**Induction and Oscillations**

Ch. 30: Faraday’s Law  
Ch. 31: AC Circuits

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**Induced EMF: Faraday’s Law**

“Time-dependent $B$ creates induced $E$”

In particular: A changing magnetic flux creates an emf in a circuit:

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**Electromagnetic Induction**

Current in secondary circuit can be produced by a changing current in primary circuit.

Ammeter or voltmeter.

Application: Transformer

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**Demonstrations**

- EMF induced in a coil by moving a bar magnet
- EMF induced in a secondary coil by changing current in primary coil

Sorry, we can't do it in this packed room… but here is the essence of it

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**EMF induced in a coil by moving a bar magnet**

EMF depends on how strong magnet and how fast we move in/out

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**EMF induced in a secondary coil by changing current in primary coil**
Magnetic Flux

We define magnetic flux $\Phi$ exactly as we defined the flux of the electric field. The idea is the number of lines of $\mathbf{B}$ that pass through an area.

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A}$$

Simple case #1: uniform $\mathbf{B}$, $\perp$ surface: $\Phi = BA$

Simple case #2: surface is closed: $\Phi = 0$

Faraday’s Law

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

The emf induced in any loop or circuit is equal to the negative rate of change of the magnetic flux through that loop.

Example 1

A circle of radius 20 cm in the $xy$ plane is formed by a wire and a 3-ohm resistor. A uniform magnetic field is in the $z$ direction; its magnitude decreases steadily from .08 tesla to 0 in a time of 4 seconds.

What emf is generated?

$$A = \pi r^2 = 0.13 \ m^2 \quad \frac{dB}{dt} = -\frac{.08T}{4 \ s} = -.02 \ T/s$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -A\frac{dB}{dt} = -(0.13)(-0.02) = 2.6 \times 10^{-3} \ V$$

Example 2

I push a rod along metal rails through a uniform magnetic field.

(a) What emf is generated?

(b) What current will flow?

(c) What power must I supply?
Example 2a
L = 20 cm
V = 3.0 m/s
B = .05 T
(a) What emf is generated?
\[ \frac{dA}{dt} = L \frac{dx}{dt} = L v = 0.6 \text{ m}^2 / \text{s} \]
\[ \mathcal{E} = -\frac{d\Phi}{dt} = -B \frac{dA}{dt} = -0.05 \times 0.6 = -30 \text{ mV} \]

Example 2b
Resistance of bar: R = 15 Ω
(b) What current will flow?
\[ i = \frac{\mathcal{E}}{R} = \frac{-30 \times 10^{-3} V}{15 \Omega} = -2 \text{ mA} \]
Which direction does current flow?
*Forget the minus sign.* Use Lenz’s Law!
Flux is increasing outward. Therefore current will resist that change by flowing clockwise.

Example 2c
(c) What power must I supply?
Magnetic force: \( \vec{F} = i \vec{L} \times \vec{B} \)
\[ F = 0.002 \times 0.2 \times 0.05 = 2 \times 10^{-5} \text{ N} \]
Power: \( P = Fv = (2 \times 10^{-5} \text{ N})(3 \text{ m/s}) = 6 \times 10^{-5} \text{ W} \)
Check Joule heating: \( P = i^2 R = 6 \times 10^{-5} \text{ W} \)

Faraday’s Law: General Form
\[ \oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} \]

Inductance
- For any coil of wire, there is a flux \( \Phi \) through the coil, which is proportional to the current.
- If that changes, Faraday’s Law requires an emf induced in the coil, proportional to the rate of change of the flux.
- Clearly \( \Phi \propto i \) and so \( \mathcal{E} = -\frac{d\Phi}{dt} = -\frac{di}{dt} \)
- Define the proportionality constant to be the inductance \( L \):
  \[ \mathcal{E} = -L \frac{di}{dt} \]
- SI unit of inductance is the henry (H).

Inductors
If current is increasing, the induced emf acts against the increase, giving a voltage drop.
If current is decreasing, the induced emf acts against the decrease, giving a voltage rise.
Energy in an Inductor

The energy stored in an inductor equals the work required to set up the current.

\[ dW = Vdq = V \frac{dq}{dt} \frac{dt}{(Li)} = Lidi \]

\[ W = \int dW = L \int_0^i idt = \frac{1}{2} LI^2 \]

So energy stored in an inductor is \( U = \frac{1}{2} Li^2 \)

Magnetic Field Energy

The energy stored in an inductor is contained in the magnetic field. The general formula for the energy density in any magnetic field is

\[ u = \frac{B^2}{2\mu_0} \]

Inductors and Resistors

 Voltage changes going clockwise around this loop:

\[ + \varepsilon - iR - L \frac{di}{dt} = 0 \]

Inductor gives voltage drop if current is increasing.

RL Circuits

\[ + \varepsilon - iR - L \frac{di}{dt} = 0 \]

\[ L \frac{di}{dt} + Ri = \varepsilon \]

Same equation as for charging a capacitor!

Try same kind of solution:

\[ i = \frac{\varepsilon}{R} \{1 - e^{-t/\tau}\} \]

This works, provided

\[ \tau = \frac{L}{R} \]

Example

\( \varepsilon = 30 V \)

\( R = 5000 \Omega \)

\( L = 15 mH \)

(a) What is the time constant?

\[ \tau = \frac{L}{R} = \frac{15 \times 10^{-3}}{5 \times 10^{-3}} = 3 \times 10^{-6} = 3 \mu s \]

(b) What is current after 1 second?

\[ i = \frac{\varepsilon}{R} \{1 - e^{-t/\tau}\} = \frac{30}{5000} (1 - 0) = 6 mA \]
Example 2: Problem 30-89

(a) What happens immediately after switch is closed?

L prevents sudden change so:

\[ i_2 = 0 \quad \therefore \quad i = i_1 = \frac{\varepsilon}{R_1} \]

So:

\[ V_{R2} = 0 \quad \therefore \quad V_L = \varepsilon \quad \text{and} \quad \frac{di_2}{dt} = \frac{\varepsilon}{L} \]

Example 2 continued

(b) What happens a long time after switch is closed?

We have reached a steady state so:

\[ \frac{di_2}{dt} = 0 \quad \therefore \quad V_L = 0 \quad \text{and} \quad V_{R2} = \varepsilon \]

So:

\[ i_1 = \frac{\varepsilon}{R_1}, \quad i_2 = \frac{\varepsilon}{R_2}, \quad i = i_1 + i_2 \]

Induction and Oscillations

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• Chapter 30 Homework for Monday:
  – Questions 1, 3, 7
  – Problems 3, 5, 29, 44

• Chapter 31 Homework for Tuesday:
  – Questions 3, 4, 7
  – Problems 5, 19, 39

• WileyPlus chapters 30, 31 for Tuesday.