Induction and Oscillations

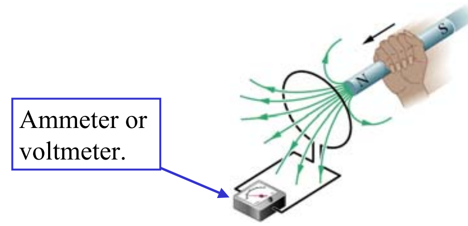
Ch. 30: Faraday's Law

Ch. 31: AC Circuits

Induced EMF: Faraday's Law

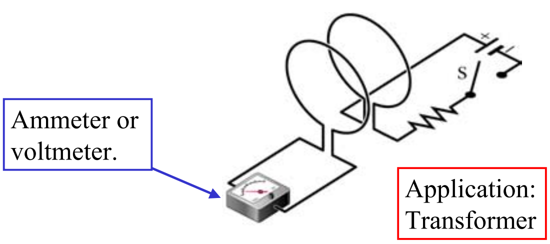
"Time-dependent *B* creates induced *E*"

In particular: A *changing magnetic flux* creates an *emf in a circuit*:



Electromagnetic Induction

Current in secondary circuit can be produced by a *changing* current in primary circuit.

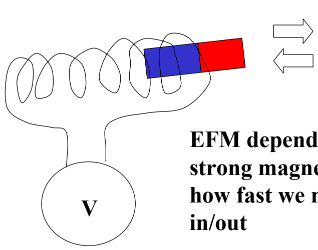


Demonstrations

- **EMF induced in a coil by moving a bar magnet**
- **EMF induced in a secondary coil by changing current in primary coil**

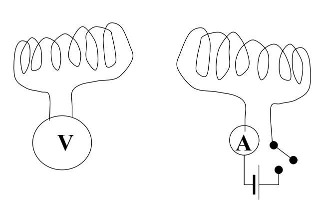
Sorry, we can't do it in this packed room … but here is the essence of it

EMF induced in a coil by moving a bar magnet



EFM depends on how strong magnet and how fast we move in/out

EMF induced in a secondary coil by changing current in primary coil



Magnetic Flux

We define ***magnetic flux*** Φ exactly as we defined the flux of the electric field. The idea is the number of lines of ***B*** that pass through an area.

$$\Phi = \int \vec{B} \cdot d\vec{A}$$

Simple case #1: uniform ***B***, $\perp$ surface: $\Phi = BA$

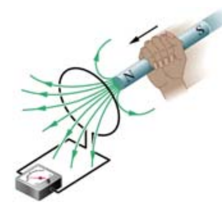
Simple case #2: surface is ***closed:*** $\Phi = 0$

Faraday's Law

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

The emf induced in any loop or circuit is equal to the negative rate of change of the magnetic flux through that loop.

Voltmeter reading gives ***rate of change*** of the number of lines linking the loop.



Changing Magnetic Flux

$$\Phi = \int \vec{B} \cdot d\vec{A}$$

How can we get a time-changing flux, so that $\mathcal{E} = -\frac{d\Phi}{dt} \neq 0$?

- **Change the field: Φ = B(t) A**
- **Change the area: Φ = B A(t)**
- **Change the angle: Φ = B A cos θ(t)**

Example 1

A circle of radius 20 cm in the *xy* plane is formed by a wire and a 3-ohm resistor. A uniform magnetic field is in the *z* direction; its magnitude decreases steadily from .08 tesla to 0 in a time of 4 seconds.

What emf is generated?

$$A = \pi r^2 = 0.13\, m^2 \qquad \frac{dB}{dt} = -\frac{.08\,T}{4\,s} = -.02\,T/s$$

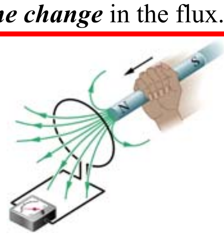
$$\mathcal{E} = -\frac{d\Phi}{dt} = -A\frac{dB}{dt} = -(.13)(-.02) = 2.6 \times 10^{-3}\, V$$

Lenz's Law

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

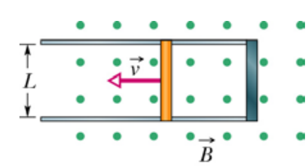
The ***direction*** of the induced emf is such as to create a current which will ***oppose the change*** in the flux.

