INTERFERENCE

- **Today Ch. 35 Interference**
  - **Review of the general idea**
  - **Two slits**
  - **Multiple slits**
  - **Intensities**

Review of Interference

Adding two waves of the same frequency:

$$E_1 = E_1^{\,0}\sin(kx - \omega t)$$

$$E_2 = E_2^{\,0}\sin(kx - \omega t + \phi)$$

$$E_T = E_1 + E_2 = ?$$

Answer: $$E_T = E_T^{\,0}\sin(kx - \omega t + \phi_T)$$

Result is a wave of the same frequency. Usually we want the ***amplitude*** $E_T^{\,0}$ or the ***intensity*** I_T.

Review: Phase & Path Differences

One way to get a *phase difference* Δϕ between two waves is to arrange for a *path difference* ΔL.

The *general relation* between phase difference and path difference is

$$\Delta\phi = k\Delta L = 2\pi\frac{\Delta L}{\lambda}$$

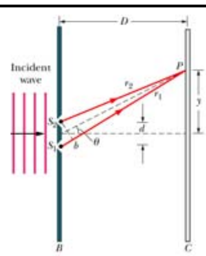
Remember k is phase per unit length: $E = E^0 \sin(kx - \omega t)$

Review

*

- **We have discussed conditions for constructive and destructive *interference* in terms of the *phase difference* Δφ:**
  - **Constructive: Δφ = 0, 360°, 720 °, …**
  - **Destructive: Δφ = 180°, 540°, …**
- **We have looked at 5 different ways to arrange for interference between two light waves:**
  - **Double slit, Reflection from glass surface, Thin films, Michelson interferometer, Different index of refraction.**
- **In most cases, we achieve a phase difference by arranging to have a *path difference* ΔL:**
  - **Constructive: ΔL = λ, 2λ, 3λ, …**
  - **Destructive: ΔL = λ/2, 3λ/2, 5λ/2, …**

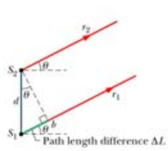
Review: Double Slit



For point P at angle θ triangle shows $\Delta L = d\sin\theta$

For constructive interference we need $\Delta L = m\lambda$

where m=0,1,2,… is any integer.



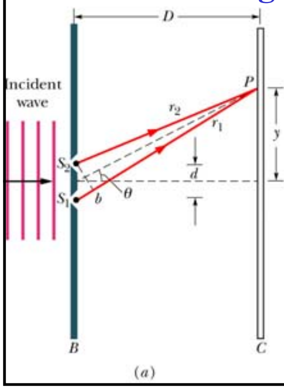
So the bright fringes are at angles given by $$d\sin\theta = m\lambda$$

Bright and Dark Fringes

So the *bright fringes* are at angles given by $$d\sin\theta = m\lambda$$

And the *dark fringes* are at angles given by $$d\sin\theta = (m + \tfrac{1}{2})\lambda$$

Locating the Fringes



For a bright spot we need $d\sin\theta = m\lambda$.

From the figure we see $\tan\theta = y/D$.

But for small angles we have $\sin\theta \approx \tan\theta$.

So the bright lines are at

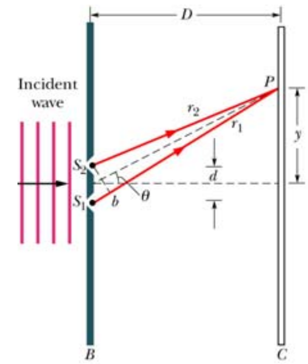
$$m\lambda = dy/D$$

$$y = m\lambda D/d$$

Q.36-1

$\lambda = 600\ nm$

$r_1 - r_2 = 900\ nm$



What is the phase difference between the two waves at P?

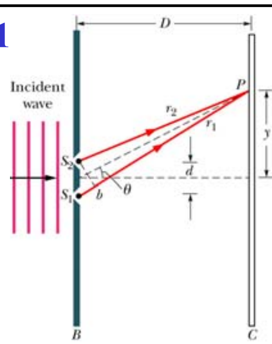
(1) $\pi/2$ (2) π (3) $3\pi/2$ (4) 2π (5) 3π

Q.36-1

$\lambda = 600\ nm$

$r_1 - r_2 = 900\ nm$

What is the phase difference between the two waves at P?



$$\Delta L = 1.5\lambda$$

$$\therefore \Delta\phi = 1.5(2\pi) = 3\pi$$

(1) $\pi/2$ (2) π (3) $3\pi/2$ (4) 2π **(5) 3π**

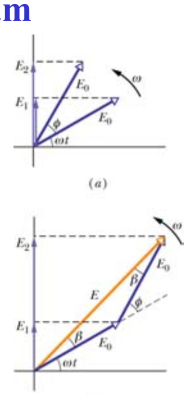
Interference: The General Case

What if the phase difference neither 0 nor 180° but something in between? How can we calculate the resultant amplitude?

