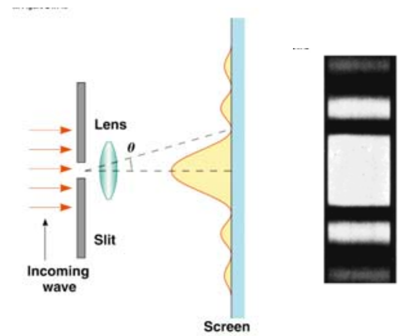
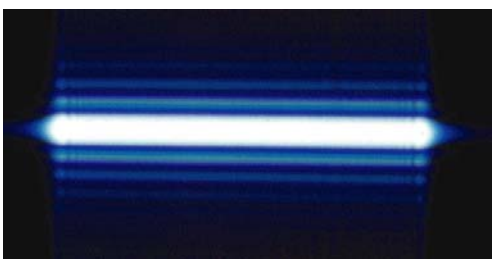
Diffraction

- **Today**
  - Single-slit diffraction
  - Diffraction by a circular aperture
  - Use of phasors in diffraction
  - Double-slit diffraction

Diffraction by a single slit



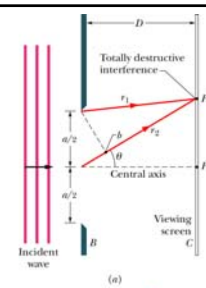
Single slit: Pattern on screen



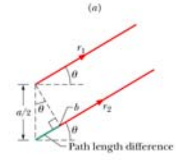
Bright and dark fringes appear behind a single very thin slit.

As the slit is made narrower the pattern of fringes becomes wider.

Single Slit: finding the minima



Divide the single slit into many tiny regions. If the *top ray* and the *middle ray* interfere *destructively*, then every pair of rays will also.



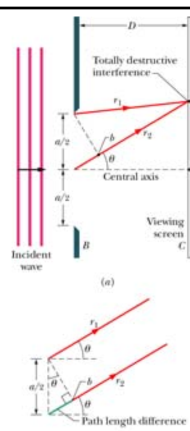
So each pair cancels each other and we have total cancellation!

Single Slit: the first minimum

So we get ***destructive*** interference if:

$$(a/2)\sin\theta = (\lambda/2)$$

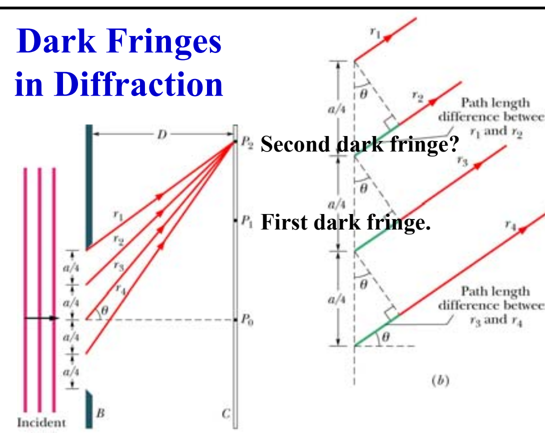
$$\boxed{a\sin\theta = \lambda}$$



Note this mustn't be confused for the condition for *constructive* interference in the two-slit case!

Also note as slit width gets smaller, angle gets larger, and vice versa.

Dark Fringes in Diffraction



Second dark fringe?

First dark fringe.

Dark fringes

First is at $a\sin\theta = \lambda$

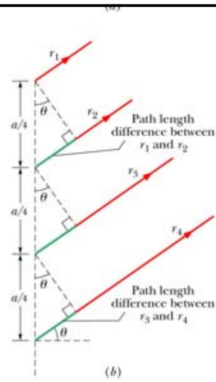
As we move up on the screen the next dark fringe occurs when the first ray interferes destructively with the one from *one-fourth* the way down the slit.

$$(a/4)\sin\theta = \lambda/2$$

$$a\sin\theta = 2\lambda$$

We can continue to *one-sixth, one-eighth etc.* to get all the dark fringes at

$$a\sin\theta = m\lambda$$



Again note as slit gets narrower, pattern gets wider.

Summary of single-slit diffraction

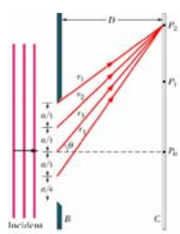
- Given light of wavelength λ passing through a slit of width a.
- There are dark fringes (diffraction minima) at angles θ given by $a \sin\theta = m\lambda$ where m is an integer.
- Note this exactly the condition for *constructive interference* between the rays from the top and bottom of the slit.
- Also note the pattern gets wider as the slit gets narrower.
- The bright fringes are roughly half-way between the dark fringes. (Not exactly but close enough.)

