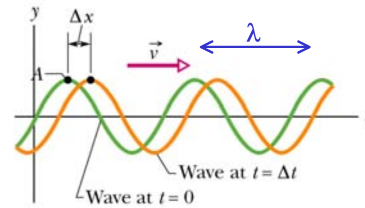
Light Waves

- **Today**
  - **Wavelengths, frequencies, polarization**
  - **Energy, momentum and photons**
  - **Reflection and refraction**

Review of Waves (Ch. 16)



Wavelength = λ

Frequency = f

Velocity = v = f λ

$$y = y_0 \sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$$

$$v = f\lambda = \omega / k$$

Electromagnetic Waves

Try wave solutions for Maxwell Equations:

$$\vec{E}(x,t) = \vec{E}_0 \sin(kx - \omega t)$$
$$\vec{B}(x,t) = \vec{B}_0 \sin(kx - \omega t)$$

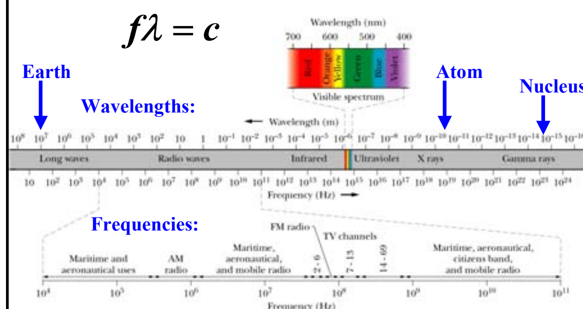
If we try these functions, for **any** given ω, we find they **do** satisfy Maxwell's Equations, and the wave speed is equal to the speed of light!

$$v = f\lambda = c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0 \times 10^8 \, m/s$$

Universal Constant

The Electromagnetic Spectrum

$$f\lambda = c$$



Radio Waves to X Rays

The range of wavelengths λ (and frequencies f = c/λ) is amazing.

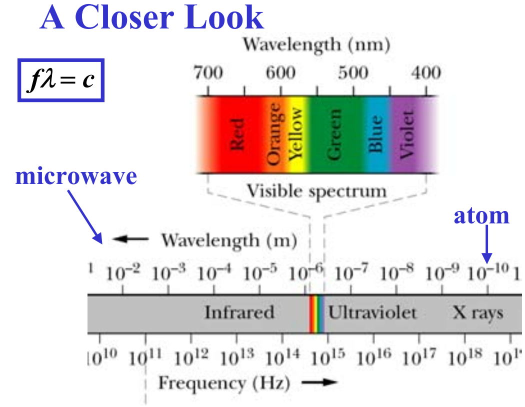
All are electromagnetic waves governed by Maxwell's Equations!

AM radio, f =1500 kHz: $\lambda = \frac{c}{f} = \frac{3\times 10^8}{1.5\times 10^6} = 200\, m$

X ray, the size of an atom, λ = 0.1 nm: $f = \frac{c}{\lambda} = \frac{3\times 10^8}{1\times 10^{-10}} = 3\times 10^{18}\, Hz$

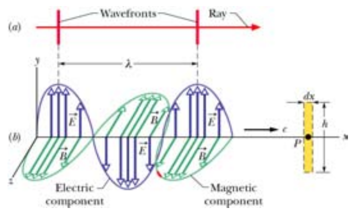
A Closer Look

$$f\lambda = c$$

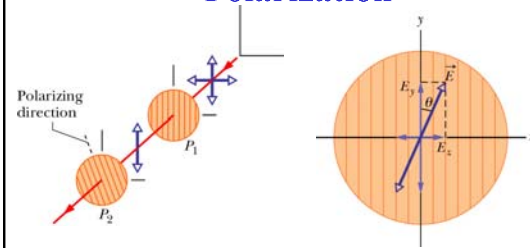


Polarization

Electromagnetic waves are ***transverse***. For a wave propagating in the ***z*** direction, the polarization can be either in the ***x*** or the ***y*** direction, depending on the direction of the oscillating electric field.



Polarization



$$E_y = E\cos\theta$$

$$I = I_0\cos^2\theta$$

Energy

$$I = Intensity \equiv \frac{Power}{Area}$$

$$= \left(\tfrac{1}{2}\varepsilon_0 E^2\right)c = \frac{E^2{}_{rms}}{c\mu_0} \propto E^2$$

Intensity of a point source

$$I(r) = \frac{P}{4\pi r^2}$$

Another inverse square law.

Momentum and Radiation Pressure

Maxwell's Equations show that a light wave carries momentum in addition to energy:

$$momentum = \frac{energy}{c}$$

NOTE: comes from Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$
For details see text section 33-5

$$force = \frac{momentum}{time} = \frac{power}{c}$$

$$radiation\ \ pressure = \frac{force}{area} = \frac{I}{c}$$

Photons (Chapter 38)

- Quantization
  - Quantize energy and momentum of wave.
- Photon energy: $E = hf$
- Photon momentum: $p = h/\lambda$

Energy-momentum relation for a particle (ch.37): $E = \sqrt{(pc)^2 + (mc^2)^2}$

But for a photon we find: $E = hf = hc/\lambda = pc$

Equations agree if m=0.

