Chapter 32

- Induced Electric Fields
- Induced Magnetic Fields
- Maxwell's Equations
- Magnetic Materials

Induced Magnetic Fields

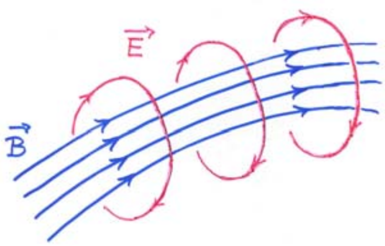
Faraday's Law gives ***induced electric fields***: namely, "Changing ***B*** creates ***E***." $\frac{d}{dt}\Phi_M$

Maxwell discovered ***induced magnetic fields:*** namely, "Changing ***E*** creates ***B***." $\frac{d}{dt}\Phi_E$

This is described by a new term in Ampere's Law (called "displacement current") of form

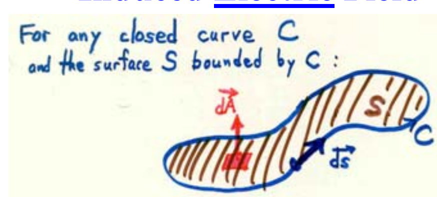
$$i_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Review: Induced E Fields



Lines of the *induced E* field make circles around the lines of the *changing B* field, just as lines of *B* circle around a wire.

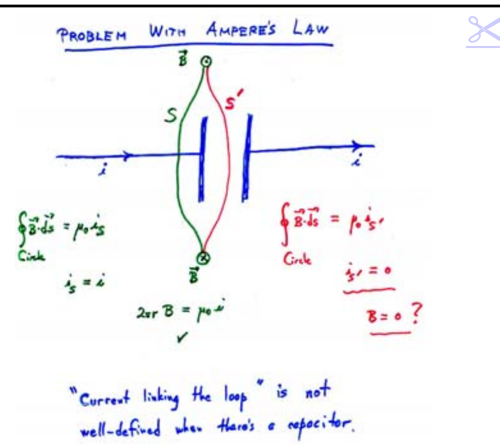
Faraday: Induced Electric Field



$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

Maxwell: Induced Magnetic Field

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$



SOLUTION TO PROBLEM

(Maxwell, 1873)

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i_S + \mu_0\epsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

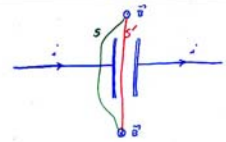
SAME IDEA AS FARADAY:

$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

New term called "displacement current"

Define $i_d = \epsilon_0 \frac{d}{dt}\Phi_E$

Then Ampere becomes $\oint \vec{B}\cdot d\vec{s} = \mu_0(i + i_d)$



Choose S:

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i_S + \mu_0\epsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

$2\pi r B \quad \mu_0 i \quad 0$ → $B = \frac{\mu_0 i}{2\pi r}$

Choose S':

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{S'} + \mu_0\epsilon_0 \frac{d}{dt}\int_{S'} \vec{E}\cdot d\vec{A}$$

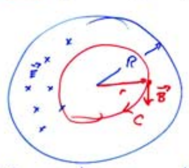
$2\pi r B \quad 0 \quad \mu_0\epsilon_0 \frac{d}{dt}(EA)$

Inside Capacitor $E = \sigma/\epsilon_0 = \frac{Q}{\epsilon_0 A}$

So: $2\pi r B = \mu_0\epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0}\right) = \mu_0 i$: $B = \frac{\mu_0 i}{2\pi r}$ ✓

Example

B inside capacitor being charged

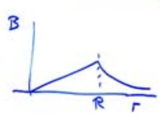


$E = \frac{Q}{\pi R^2 \epsilon_0}$

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0\epsilon_0 \frac{d}{dt}\Phi_E = \mu_0\epsilon_0 \frac{d}{dt}\left[E\pi r^2\right]$$

$$B\cdot 2\pi r = \mu_0\epsilon_0 \pi r^2 \frac{dE}{dt} = \mu_0 \frac{r^2}{R^2}\frac{dQ}{dt}$$

If $\frac{dQ}{dt} = i$: $B = \frac{\mu_0 i r}{2\pi R^2}$



Maxwell's Equations

Maxwell modified Ampere's Law to account for this new effect of the induced magnetic field, and thereby got the set of four equations which provide the complete theory of the electromagnetic field.

Maxwell's Equations (1873)

$$\oint \vec{E}\cdot d\vec{A} = Q_{in}/\epsilon_0$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$

} Gauss

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\Phi_M$$

Faraday

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{in} + \mu_0\epsilon_0 \frac{d}{dt}\Phi_E$$

Ampere – Maxwell

Summary

Gauss for E: Lines of *E* begin and end on charges.

Gauss for B: Lines of *B* never begin or end.

Faraday: Changing *B* creates circular *E*.

Ampere: Current or changing *E* creates circular *B*.

We now have the complete theory of the electromagnetic field.

Classical Electrodynamics

This is the most successful theory in all of science. In over 100 years of constant testing, no disagreements with experiment have ever been found. It is the basis for Einstein's theory of relativity and is an essential ingredient in atomic physics and quantum field theory.