Example: Problem 37-2

Light of wavelength 441 nm is incident on a narrow slit. On a screen 2 meters away, the distance between the *second diffraction minimum* and the central maximum is 1.5 cm.

(a) Calculate the angle of diffraction θ of the second minimum.

(b) Find the width of the slit.

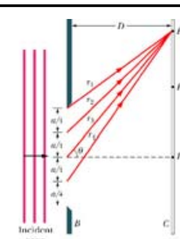


Example: (cont'd)

(a) Calculate the angle of diffraction θ of the second minimum.

$$\theta \cong \tan\theta = \frac{y}{D} = \frac{1.5\times10^{-2}}{2}$$

$$= 7.5\times10^{-3}\ rad$$



$$a\sin\theta = m\lambda$$

(b) Find the width of the slit.

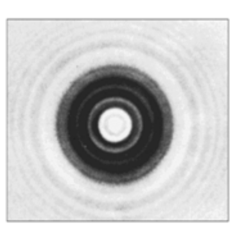
$$\theta \cong \sin\theta = m\frac{\lambda}{a}$$

$$a = m\frac{\lambda}{\theta} = 2\frac{441\times10^{-9}\ m}{7.5\times10^{-3}} = 117.6\ \mu m$$

Diffraction by a circular aperture

Consider a round hole of diameter *d*. Same idea as a long slit – only the geometry is different.

Serway, College Physics, 5/e
Text Figure 25.12



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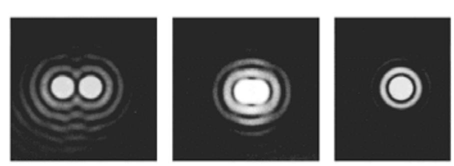
Angle for *first minimum*:

Long slit: $a\sin\theta = \lambda$

Circular hole: $d\sin\theta = 1.22\ \lambda$

The Rayleigh Criterion

Using a circular instrument (telescope, human eye), when can we just resolve two distant objects?



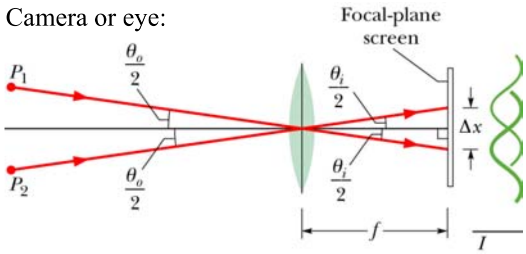
(a) (b) (c)

When images are separated by distance to first minimum:

$$\theta_R = 1.22\ \lambda/d$$

Sample Problem 36-4

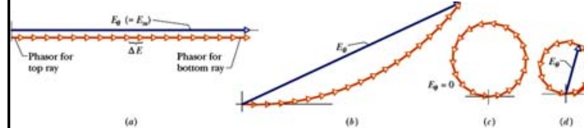
Camera or eye:



Smallest resolvable angle to two distant objects: $\theta_R = 1.22\,\lambda / d$

Single slit phasor diagram

Divide slit into many tiny slits. Use a tiny phasor for each. Add them together graphically.



See we get *destructive* interference if first and last phasors interfere *constructively!*

So the condition for the m^{th} dark fringe is: $a\sin\theta = m\lambda$

Multiple-Slit Diffraction

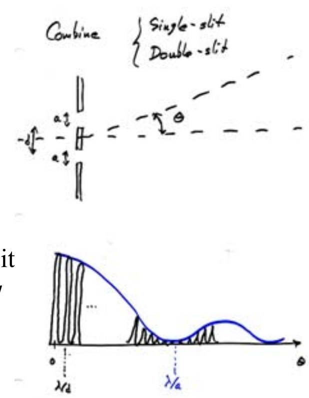
Now we can finally put together our interference and diffraction results to see what really happens with two or more slits.

RESULT: We get the two-slit (or multiple-slit) pattern as in chapter 35, but modified by the single-slit intensity as an ***envelope.***

Instead of all peaks being of the same height, they get weaker at larger angles.