Motion as shown produces clockwise current which makes B field opposing the increase.



Example 2

I push a rod along metal rails through a uniform magnetic field.

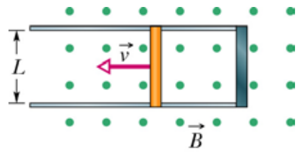


(a) What emf is generated?

(b) What current will flow?

(c) What power must I supply?

Example 2a



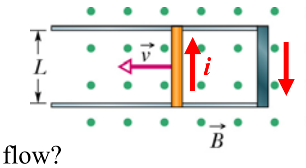
L = 20 cm
V = 3.0 m/s
B = .05 T

(a) What emf is generated?

$$\frac{dA}{dt} = L\frac{dx}{dt} = Lv = 0.6\,m^2/s$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -B\frac{dA}{dt} = -.05\times 0.6 = -30\,mV$$

Example 2b



Resistance of bar: R = 15 Ω

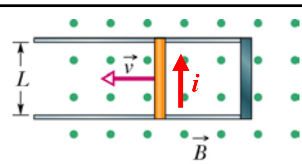
(b) What current will flow?

$$i = \frac{\mathcal{E}}{R} = \frac{-30\times 10^{-3}\,V}{15\Omega} = -2\,mA$$

Which direction does current flow?
Forget the minus sign. **Use Lenz's Law!**

Flux is ***increasing outward.*** Therefore current will ***resist that change*** by flowing ***clockwise.***

Example 2c



(c) What power must I supply?

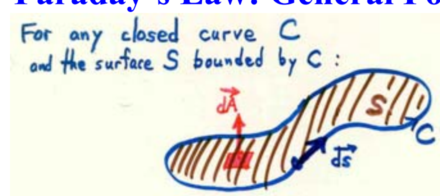
Magnetic force: $\vec{F} = i\vec{L}\times\vec{B}$

$$F = .002\times .2\times .05 = 2\times 10^{-5}\,N$$

Power: $P = Fv = (2\times 10^{-5}\,N)(3\,m/s) = 6\times 10^{-5}\,W$

Check Joule heating: $P = i^2 R = 6\times 10^{-5}\,W$

Faraday's Law: General Form



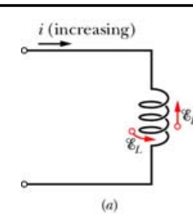
$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

$\mathcal{E}$ $\qquad$ Φ

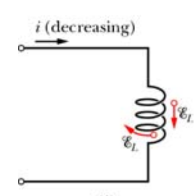
Inductance

- For any coil of wire, there is a flux Φ through the coil, which is proportional to the current.
- If that changes, Faraday's Law requires an emf ***induced*** in the coil, proportional to the ***rate of change*** of the flux.
- Clearly $\Phi \propto$ i and so $\mathcal{E} = -\frac{d\Phi}{dt} \propto -\frac{di}{dt}$
- Define the proportionality constant to be the ***inductance L:*** $\mathcal{E} = -L\frac{di}{dt}$
- SI unit of inductance is the **henry** (**H**).

