Diffraction Review

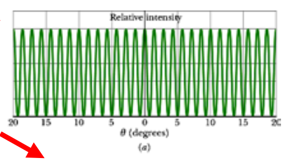
- **Today**
  - Single-slit diffraction review
  - Multiple slit diffraction review
  - Diffraction intensities
  - Diffraction grating and spectroscopy

Summary of single-slit diffraction

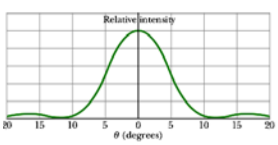
- Given light of wavelength λ passing through a slit of width a.
- There are dark fringes (diffraction minima) at angles θ given by $a \sin \theta = m\lambda$ where m is an integer.
- Note this exactly the condition for *constructive interference* between the rays from the top and bottom of the slit.
- Also note the pattern gets wider as the slit gets narrower.
- The bright fringes are roughly half-way between the dark fringes. (Not exactly but close enough.)

Double-slit diffraction

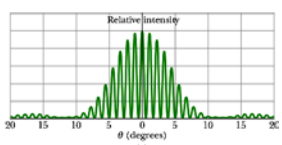
2 slits of zero width



1 slit of width a = 5λ

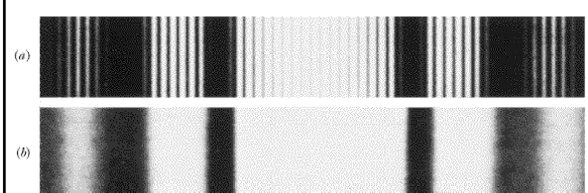


2 slits of width a = 5λ



Two-slit and one-slit patterns

Actual photograph:
(a)= two slits
(b)=one list covered



(Figure 36-15 from text page 1003.)

Scaling of diffraction patterns

Notice a common feature of interference and diffraction patterns: The large-scale features of the pattern are determined by the small-scale regularities of the object, and vice-versa.

Holograms and X-ray diffraction patterns are examples.

Single-slit Intensity

- We know where to find the dark fringes in the single-slit pattern. But can we calculate the actual intensity at a general point?
- Yes, using the phasor method.
- Book gives result on page 998: $\dfrac{I_\theta}{I_m} = \left(\dfrac{\sin\alpha}{\alpha}\right)^2$

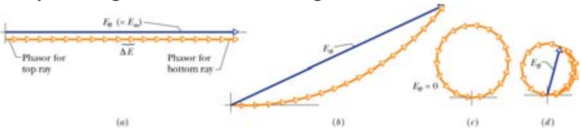
Here I_θ is the intensity at angle θ on the screen.

I_m is the intensity at the central maximum.

The angle $\alpha = \varphi/2$, and φ is the phase difference between the rays from top and bottom of slit.

Phasors for Single Slit

Break up the slit into ***many*** tiny zones, giving ***many*** rays of light, which come together on the screen.



$\Delta\phi =$ Phase difference between ***adjacent*** rays

$\phi =$ Phase difference between top and bottom rays

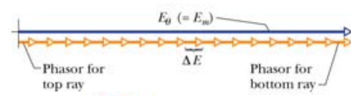
$E_m =$ **Amplitude at center = Sum of all phasors**

$E_\theta =$ **Amplitude at angle θ, get from diagram**

First Maximum and Minimum

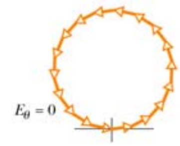
Remember of course the relation between phase difference and path difference

$$\phi = \left(\frac{2\pi}{\lambda}\right)(a\sin\theta)$$



$$\phi = 0$$

$$E_\theta = E_m$$

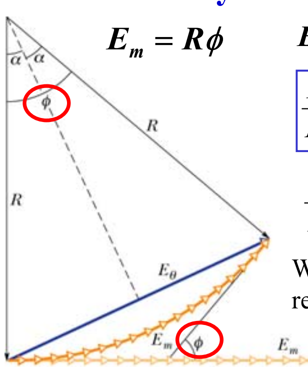


$$E_\theta = 0$$

$$\phi = m(2\pi)$$

$$a\sin\theta = m\lambda$$

Intensity for Single Slit



$$E_m = R\phi \qquad E_\theta = 2R\sin(\phi/2)$$

$$\frac{E_\theta}{E_m} = \frac{2\sin(\phi/2)}{\phi}$$

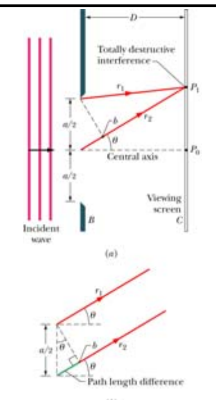
$$\frac{I_\theta}{I_m} = \frac{4\sin^2(\phi/2)}{\phi^2}$$

Which gives the textbook result:

$$\frac{I_\theta}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2$$

Q.36-3

Light of wavelength 600 nm is incident on a slit of width 30 µm. What is the diffraction angle θ for the *first diffraction minimum* (dark fringe)?



