**INTERFERENCE**

- Today Ch. 35 Interference
  - The general idea
  - Examples
    - Two slits
    - Phase change on reflection
    - Thin films
    - Interferometers
  - Intensities

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**Review of Waves (Ch. 16)**

\[
y = y_0 \sin(kx - \omega t)
\]

Wavelength = \( \lambda \)
Frequency = \( f \)
Velocity = \( v = f \lambda \)

\[
k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \quad v = \frac{\omega}{k}
\]

Amplitude = \( y_0 \)
Intensity \( \propto y_0^2 \)

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**Interference of Two Waves**

Adding two waves of the same frequency:

\[
E_1 = E_0^\phi \sin(kx - \omega t)
\]

\[
E_2 = E_0^\phi \sin(kx - \omega t + \phi)
\]

\[
E_T = E_1 + E_2 = ?
\]

Answer:

\[
E_T = E_0^\phi \sin(kx - \omega t + \phi_T)
\]

Result is a wave of the same frequency. Usually we want the amplitude \( E_T \) or the intensity \( I_T \).

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**Phase and Path Differences**

One way to get a phase difference \( \Delta \phi \) between two waves is to arrange for a path difference \( \Delta L \).

The general relation between phase difference and path difference is

\[
\Delta \phi = k\Delta L = 2\pi \frac{\Delta L}{\lambda}
\]

Remember k is phase per unit length: \( E = E_0^\phi \sin(kx - \omega t) \)

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**Simple Interference**

**Constructive case:**

\[
E_T = E_1^\phi + E_2^\phi
\]

\[
\phi = m(2\pi) \quad \Delta L = m\lambda
\]

Note if \( E_1^\phi = E_2^\phi \) then \( E_T^\phi = 2E_1^\phi \) and \( I_T = 4I_1 \)

**Destructive case:**

\[
E_T = E_1^\phi - E_2^\phi
\]

\[
\phi = (m + \frac{1}{2})2\pi \quad \Delta L = (m + \frac{1}{2})\lambda
\]

Note if \( E_1^\phi = E_2^\phi \) then \( E_T^\phi = 0 \) and \( I_T = 0 \)

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**The Double Slit Experiment**

Interference “fringes” due to alternating constructive and destructive interference between rays from \( S_1 \) and \( S_2 \).
Double Slit Path Differences
For point P at angle $\theta$ triangle shows $\Delta L = d \sin \theta$

For constructive interference we need $\Delta L = m\lambda$
where $m=0,1,2,...$ is any integer.

So the bright fringes are at angles given by $d \sin \theta = m\lambda$

Bright and Dark Fringes
So the bright fringes are at angles given by $d \sin \theta = m\lambda$

And the dark fringes are at angles given by $d \sin \theta = (m + \frac{1}{2})\lambda$

Locating the Fringes
For a bright spot we need $d \sin \theta = m\lambda$.

From the figure we see $\tan \theta = y / D$.
But for small angles we have $\sin \theta \approx \tan \theta$.

So the bright lines are at $m\lambda = dy / D$
$y = m\lambda D / d$

Double Slit Example
Given a double slit experiment with wavelength 450 nm, slit separation 0.3 mm, distance to screen 2 m, where will be the bright fringes?

$y = m\lambda D / d$ with $m = 0, 1, 2, ...$

$\lambda D / d = \frac{45 \times 10^{-6}}{.3 \times 10^{-3}} = 3 \text{ mm}$

So the bright lines are at $y = 0, 3 \text{ mm}, 6 \text{ mm}, 9 \text{ mm}, etc.$

Note the angles really are small: $\theta = \frac{6 \text{ mm}}{2 m} = .003 \text{ rad} = 0.17^\circ$

Phase Change on Reflection
- Two rays from S arrive at P. Path difference gives interference. Expect as angle $\to 0$, get constructive.
- Wrong! It’s destructive. Why?
- Because reflection from medium of higher $n$ always gives a phase change of 180°.
- (Maxwell says so!)

Michelson Interferometer
Another device for getting optical interference.

$\Delta L = 2d_1 - 2d_2$

As $d_2$ is changed, we see series of bright and dark fringes.