Double-Slit Diffraction

a = slit width
d = slit separation
θ = angle on screen

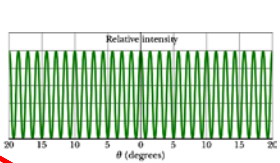


Bright fringes due to 2-slit interference: $\theta = m\lambda / d$

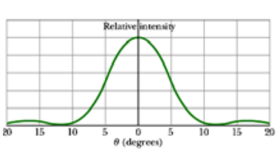
Zero due to diffraction: $\theta = \lambda / a, 2\lambda / a, \cdots$

Double-slit diffraction

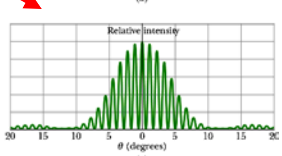
2 slits of zero width



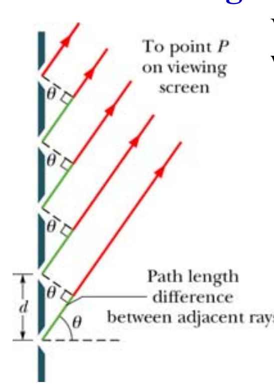
1 slit of width a = 5λ



2 slits of width a = 5λ



Diffraction grating: Many Slits



Very sharp maximum when *all rays* are in phase.

$$d\sin\theta = m\lambda$$

m = "order" = 1, 2, 3, …

First-order maximum:

$$\sin\theta = \lambda / d$$

Second-order maximum:

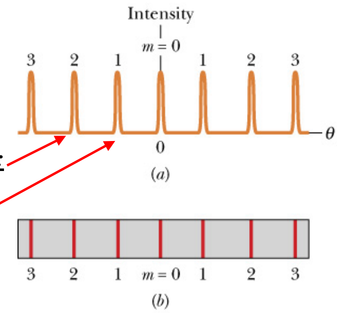
$$\sin\theta = 2\lambda / d$$

Diffraction grating bright lines

$$d \sin\theta = m\lambda$$

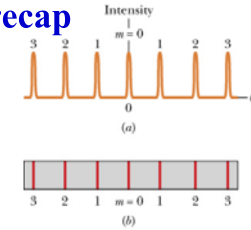
Second-order maximum:

First-order maximum:



Spectroscopy: separate lines of different wavelengths.

Diffraction grating recap



Position **of lines is determined by** ***separation*** **of rulings.**

$$d \sin\theta = m\lambda$$

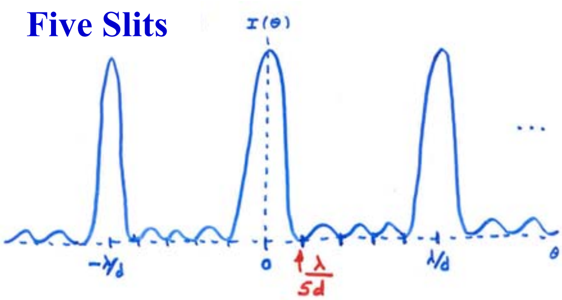
Sharpness **of lines is determined by** ***number*** **of rulings.**

$$\Delta\theta = \lambda / Nd$$

Resolving power **is determined by** ***number*** **of rulings and** ***order*** **of line.**

$$R = mN$$

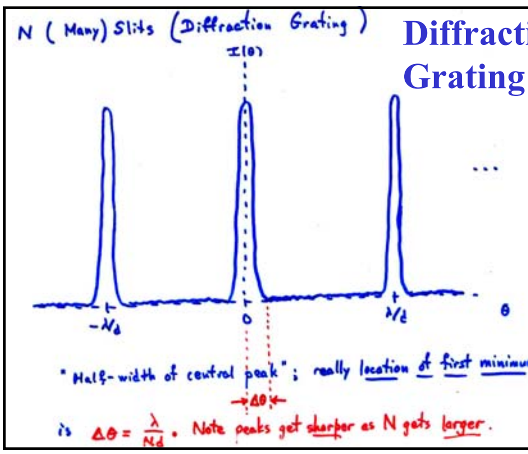
Five Slits



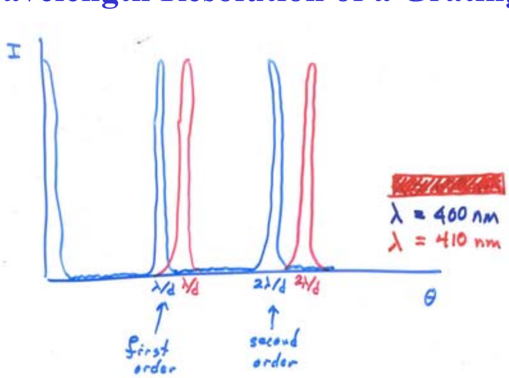
Note we still have $\theta_{max} = m\lambda / d$
But more slits makes the peaks *sharper*.

For *many* slits, we get a diffraction grating.