(1) .01 rad (2) .02 rad (3) .03 rad (4) .04 rad

Q.36-3

Light of wavelength 600 nm is incident on a slit of width 30 µm. What is the diffraction angle for the *first diffraction minimum* (dark fringe)?

Solution:

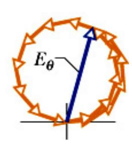
$$a\sin\theta = \lambda$$

$$\sin\theta = \frac{\lambda}{a} = \frac{600\times10^{-9}\ m}{30\times10^{-6}\ m} = 20\times10^{-3}$$

$$\theta \approx \sin\theta = .02\ rad = 1.15°$$

(1) .01 rad (2) .02 rad (3) .03 rad (4) .04 rad

Q.36-4

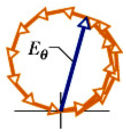


In the same diffraction experiment, where the angle for the first minimum is θ=.02 rad, consider a point on the screen where the phasor diagram for the interfering waves is as shown.

Roughly **what diffraction angle θ is this point found at?**

(1) .01 rad (2) .02 rad (3) .03 rad (4) .04 rad

Q.36-4



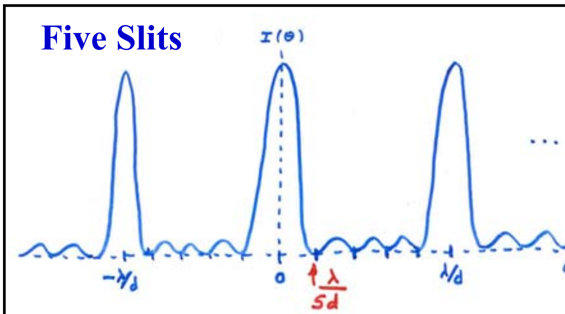
In the same diffraction experiment, where the angle for the first minimum is .02 rad, consider a point on the screen where the phasor diagram for the interfering waves is as shown.

Roughly what is this diffraction angle?

Solution: For θ=.02 rad, we have a closed circle. Thus for θ=.04 rad, we would have the second closed circle. So this point is about midway between: θ=.03 rad.

(1) .01 rad (2) .02 rad **(3) .03 rad** (4) .04 rad

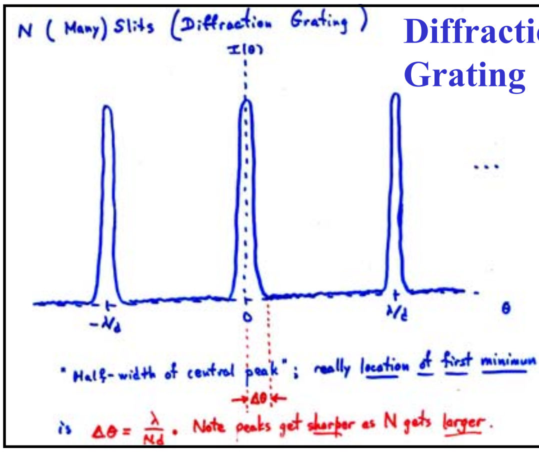
Five Slits



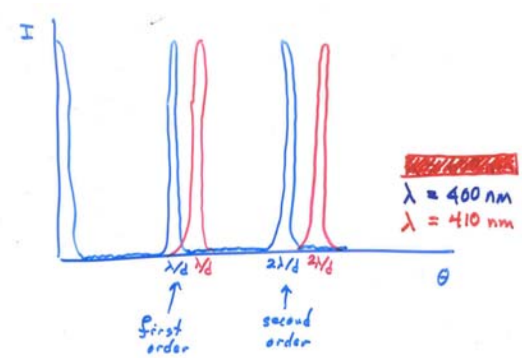
Note we still have $\theta_{max} = m\lambda / d$
But more slits makes the peaks *sharper*. ⊗

For *many* slits, we get a diffraction grating.

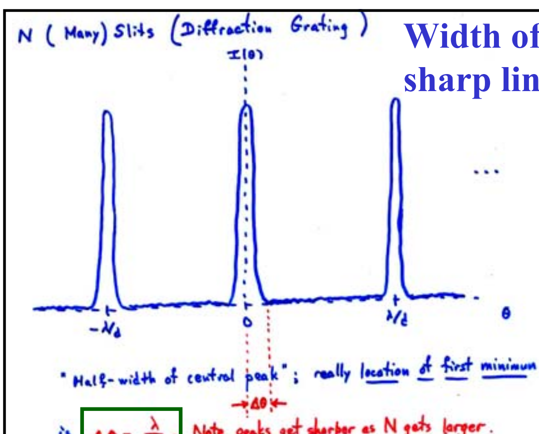
Diffraction Grating



Wavelength Resolution of a Grating



Width of sharp lines



Resolving Power

Positions of maximums: $d\sin\theta = m\lambda$

Small angles: $\theta = m\lambda / d$

$\Delta\theta = m\Delta\lambda / d$

Widths of sharp maximums: $\Delta\theta = \lambda / Nd$

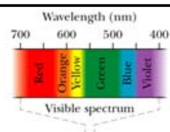
Wavelengths just resolved: $m\Delta\lambda / d = \lambda / Nd$