Inductors



If current is ***increasing,*** the induced emf acts against the increase, giving a voltage ***drop.***



If current is ***decreasing,*** the induced emf acts against the decrease, giving a voltage ***rise.***

Energy in an Inductor

The energy stored in an inductor equals the work required to set up the current.

$$dW = Vdq = V\frac{dq}{dt}dt = (L\frac{di}{dt})idt = Lidi$$

$$W = \int dW = L\int_0^I idi = \tfrac{1}{2}LI^2$$

So energy stored in an inductor is $U = \tfrac{1}{2}Li^2$

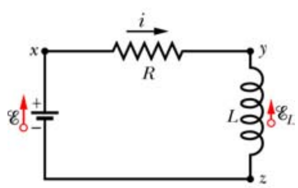
Magnetic Field Energy

The energy stored in an inductor is contained in the ***magnetic field.*** The general formula for the ***energy density*** in any magnetic field is

$$u = \frac{B^2}{2\mu_0}$$

Inductors and Resistors

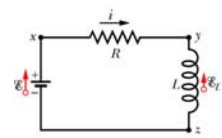
Voltage changes going clockwise around this loop:



$$+\mathcal{E} - iR - L\frac{di}{dt} = 0$$

Inductor gives voltage ***drop*** if current is ***increasing***.

RL Circuits



$$+\mathcal{E} - iR - L\frac{di}{dt} = 0$$

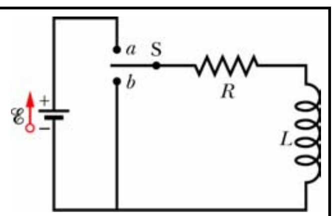
$$L\frac{di}{dt} + Ri = \mathcal{E}$$

Same equation as for charging a capacitor!

Try same kind of solution: $i = \frac{\mathcal{E}}{R}\left\{1 - e^{-t/\tau}\right\}$

This works, provided $\tau = L/R$

RL Summary



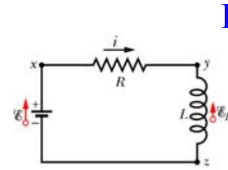
Set switch to position a:

$$i = \frac{\mathcal{E}}{R}\left\{1 - e^{-t/\tau}\right\}$$

Set switch to position b: $i = \frac{\mathcal{E}}{R}e^{-t/\tau}$

In either case time constant is: $\tau = L/R$

Example



$\mathcal{E} = 30\,V$

$R = 5000\,\Omega$

$L = 15\,mH$

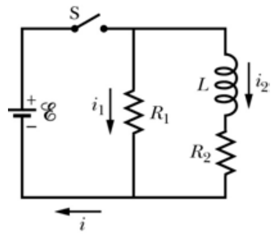
(a) What is the time constant?

$$\tau = L/R = \frac{15\times10^{-3}}{5\times10^{3}} = 3\times10^{-6} = 3\,\mu s$$

(b) What is current after 1 second?

$$i = \frac{\mathcal{E}}{R}\left\{1 - e^{-t/\tau}\right\} = \frac{30}{5000}(1-0) = 6\,mA$$

Example 2:
Problem 30-89



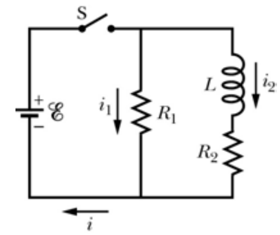
(a) What happens *immediately* after switch is closed?

L prevents sudden change so:

$$i_2 = 0 \quad \therefore \quad i = i_1 = \mathcal{E} / R_1$$

So: $V_{R2} = 0 \quad \therefore \quad V_L = \mathcal{E}$ and $\dfrac{di_2}{dt} = \mathcal{E} / L$

Example 2
continued



(b) What happens a *long time* after switch is closed?

We have reached a steady state so:

$$\frac{di_2}{dt} = 0 \quad \therefore \quad V_L = 0 \quad \text{and} \quad V_{R2} = \mathcal{E}$$

So: $i_1 = \mathcal{E} / R_1$, $i_2 = \mathcal{E} / R_2$, $i = i_1 + i_2$

Induction and Oscillations

Ch. 30: Faraday's Law

Ch. 31: AC Circuits

- **Chapter 30 Homework for Monday:**
  - **Questions 1, 3, 7**
  - **Problems 3, 5, 29, 44**
- **Chapter 31 Homework for Tuesday:**
  - **Questions 3, 4, 7**
  - **Problems 5, 19, 39**
- **WileyPlus chapters 30, 31 for Tuesday.**