Light in Matter

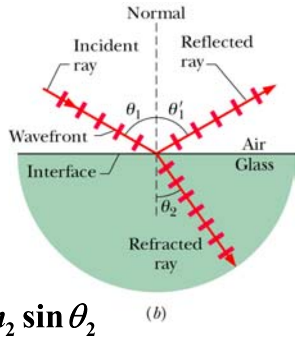
In vacuum light travels with speed c = 3×10^8 m/s. But light also travels through many materials such as glass, water, etc. In such a medium, its speed is reduced to ***v = c/n***, where ***n*** is called the **index of refraction**.

$$v = \frac{c}{n} < c$$

(In fact, no physical object can travel faster than c. There are no "warp drives" on real space ships.)

Reflection and Refraction

When a light ray passes from one medium to another, it splits into two beams, the ***reflected*** and ***refracted*** rays.



- $\theta_1' = \theta_1$
- $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Snell's Law

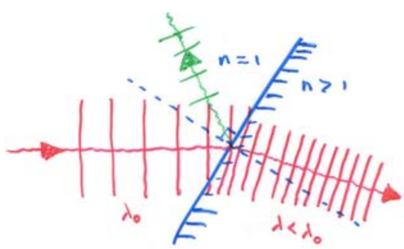
Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

What causes this? It's because the wavelength changes. For example, going from vacuum to glass the wavelength decreases along with the speed.

Vacuum: $n = 1, \quad v = c, \quad \lambda_0 = \dfrac{c}{f}$

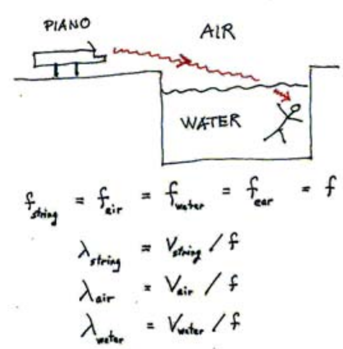
Glass: $n > 1, \quad v < c, \quad \lambda = \dfrac{v}{f} < \lambda_0$

Refraction



So ray is bent ***toward the normal*** in this case. But if we go from large n to small n, ray is bent ***away from*** the normal.

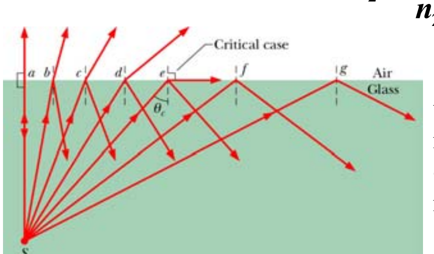
Wavelength changes in Refraction



Total Internal Reflection

If $n_1 > n_2$ then there may be ***no solution*** of Snell's Law for the angle of refraction.

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 > 1$$



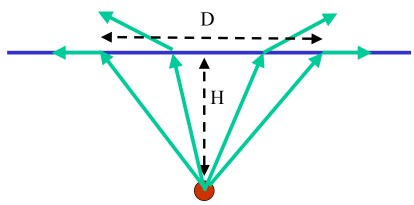
So for $\theta_1 > \theta_c$ there is no refracted ray, only the reflected ray.

Optical Fibers

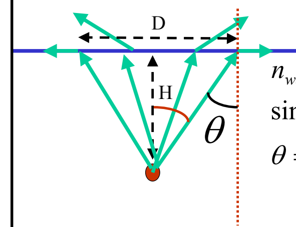
Light is trapped inside tube of transparent material with high ***n***. Kept inside by repeated total internal reflections. Propagates without loss of energy.

Example: Problem 33-55

A point source of light is 80 cm below the surface of a pool. Find the diameter of the circle at the surface through which light emerges from the water.



Problem 33-55 (cont'd)



$$n_{water} \sin\theta = n_{air} \sin 90°$$

$$\sin\theta = 1/n_{water} = 1/1.33 = 0.75$$

$$\theta = \sin^{-1}(0.75) = 48.6°$$

$$\frac{D/2}{H} = \tan\theta$$

$$D = 2H \tan\theta = 2(80cm)\tan(48.6°) = \underline{181cm}$$

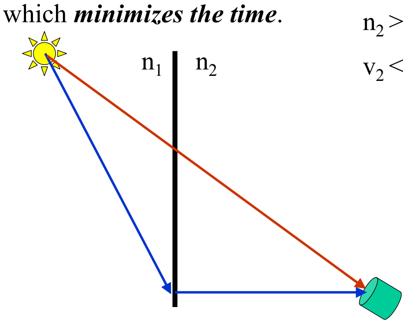
Fermat's Principle

The path chosen by a light ray will be the one which ***minimizes the time***.

$v = c/n$

$n_2 > n_1$

$v_2 < v_1$



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Result:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$