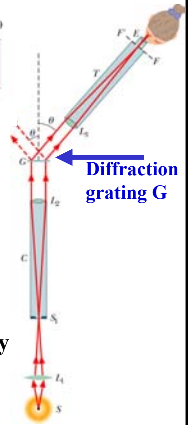
$m\Delta\lambda = \lambda / N$

Resolving power definition:

$R = \dfrac{\lambda}{\Delta\lambda}$ So we get: $R = mN$

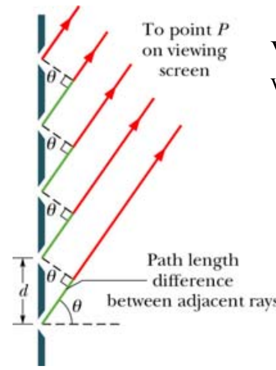
Spectroscopy





- The grating *spectrometer*
- Many grooves gives sharp lines
- High resolution:
  - Measure wavelengths precisely
  - Distinguish between almost-equal wavelengths
  - Many uses in science and technology

Resolving Power



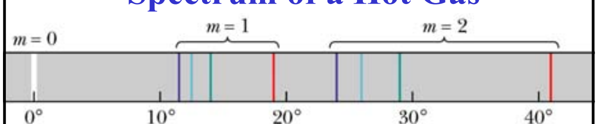
Very sharp maximum when all rays are in phase.

$$d \sin\theta = m\lambda$$

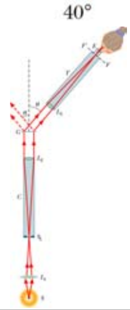
For N rulings, the ***resolving power*** is

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

Spectrum of a Hot Gas



Using a grating spectrometer to look at light from a hot gas we see ***spectrum lines***. Only certain specific wavelengths (colors) are emitted by any particular gas. This characteristic spectrum can be used as an identification tag for specific atoms and molecules. The figure shows four bright lines of the hydrogen atom.



Example: Yellow sodium vapor lines

Problem 36-50

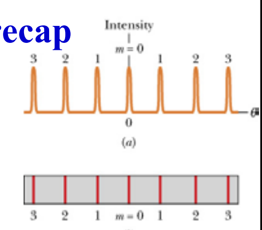
The strong yellow lines in the sodium spectrum are at wavelengths 589.0 nm and 589.6 nm.

How many rulings are needed in a diffraction grating to resolve these lines in second order?

We need $R = \frac{\lambda}{\Delta\lambda} = \frac{589\ nm}{0.6\ nm} = 982$

But $R = mN$ so $N = \frac{R}{m} = \frac{982}{2} = 491$

Diffraction grating recap



Position **of lines is determined by** ***separation*** **of rulings.**

$$d \sin\theta = m\lambda$$

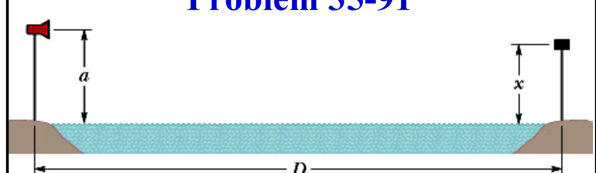
Sharpness **of lines is determined by** ***number*** **of rulings.**

$$\Delta\theta = \lambda / Nd$$

Resolving power **is determined by** ***number*** **of rulings and** ***order*** **of line.**

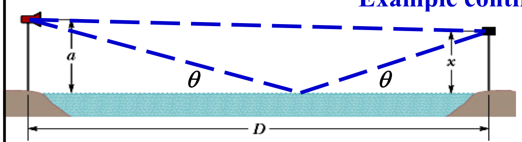
$$R = mN$$

One last example: Problem 35-91



A microwave transmitter sends waves across a lake to a receiver on the other side. D>>a and λ>a. Find x to make the signal a maximum. (Hint: reflection from surface gives phase change.)

Example continued



Direct ray: $L_d = \sqrt{D^2 + (a - x)^2} \approx D + \dfrac{(a - x)^2}{2D}$

Reflected ray: $L_r = \sqrt{D^2 + (a + x)^2} \approx D + \dfrac{(a + x)^2}{2D}$

Path difference: $\Delta L = L_r - L_d = \dfrac{(a+x)^2}{2D} - \dfrac{(a-x)^2}{2D} = \dfrac{2ax}{D}$

After accounting for phase change, condition for maximum is: $\Delta L = \dfrac{2ax}{D} = \dfrac{\lambda}{2} \quad so \quad x = \dfrac{\lambda D}{4a}$