AC Circuits

- Ch.30: Faraday's Law
- Ch.30: Inductors and RL Circuits:
- Ch.31: AC Circuits

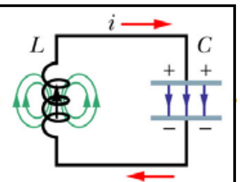
Review: Inductance

- If the current through a coil of wire changes, there is an induced emf proportional to the rate of change of the current.
- Define the proportionality constant to be the *inductance L*:

$$\mathcal{E} = -L\frac{di}{dt}$$

- SI unit of inductance is the **henry** (**H**).

Review: LC Circuits



$$q = Q\cos(\omega t)$$

$$i = \frac{dq}{dt} = -\omega Q\sin(\omega t)$$

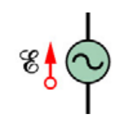
Loop rule gives differential equation: $$\frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right)q$$

Solution if frequency is correct: $$\omega^2 = \left(\frac{1}{LC}\right)$$

Natural frequency of oscillations!

Review: AC Voltage Sources

- For an AC circuit, we need an *alternating emf*, or AC power supply.
- This is characterized by its *amplitude* and its *frequency*.

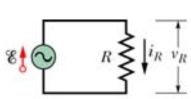


$$\mathcal{E} = \mathcal{E}_m \sin\omega t$$

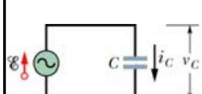
amplitude — *angular frequency*

Review: R, C, L Separately

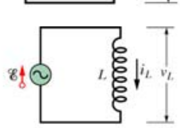
"ELI the ICE man"



$V_R = I_R R$
v_R and i_R are in phase



$V_C = I_C X_C \quad with\ X_C = \frac{1}{\omega C}$
i_C leads v_C by 90°.

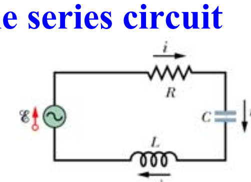


$V_L = I_L X_L \quad with\ X_L = \omega L$
v_L leads i_L by 90°.

Review: Simple series circuit

$$\mathcal{E}(t) = \mathcal{E}_m \sin\omega t$$

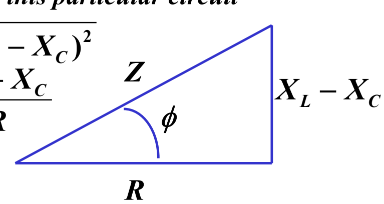
$$i(t) = I\sin(\omega t - \phi)$$



It turns out that for *this particular circuit*

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

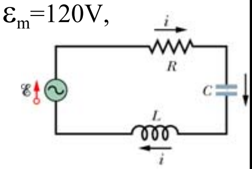


Series Circuit Example

Given L = 50 mH, C = 60 μF, $\mathcal{E}_m$=120V, f=60Hz, and I=4.0A.

(a) What is the impedance?

(b) What is the resistance R?



Solution to (a) is easy:

$$Z = \frac{\mathcal{E}_m}{I} = \frac{120}{4} = 30\,\Omega$$

Example (part b)

(a) $Z = 30\,\Omega$

L=50 mH, C = 60 μF, $\mathcal{E}_m$=120V, f=60Hz, I=4.0A

(b) What is the resistance R?

$$\omega = 2\pi f = 2\times 3.14\times 60 = 377\ rad/s$$

$$X_L = \omega L = 377\times 50\times 10^{-3} = 18.8\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{377\times 60\times 10^{-6}} = 44.2\,\Omega$$

$$X_C - X_L = 44.2 - 18.8 = 25.4\,\Omega$$

$$R = \sqrt{Z^2 - (X_L - X_C)^2} = \sqrt{30^2 - 25.4^2} = 16\,\Omega$$

Q.31-3

A certain series RLC circuit is driven by an applied emf with angular frequency 20 radians per second. If the maximum charge on the capacitor is 0.03 coulomb, what is the maximum current in the circuit?

(1) 6 A (2) 1.5 A (3) 0.6 A (4) 0.15 A

(5) Not enough information

Q.31-3

A certain series RLC circuit is driven by an applied emf with angular frequency 20 radians per second. If the maximum charge on the capacitor is 0.03 coulomb, what is the maximum current in the circuit?

$$q = Q\sin(\omega t) \qquad i = \frac{dq}{dt} = \omega Q\cos(\omega t)$$

$$I = \omega Q = 20\times 0.03 = 0.6\ A$$

(1) 6 A (2) 1.5 A (3) 0.6 A (4) 0.15 A

(5) Not enough information

Q.31-4

In a series RLC circuit, find the resistance, given the impedance and phase constant:

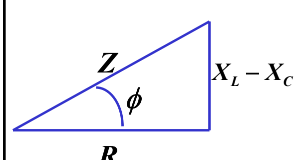
Z = 500 Ω ϕ = 60° R = ?

(1) 400 Ω (2) 350 Ω (3) 300 Ω (4) 250 Ω

Q.31-4

In an RLC circuit:

Z = 500 Ω ϕ = 60° R = ?



$$R = Z\cos\phi$$

$$\cos(60°) = \sin(30°) = \frac{1}{2}$$

$$R = Z/2 = 250\,\Omega$$

(1) 400 Ω (2) 350 Ω (3) 300 Ω (4) 250 Ω

Resonance

For a given series RLC circuit, what applied frequency will give the biggest current?