Use phasors!

Phasor Diagram

Just as for AC circuits, we can add two oscillating functions using phasors. The ***lengths*** of the phasors are the ***amplitudes*** of the waves and the ***angle*** between the phasors is the ***phase difference*** between the waves. Then the ***length*** of the resultant phasor is the ***amplitude*** of the total wave.



Adding Vectors

A good way to get the length of the sum of two vectors is to use the ***dot product:***

If $\vec{C} = \vec{A} + \vec{B}$

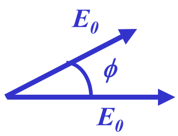
Then $C^2 = \vec{C}\cdot\vec{C} = \left(\vec{A}+\vec{B}\right)\cdot\left(\vec{A}+\vec{B}\right)$

$$= A^2 + B^2 + 2\vec{A}\cdot\vec{B}$$

$$= A^2 + B^2 + 2AB\cos\theta$$

Intensity Formula

Suppose two light waves have ***equal intensities*** I_0 and a ***phase difference*** of ϕ. When these waves interfere, what will be the total intensity I?



$$E^2 = E_1^{\ 2} + E_2^{\ 2} + 2\vec{E}_1 \cdot \vec{E}_2$$

$$E^2 = 2E_0^{\ 2} + 2E_0^{\ 2}\cos\phi$$

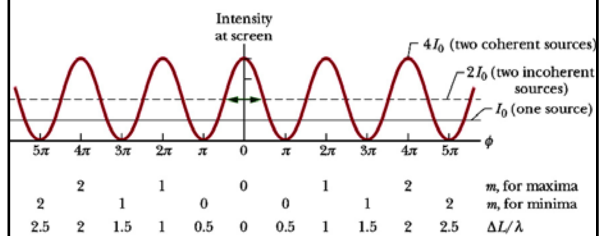
$$= 4E_0^{\ 2}\cos^2\left(\frac{1}{2}\phi\right)$$

But $I \propto E^2$ ***so:***

$$I = 4I_0\cos^2\left(\frac{1}{2}\phi\right)$$

Text Eq. 36-21

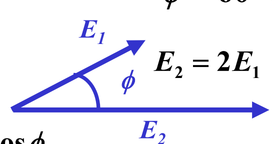
Double-slit intensity



Intensity Example

Problem 35-29.

Two waves interfere with phase difference $\phi = 60°$. One wave has intensity I_0, the other $4I_0$. What is the resulting intensity?



$$E^2 = E_1^{\ 2} + E_2^{\ 2} + 2E_1E_2\cos\phi$$

$$E^2 = E_0^{\ 2} + 4E_0^{\ 2} + 2E_0(2E_0)(1/2)$$

$$= 5E_0^{\ 2} + 2E_0^{\ 2}$$

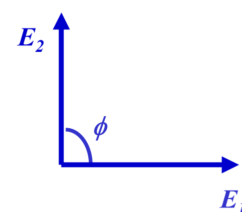
But $I = (Const)E^2$

so $I = 5I_0 + 2I_0 = 7I_0$

Q.36-2

Two waves interfere with phase difference $\phi = 90°$

Each wave individually has intensity I_0. What is the resulting intensity?