Bright when $\Delta L = m\lambda$

And dark when $\Delta L = (m + \frac{1}{2})\lambda$
Phase Difference Due To Different Index of Refraction

Yet another way to get 2 light waves out of phase.

$$\phi_2 = k_2 L = \left(\frac{2\pi}{\lambda_2}\right)L = \left(\frac{2\pi}{\lambda_0}\right)n_2 L$$

$$\phi_1 = k_1 L = \left(\frac{2\pi}{\lambda_1}\right)L = \left(\frac{2\pi}{\lambda_0}\right)n_1 L$$

$$\Delta\phi = \phi_2 - \phi_1 = \left(\frac{2\pi}{\lambda_0}\right)L(n_2 - n_1)$$

Thin Film Interference

There are many ways to get a phase difference between two rays of light and so get interference.

When do rays $r_1$ and $r_2$ interfere destructively so there is no reflection?

For $\theta = 0$ and $n_1 < n_2 < n_3$, the answer is easy: when the path difference $2L$ equals $\lambda/2$ (Or $3\lambda/2$, …)

Thin Film Example

Problem 36-33. Reflection of red light from a soap film with air on both sides. What thickness will give strong reflection? $\lambda_0 = 624 \text{ nm}$ $n = 1.33$

Wavelength in film: $\lambda = \lambda_0 / n = 624 / 1.33 = 469 \text{ nm}$

Phase change on reflection at front surface but not at back. So condition for strong reflection is

$$2L = (m + \frac{1}{2})\lambda$$

Solution: $L = \lambda / 4 = 117 \text{ nm}$

$L = 3(117) = 352 \text{ nm}$

Interference: The General Case

What if the phase difference is neither 0 nor 180° but something in between? How can we calculate the resultant amplitude?

Use phasors!

Recap

• We have discussed conditions for constructive and destructive interference in terms of the phase difference $\Delta\phi$:
  - Constructive: $\Delta\phi = 0, 360°, 720°, …$
  - Destructive: $\Delta\phi = 180°, 540°, …$

• We have looked at 5 different ways to arrange for interference between two light waves:
  - Double slit, Reflection from glass surface, Thin films, Michelson interferometer, Different index of refraction.

• In most cases, we achieve a phase difference by arranging to have a path difference $\Delta L$:
  - Constructive: $\Delta L = \lambda, 2\lambda, 3\lambda, …$
  - Destructive: $\Delta L = \lambda/2, 3\lambda/2, 5\lambda/2, …$

Phasor Diagram

Just as for AC circuits, we can add two oscillating functions using phasors. The lengths of the phasors are the amplitudes of the waves and the angle between the phasors is the phase difference between the waves. Then the length of the resultant phasor is the amplitude of the total wave.
Adding Vectors

A good way to get the length of the sum of two vectors is to use the dot product:

If \( \vec{C} = \vec{A} + \vec{B} \)

Then \( C^2 = \vec{C} \cdot \vec{C} = (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) \)

\[ = A^2 + B^2 + 2\vec{A} \cdot \vec{B} \]

\[ = A^2 + B^2 + 2AB \cos \theta \]

Intensity Formula

Suppose two light waves have equal intensities \( I_0 \) and a phase difference of \( \phi \). When these waves interfere, what will be the total intensity \( I \)?

\[ E^2 = E_1^2 + E_2^2 + 2E_1 \cdot E_2 \cos \phi \]

\[ E^2 = 2E_0^2 + 4E_0^2 \cos^2 \left( \frac{1}{2} \phi \right) \]

But \( I \propto E^2 \) so:

\[ I = 4I_0 \cos^2 \left( \frac{1}{2} \phi \right) \]

Text equations 35-22, 23; proved on p. 970.

Double-slit intensity

Intensity Example

Problem 35-29 revised.

Two waves interfere with phase difference \( \phi = 60^\circ \). One wave has intensity \( I_0 \), the other 4\( I_0 \). What is the resulting intensity?

\[ E^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos \phi \]

\[ E^2 = E_0^2 + 4E_0^2 + 2E_0 \left( 2E_0 \right) \left( \frac{1}{2} \right) \]

\[ = 5E_0^2 + 2E_0^2 \]

But \( I = (\text{Const})E^2 \)

so \( I = 5I_0 + 2I_0 = 7I_0 \)