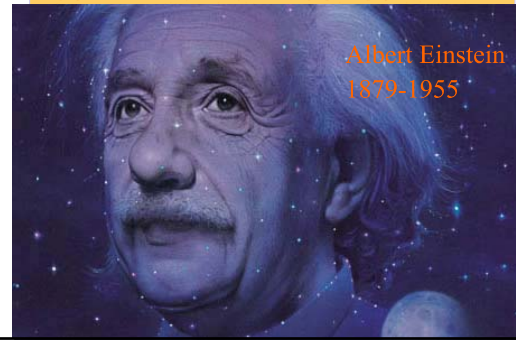
Relativity

- Einstein in 1905
- Special *vs* General Relativity
- Galileo *vs* Maxwell
- Time dilation
- Motion at high speed

TIME 100

PERSON OF THE CENTURY



TIME 100

PERSON OF THE CENTURY

Albert Einstein (1879-1955)

He was the pre-eminent scientist in a century dominated by science. The touchstones of the era--the Bomb, the Big Bang, quantum physics and electronics--all bear his imprint.

1905

1. **First Jazz Band**
2. **Major league debut of Ty Cobb**
3. **Cedar Point hotel: The Breakers**
4. **Wilbur Wright flight of 24 miles**
5. **Einstein's miraculous year.**

Einstein's great year

1. Light energy is discrete: **the photon**.
2. Molecular motion: **atoms** are real.
3. **Relativity**: 4-dimension space-time.
4. Mass-energy equivalence: $E=mc^2$.
5. Dissertation: size of molecules.

He was 26 years old at the time.

Nobel Prize for (1) in 1921.

RELATIVITY

SPECIAL THEORY OF RELATIVITY

High speed motion
Very well-tested
CERTAINLY CORRECT

GENERAL THEORY OF RELATIVITY

Theory of gravity
Large masses and distances
Not very well tested
CORRECT AS FAR AS WE KNOW — BUT COMPETING THEORIES EXIST
(Black Holes ?)

"Everything is relative!?"

Wrong:

Some things which were previously thought to be absolute, we now know to be relative. Some things which were thought to be relative, we now know to be absolute.

But this is not the main thing. The reason we must learn relativity is to get the right equations for high-speed motion.

Galileo 1638, Newton 1687: The laws of physics are the same for all inertial observers.

Maxwell 1865: Electromagnetic equations predict light travels at speed ***c*** in vacuum.

Einstein 1905: Combining these two ***is possible***: the speed of light in vacuum is the same for all observers!

The result is the Special Theory of Relativity.

Inertial Reference Frames

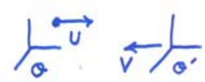
- **An inertial reference frame is a coordinate system (x,y,z,t) which is at rest or moving with constant speed in a straight line.**
- **An inertial observer *O* is a physicist using an inertial reference frame.**
- **Suppose *O* and *O'* are inertial observers, with *O'* moving at velocity *v* relative to *O*.**
- **If *O* and *O'* observe the same events, which measurements to they agree upon?**

"Classical" theory (Newton, Galileo) states that all inertial observers will agree on the following:

- The distance between two events (dx)
- The time between two events (dt)
- A free body moves with constant velocity
- The equations of motion (Newton's Laws)

Inertial observers will disagree about velocities, but the relation is "obvious":

$u' = u + v$



Special Relativity (Einstein, 1905) states that all inertial observers will agree on the following:

- A free body moves with constant velocity
- All the laws of physics
- The velocity of light in vacuum (c)
- The "spacetime interval" between events (ds)

But they will disagree about:

- The distance between two events (dx)
- The time between two events (dt)
- The velocity of an object

Furthermore the simple "obvious" addition of velocities law must be changed to a more complicated form:

$$u' = \frac{u + v}{1 + uv/c^2}$$

Notice if u,v << c we get familiar u' = u + v

Notice if u=c we get u' = (c+v)/(1+v/c) = c (!)

Where did Einstein get this?

- **He saw that there seemed to be a contradiction between his two favorite physicists: Galileo and Maxwell.**
- **Galileo: the laws of physics are the same for all inertial observers.**
- **Maxwell: the speed of light is determined by electrodynamics to be** $c = 3.0 \times 10^8\ m/s.$

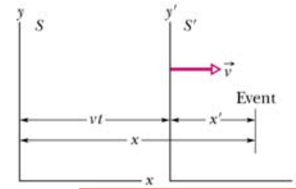
He worried about this for *10 years*: from age 16 to 26 (1895-1905).

How did he solve the problem?