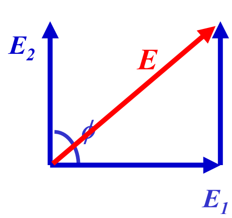


(1) $2I_0$ (2) $\sqrt{2}I_0$ (3) I_0 (4) $I_0/\sqrt{2}$ (5) $I_0/2$

Q.36-2

Two waves interfere with phase difference $\phi = 90°$

Each wave individually has intensity I_0. What is the resulting intensity?



$$E^2 = E_1^{\ 2} + E_2^{\ 2}$$

But $I \propto E^2$

$$\therefore \quad I = I_0 + I_0 = 2I_0$$

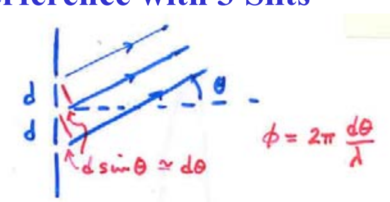
(1) $2I_0$ (2) $\sqrt{2}I_0$ (3) I_0 (4) $I_0/\sqrt{2}$ (5) $I_0/2$

Interference with 3 Slits

Path difference between rays from adjacent slits:

$\Delta L = d\sin\theta$

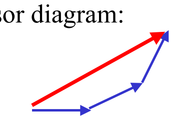
$\theta \approx \Delta L / d$



Phase difference between rays from adjacent slits:

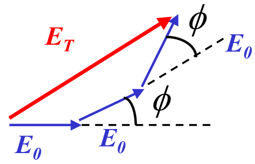
$$\phi = 2\pi\frac{\Delta L}{\lambda} = 2\pi\frac{d\theta}{\lambda}$$

Get intensity from phasor diagram:



Intensity for 3 Slits

What is intensity as a function of angle?



$I = E_T^{\,2}$

$I_0 = E_0^{\,2}$

$\phi = 2\pi \dfrac{\Delta L}{\lambda} = 2\pi \dfrac{d \sin\theta}{\lambda}$

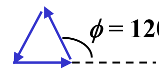
We could do a lot of trigonometry and figure out the general result for I as a function of I_0 and ϕ but let's not. Just get location of fringes using the phasor diagram, as θ and ϕ increase.

3 Slits Continued

1. Central maximum: $\phi = 0 \qquad \theta = 0$

$E_T = 3E_0 \qquad I_T = 9I_0$

2. First minimum:

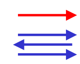


$\phi = 120°$

$\phi = \dfrac{2\pi}{3} \qquad d\sin\theta = \dfrac{\lambda}{3}$

$E_T = 0 \qquad I_T = 0$

3. Next maximum: $\phi = \pi \qquad d\sin\theta = \lambda / 2$

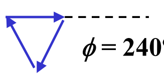


$\phi = 180°$

$E_T = E_0 \qquad I_T = I_0$

3 Slits Continued

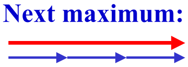
4. Next minimum: $\phi = \dfrac{2}{3} 2\pi \qquad d\sin\theta = \dfrac{2}{3}\lambda$



$\phi = 240°$

$E_T = 0 \qquad I_T = 0$

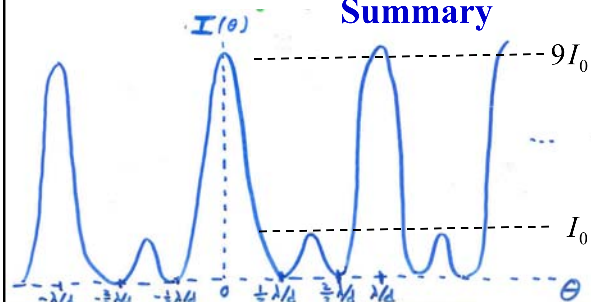
5. Next maximum: $\phi = 2\pi \qquad d\sin\theta = \lambda$



$E_T = 3E_0 \qquad I_T = 9I_0$

$\phi = 360°$

Summary

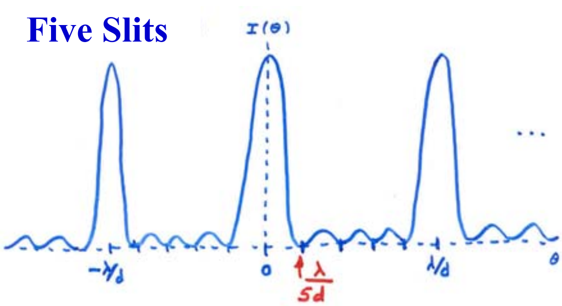


$\theta = \Delta L / d$

$\phi =$	0	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	2π	
$\Delta L =$	0	$\frac{\lambda}{3}$	$\frac{\lambda}{2}$	$\frac{2\lambda}{3}$	λ	

$\theta_{\max} = m\lambda / d$

Five Slits



Note we still have $\theta_{\max} = m\lambda / d$
But more slits makes the peaks *sharper*.

⊗

For *many* slits, we get a diffraction grating.