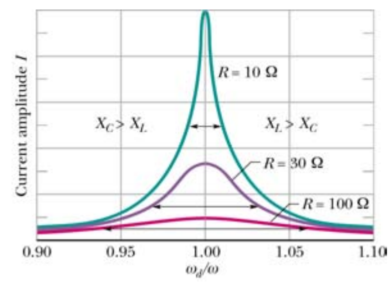
$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

I is biggest when denominator is smallest, which is when reactances cancel:

$$\omega L = \frac{1}{\omega C} \qquad \text{Which gives} \quad \omega^2 = \frac{1}{LC}$$

Same as natural frequency for oscillations!

Resonance Plot: I *vs* ω



Peak at $\omega = 1/\sqrt{LC}$ Becomes sharper as $R \to 0$

Power in AC Circuits

To know how much power will be consumed in an AC circuit we need more than the impedance Z; we also need the phase angle ϕ.

AC Power

What is the power provided by an AC source?

- Only interested in the ***time average!***
- Time average power to L and C is ***zero.***
- So P_{ave}(from source) = P_{ave}(to resistor).

$$P_{ave} = \langle v_R i_R \rangle = \langle (V_R \sin\omega t)(I_R \sin\omega t) \rangle$$

$$= V_R I_R \langle \sin^2 \omega t \rangle = \frac{1}{2} V_R I_R$$

RMS Values

$$V_{RMS} = \sqrt{\langle v^2 \rangle} = \sqrt{\langle V^2 \sin^2 \omega t \rangle} = V\sqrt{\frac{1}{2}} = V/\sqrt{2}$$

$$\textit{Likewise} \quad I_{RMS} = \sqrt{\langle i^2 \rangle} = I/\sqrt{2}$$

So for a resistor:

$$P_{ave} = V_{RMS} I_{RMS} = \frac{V}{\sqrt{2}} \frac{I}{\sqrt{2}} = \frac{1}{2} VI$$

Power and Phase Angle

Power to resistor $\quad P = i_R{}^2 R = I_{R^2} R \sin^2(\omega t - \phi)$

$$P_{avg} = I_R{}^2 R \langle \sin^2 \rangle_{avg} = \frac{1}{2} I_R{}^2 R$$

But we want result in terms of impedance and phase angle:

$$R = Z\cos\phi$$

$$\textit{So} \quad P_{avg} = \frac{1}{2} I^2 Z \cos\phi$$

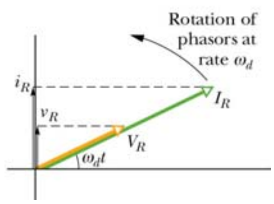
Power factor

What about more complicated circuits?

Method of Phasors

- **Useful mathematical trick for working with oscillating functions having different phase relationships.**
- **We'll use it to derive the known result for the series circuit, to show how it works. Good for general case.**

$$i_R = I_R \sin(\omega t)$$



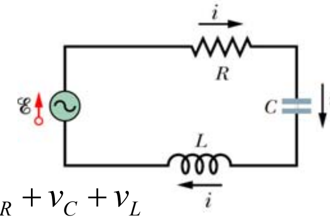
I_R = Length of phasor

i_R = Vertical component of phasor

Series RLC Circuit

1. The currents are all equal.

2. The voltage drops add up to the applied emf:



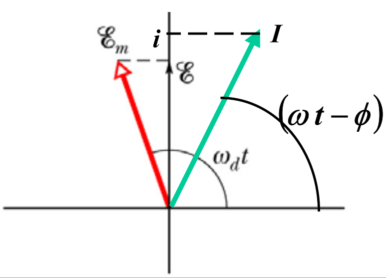
$$\mathcal{E} = v_R + v_C + v_L$$

Because of the phase differences, the amplitudes do not add, **but** in the phasor diagram, this means that the vertical components **do** add, so we get the right answer if we add the phasors **as vectors.**

Phasors for Series RLC Circuit

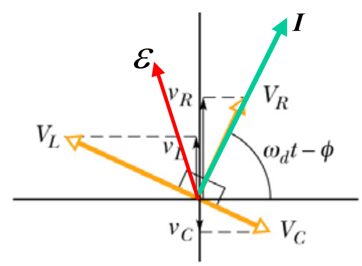
$$\mathcal{E} = v_R + v_C + v_L = \mathcal{E}_m \sin \omega t$$

$$i_R = i_C = i_L = I \sin(\omega t - \phi)$$



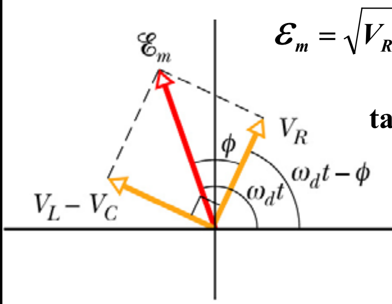
Adding the Voltage Phasors

$$\mathcal{E} = v_R + v_C + v_L = \mathcal{E}_m \sin \omega t$$



Adding the Voltage Phasors

$$\mathcal{E} = v_R + v_C + v_L = \mathcal{E}_m \sin \omega t$$



$$\mathcal{E}_m = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

Impedance and Phase Angle

$$\mathcal{E}_m = \sqrt{V_R^2 + (V_L - V_C)^2} \qquad \tan\phi = \frac{V_L - V_C}{V_R}$$

Divide by the common current amplitude ***I:***

$$\mathcal{E}_m = IZ, \quad V_R = IR, \quad V_L = IX_L, \quad V_C = IX_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

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