He found that the old "self-evident" laws for transformations between inertial frames must be replaced by new ones.

Galileo: $x' = x - vt$

$t' = t$



Einstein: use the ***Lorentz transformation:***

$x' = \gamma(x - vt)$

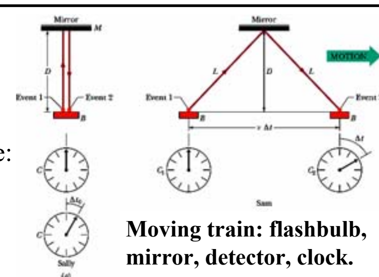
$t' = \gamma(t - vx/c^2)$

where $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}}$

Everything Follows

- Lorentz transformation equations
- Doppler shift for light
- Addition of velocities
- Length contraction
- Time dilation (twin paradox)
- Equivalence of mass and energy (E=mc²)
- Correct equations for kinetic energy
- Nothing can move faster than c

Time Dilation



Comoving Frame:

$\Delta t_0 = 2D/c$

Lab Frame:

$\Delta t = 2L/c$

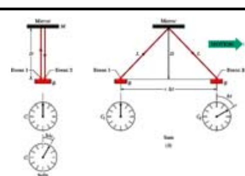
Moving train: flashbulb, mirror, detector, clock. On ground: two clocks.

Pythagoras:

$D^2 = L^2 - (v\Delta t/2)^2$

"Moving clocks run

Time Dilation



$D^2 = L^2 - (v\Delta t/2)^2$

$(c\Delta t_0/2)^2 = (c\Delta t/2)^2 - (v\Delta t/2)^2$

$(\Delta t_0)^2 = (\Delta t)^2 - (v\Delta t/c)^2$

So:

$(\Delta t_0)^2 = (\Delta t)^2\{1-(v/c)^2\}$

$\Delta t = \Delta t_0/\sqrt{1-(v/c)^2} = \gamma\Delta t_0$

$\Delta t \geq \Delta t_0$

Time Dilation

$$\Delta t = \gamma\Delta t_0 \geq \Delta t_0$$

Time measured in lab (Δt) is greater than proper time Δt_0 (measured by co-moving observer).

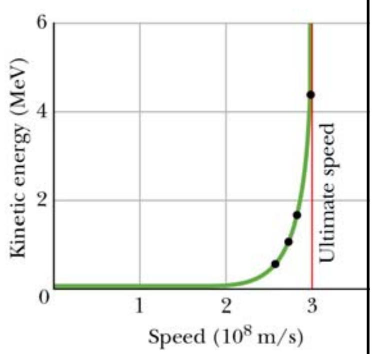
"Moving clocks run slow".

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \geq 1$$

The Gamma Factor

$$E = \gamma \, mc^2$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



$$c = 3 \times 10^8 \; m/s = 300 \; km/ms$$

Uses of gamma

- **Time dilation:** $\Delta t = \gamma \, \Delta t_0$
- **Length contraction:** $\Delta x = \Delta x_0 / \gamma$
- **Energy:** $E = \gamma \, mc^2$

Kinetic Energy of a Fast Particle

General relation for ***total*** energy: $E = \gamma \, mc^2$

Rest energy, v=0: $E = mc^2$

Kinetic energy: $K = E - mc^2 = (\gamma - 1)mc^2$

Momentum: $p = \gamma \, mv$

Relation between momentum and energy: $E^2 = (mc^2)^2 + (pc)^2$

Recap

- Relativistic equations involving the "gamma factor" such as $E = \gamma mc^2$ are **essential** for any description of high-speed motion.
- All these equations follow directly from:
  - ***The principle of relativity.***
  - ***The invariance of the speed of light.***
- But the most powerful and elegant ideas involve the ***four-dimensional spacetime continuum.***

The spacetime continuum

Another way of expressing laws of special relativity which is often simpler than using the Lorentz transformation equations.

- Instead of thinking of space and time separately, think of a ***four-dimensional spacetime.*** The "points" in this spacetime are really ***events.***
- Then the "distance" between events is called the ***spacetime interval.***
- Now relativity follows from the fundamental assumption that the ***spacetime interval*** is ***invariant:*** **the same for all inertial observers.**

Doppler shift for light

Frequency shift for motion along the line of sight: $f = f_0 \sqrt{\dfrac{1 \mp v/c}{1 \pm v/c}}$

Approximation for v << c: $\dfrac{\Delta \lambda}{\lambda} = \dfrac{v}{c}$

For motion ***transverse*** to line of sight: $f = f_0 \sqrt{1 - (v/c)^2} = f_0 / \gamma$