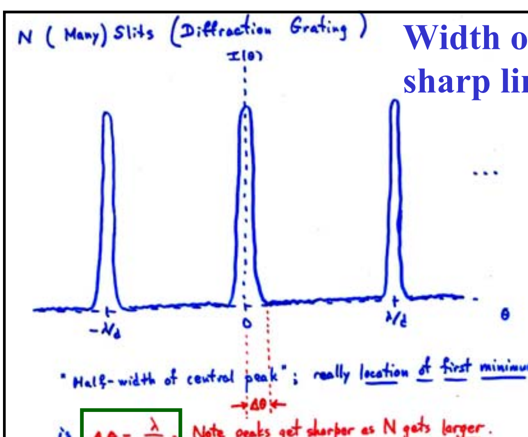
Diffraction Grating



Wavelength Resolution of a Grating



Width of sharp lines



Resolving Power

Positions of maximums: $d \sin\theta = m\lambda$

Small angles: $\theta = m\lambda / d$

$\Delta\theta = m\Delta\lambda / d$

Widths of sharp maximums: $\Delta\theta = \lambda / Nd$

Wavelengths just resolved: $m\Delta\lambda / d = \lambda / Nd$

$m\Delta\lambda = \lambda / N$

Resolving power definition:

$R = \dfrac{\lambda}{\Delta\lambda}$ **So we get:** $R = mN$

Example: Yellow sodium vapor lines

Problem 36-50

The strong yellow lines in the sodium spectrum are at wavelengths 589.0 nm and 589.6 nm.

How many rulings are needed in a diffraction grating to resolve these lines in second order?

We need $R = \dfrac{\lambda}{\Delta\lambda} = \dfrac{589\ nm}{0.6\ nm} = 982$

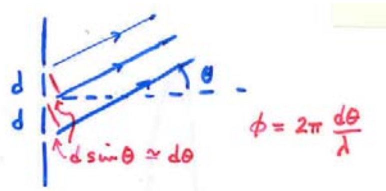
But $R = mN$ **so** $N = \dfrac{R}{m} = \dfrac{982}{2} = 491$

Interference with 3 Slits

Path difference between rays from adjacent slits:

$\Delta L = d \sin\theta$

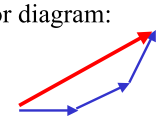
$\theta \approx \Delta L / d$



Phase difference between rays from adjacent slits:

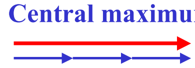
$$\phi = 2\pi \frac{\Delta L}{\lambda} = 2\pi \frac{d\theta}{\lambda}$$

Get intensity from phasor diagram:



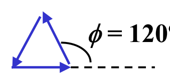
3 Slits Continued

1. **Central maximum:** $\phi = 0 \quad \theta = 0$



$E_T = 3E_0 \quad I_T = 9I_0$

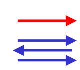
2. **First minimum:**



$\phi = \dfrac{2\pi}{3} \quad d\sin\theta = \dfrac{\lambda}{3}$

$E_T = 0 \quad I_T = 0$

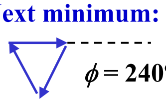
3. **Next maximum:** $\phi = \pi \quad d\sin\theta = \lambda / 2$



$E_T = E_0 \quad I_T = I_0$

3 Slits Continued

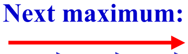
4. **Next minimum:**



$\phi = \dfrac{2}{3} 2\pi \quad d\sin\theta = \dfrac{2}{3}\lambda$

$E_T = 0 \quad I_T = 0$

5. **Next maximum:**

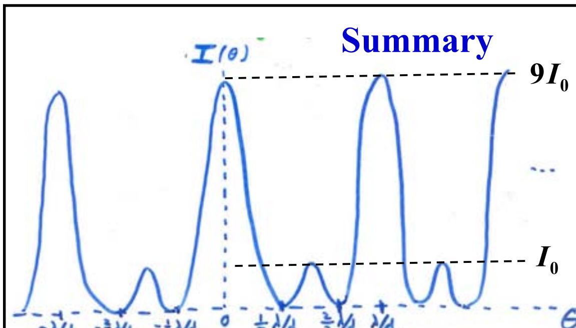


$\phi = 2\pi \quad d\sin\theta = \lambda$

$E_T = 3E_0 \quad I_T = 9I_0$

$\theta \approx \lambda / d$

Summary



$\theta \approx \Delta L / d \qquad \Delta L = \quad 0 \quad \dfrac{\lambda}{3} \quad \dfrac{\lambda}{2} \quad \dfrac{2\lambda}{3} \quad \lambda$

$\theta_{max} \approx m\lambda / d$