt_0 measured by co-moving observer.

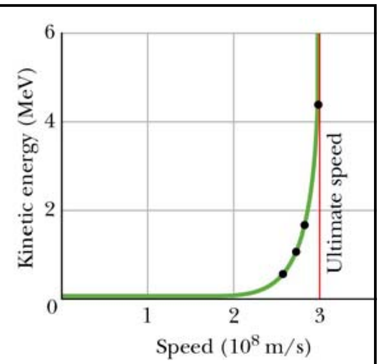
"Moving clocks run slow".

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \geq 1$$

The Gamma Factor

$$\boxed{E = \gamma mc^2}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



$$c = 3 \times 10^8 \ m/s = 300 \ km/ms$$

Uses of gamma

- **Time dilation:** $\Delta t = \gamma\, \Delta t_0$
- **Length contraction:** $\Delta x = \Delta x_0 / \gamma$
- **Energy:** $E = \gamma\, mc^2$

Kinetic Energy of a Fast Particle

General relation for ***total*** energy: $E = \gamma\, mc^2$

Rest energy, v=0: $E = mc^2$

Kinetic energy: $K = E - mc^2 = (\gamma - 1)mc^2$

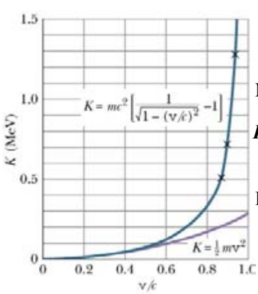
Momentum: $p = \gamma\, mv$

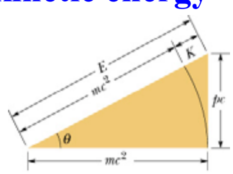
Relation between momentum and energy: $E^2 = (mc^2)^2 + (pc)^2$

Graphical views of kinetic energy

$K = E - mc^2 = (\gamma - 1)mc^2$

$E^2 = (mc^2)^2 + (pc)^2$





Non-relativistic limit (NR):

For $v/c \to 0 \quad K = E - mc^2 \to \frac{1}{2}mv^2$

Extreme-relativistic limit (ER):

For $pc >> mc^2 \quad E \to pc$

Q.37-2

An electron is moving with a velocity v = 0.94c, which means that it has $\gamma = 3$.

What is its kinetic energy?

(Recall that the electron rest energy mc² is about 0.5 MeV.)

(1) 0.17 MeV (2) 0.5 MeV (3) 1.0 MeV (4) 1.5 MeV

Q.37-2

An electron is moving with a velocity v = 0.94c, which means that it has $\gamma = 3$.

What is its kinetic energy?

(Recall that mc² is about 0.5 MeV.)

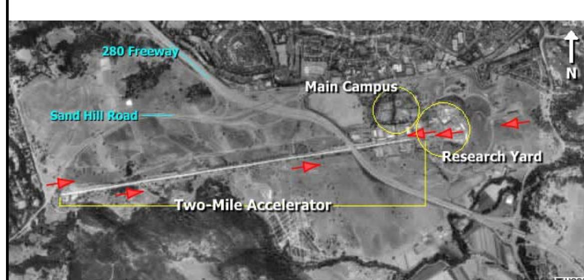
Total energy is $E = \gamma\, mc^2 = 3 \times 0.5 MeV$

Kinetic energy is

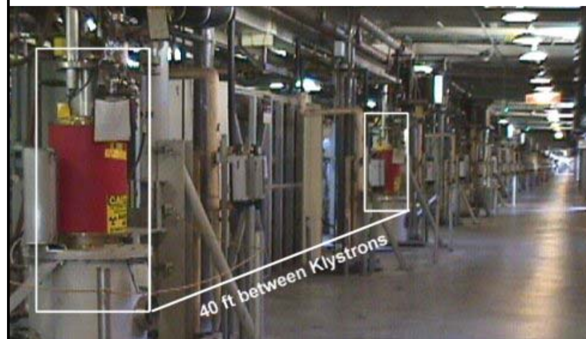
$$K = E - mc^2 = 1.5 MeV - 0.5 MeV = 1.0 MeV$$