Maxwell's Waves: Preview of Chapter 33

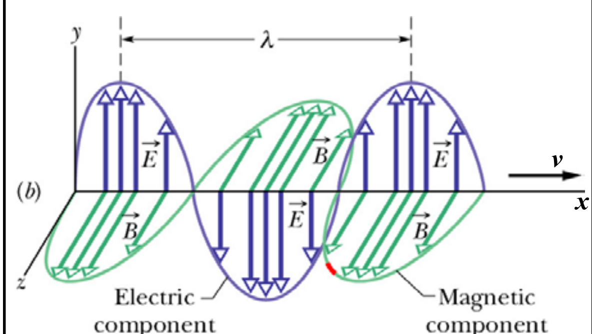
PREDICTION: Electromagnetic waves propagate in a vacuum with speed $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0\times10^8$ m/s

Electromagnetic Waves

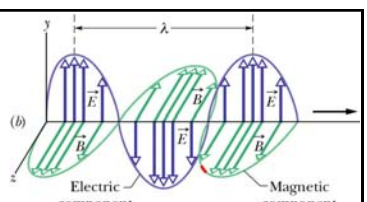
- Maxwell's Equations have wave solutions.
- Wave must have both ***E*** and ***B*** fields.
- ***E*** and ***B*** have same wavelength and velocity.
- ***E***, ***B***, ***v*** are three perpendicular vectors
- Their magnitudes are related by $E/B = v$.
- The speed must be

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0\times10^8 \ m/s$$

Electromagnetic Waves



Wave Equations



Recall equations for waves from Ch. 16:

$$f(x,t) = f_m \sin(kx - \omega t) \qquad k = \frac{2\pi}{\lambda}$$

with the wave velocity $v = \omega / k$

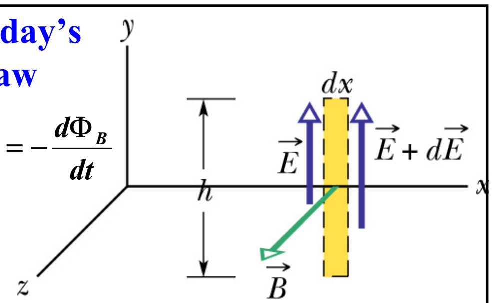
Could ***E*** and ***B*** both have this same form?

$$E = E_m \sin(kx - \omega t) \quad and \quad B = B_m \sin(kx - \omega t)$$

YES! These are solutions if: $v = 1/\sqrt{\varepsilon_0 \mu_0}$

Faraday's Law

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$



$$h\,dE = -h\,dx\,\frac{dB}{dt} \qquad \frac{dE}{dx} = -\frac{dB}{dt}$$

Apply Faraday to Wave $\frac{dE}{dx} = -\frac{dB}{dt}$

$E = E_m \sin(kx - \omega t)$ $B = B_m \sin(kx - \omega t)$

$\frac{dE}{dx} = E_m k \cos(kx - \omega t)$ $\frac{dB}{dt} = -\omega B_m \cos(kx - \omega t)$

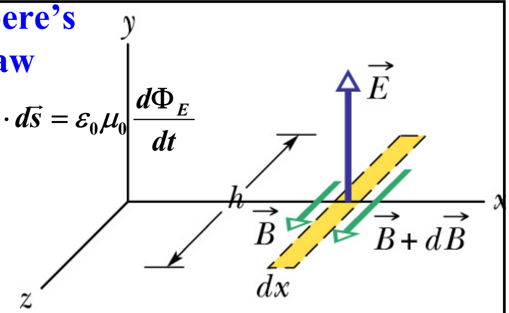
So Faraday gives

$E_m k = -(-\omega B_m)$ $\frac{E_m}{B_m} = \frac{\omega}{k} = v$

Or finally $E / B = v$

Ampere's Law

$\oint \vec{B} \cdot d\vec{s} = \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$



$h\, dB = \varepsilon_0 \mu_0 \left(-h\, dx \frac{dE}{dt}\right)$ $\frac{dB}{dx} = -\varepsilon_0 \mu_0 \frac{dE}{dt}$

Apply Ampere to Wave $\frac{dB}{dx} = -\varepsilon_0 \mu_0 \frac{dE}{dt}$

$E = E_m \sin(kx - \omega t)$ $B = B_m \sin(kx - \omega t)$

$\frac{dB}{dx} = B_m k \cos(kx - \omega t)$ $\frac{dE}{dt} = -\omega E_m \cos(kx - \omega t)$

$$B_m k = -\varepsilon_0 \mu_0 (-\omega E_m) = \varepsilon_0 \mu_0 \omega E_m$$

$\frac{B_m}{E_m} = \frac{\omega}{k} \varepsilon_0 \mu_0$ $\frac{1}{v} = v\, \varepsilon_0 \mu_0$ $v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

Introduction to Magnetic Materials

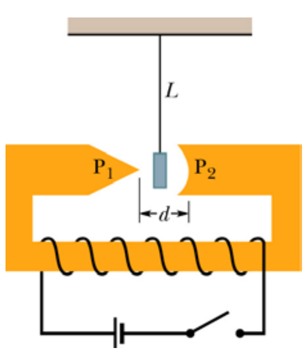
- **Three main classes:**
  - **Ferromagnetic**
  - **Paramagnetic**
  - **Diamagnetic**

$\vec{M} = \vec{\mu} / (Volume)$

- **Define *magnetization*, the *density of dipole moments.***

Magnetic effects occur when dipoles *line up* with an applied field, or with each other, to produce large magnetization.

Materials in an External Field



Place sample in external magnetic field. Measure its magnetization.