(1) 0.17 MeV (2) 0.5 MeV $\boxed{\text{(3) 1.0 MeV}}$ (4) 1.5 MeV

SLAC: Stanford Linear Accelerator



Accelerates electrons for *2 miles*: $E = \gamma\, mc^2 = 20\ GeV$



Example: SLAC electron beam

- **The Stanford Linear Accelerator (SLAC) accelerates a beam of electrons for two miles.**
- **The final kinetic energy is 20 GeV.**
- ***What is the electrons' final velocity?***

Step 1: find the γ factor

We know $E = \gamma mc^2$ so given E,m solve for γ.

$$1\ eV = (1V)\times(e) = 1.6\times10^{-19}\ J$$
$$20\ GeV = 20\times10^{9}\times1.6\times10^{-19} = 3.2\times10^{-9}\ J$$
$$m = 9.1\times10^{-31}\ kg$$

So $\gamma = \dfrac{E}{mc^2} = \dfrac{3.2\times10^{-9}}{9.1\times10^{-31}\times9.0\times10^{16}} = 3.9\times10^{4}$

Step 1 (cont'd)

Actually nobody would do it that way.

Don't use mass in *kg* and energy in *J*.

$$E = 20\ GeV = 2\times10^{4}\ MeV \quad and \quad mc^2 = 0.51\ MeV$$

$$\gamma = \frac{E}{mc^2} = \frac{2\times10^{4}}{0.51} = 3.9\times10^{4}$$

Solution

Now that we have γ we can solve for v.

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \quad \text{so} \quad 1-(v/c)^2 = 1/\gamma^2, \quad v/c = \sqrt{1-1/\gamma^2}$$

$$v/c = \sqrt{1-1/(3.9\times10^{4})^2} = \sqrt{1-6.6\times10^{-10}}$$
$$= \sqrt{0.999999934} = 0.999999967$$

Example: "Newtonian" SLAC

- **SLAC gives an electron an energy of 20 GeV by providing a constant force acting over a distance of about 2 miles, or about 3 km.**
- **What work is done during one meter of flight?**
- **Using *Newtonian* kinetic energy formula, how long would the acceleration tube need to be to bring the electrons to the speed of light?**

Newtonian SLAC Solution

Work = Force × Distance

Acceleration for 3 km: $20\ GeV = 3.2 \times 10^{-9}\ J$

Acceleration for 1 m: $\dfrac{3.2 \times 10^{-9}}{3000} \cong 1 \times 10^{-12}\ J$

Required to reach $v=c$ using $W = K = \frac{1}{2} m v^2$

$$W = \frac{1}{2} m c^2 = 0.5 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \cong 4 \times 10^{-14}\ J$$

Distance needed is: $\dfrac{4 \times 10^{-14}}{1 \times 10^{-12}} \cong 4 \times 10^{-2}\ m = \underline{4\ cm}$

Error in Newtonian Solution

Newtonian answer for the distance needed is:

$$4 \times 10^{-2}\ m = 4\ cm$$

Distance needed in real life is:

$$3\ km$$

Newtonian error factor is:

$$\frac{3\ km}{4\ cm} = \frac{3 \times 10^3\ m}{4 \times 10^{-2}\ m} = .75 \times 10^5 = \underline{75000}$$

Recap

- This example shows that relativistic equations involving the "gamma factor" such as $E = \gamma mc^2$ are **essential** for any description of high-speed motion.
- All these equations follow directly from:
  - ***The principle of relativity.***
  - ***The invariance of the speed of light.***
- But the most powerful and elegant ideas involve the ***four-dimensional spacetime continuum.***

Relativity

M 4/3/06

- **Today: Relativistic Mechanics**
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  - **Examples**
- **Homework for Tomorrow**
  - **Ch. 37 Questions 3, 10**
  - **Ch. 37 Problems 1, 27, 35, 43, 44**
  - **WileyPlus Ch 37 assignment**
- **Wednesday**
  - **The invariant spacetime interval**
- **Exam Thursday Chapters 33-37**