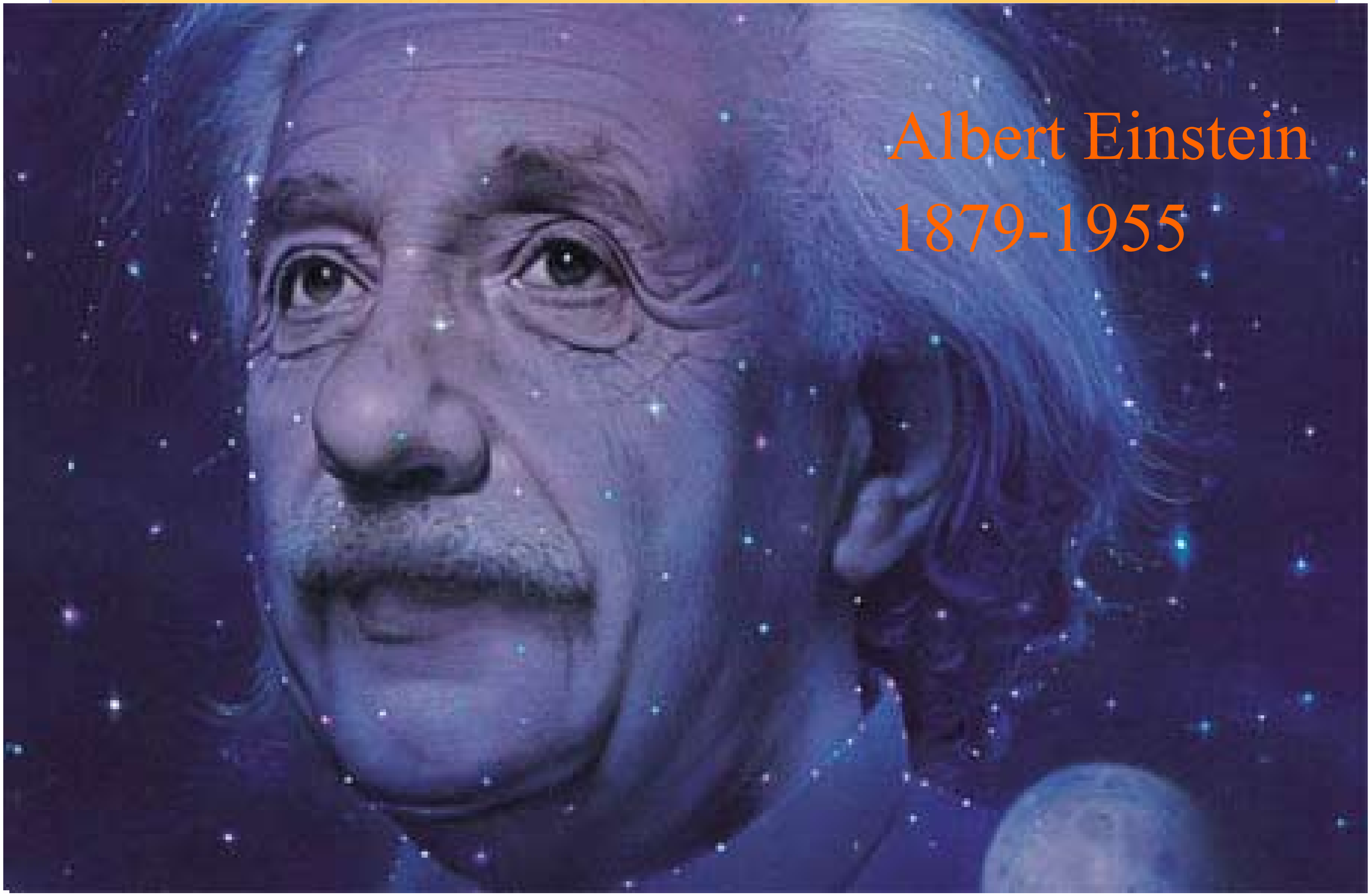
Relativity

TIME 100

PERSON OF THE CENTURY



TIME 100

PERSON OF THE CENTURY

Albert Einstein (1879-1955)

He was the pre-eminent scientist in a century dominated by science. The touchstones of the era--the Bomb, the Big Bang, quantum physics and electronics--all bear his imprint.

1905

1. **First Jazz Band**
2. **Major league debut of Ty Cobb**
3. **Cedar Point hotel: The Breakers**
4. **Wilbur Wright flight of 24 miles**
5. **Einstein's miraculous year.**

Einstein's great year

1. Light energy is discrete: **the photon**.
2. Molecular motion: **atoms** are real.
3. **Relativity**: 4-dimension space-time.
4. Mass-energy equivalence: $\mathbf{E=mc^2}$.
5. Dissertation: size of molecules.

He was 26 years old at the time.

Nobel Prize for (1) in 1921.

World Year of PHYSICS 2005

www.physics2005.org

RELATIVITY

SPECIAL THEORY OF RELATIVITY

High speed motion

Very well-tested

CERTAINLY CORRECT

GENERAL THEORY OF RELATIVITY

Theory of gravity

Large masses and distances

Not very well tested

CORRECT AS FAR AS WE KNOW — BUT COMPETING THEORIES EXIST

(Black Holes ?)

“Everything is relative!?”

Wrong:

Some things which were previously thought to be absolute, we now know to be relative. Some things which were thought to be relative, we now know to be absolute.

But this is not the main thing. The reason we must learn relativity is to get the right equations for high-speed motion.

Galileo 1638, Newton 1687:

The laws of physics are the same for all inertial observers.

Maxwell 1865:

Electromagnetic equations predict light travels at speed ***c*** in vacuum.

Einstein 1905:

Combining these two ***is possible***: the speed of light in vacuum is the same for all observers!

The result is the Special Theory of Relativity.

Inertial Reference Frames

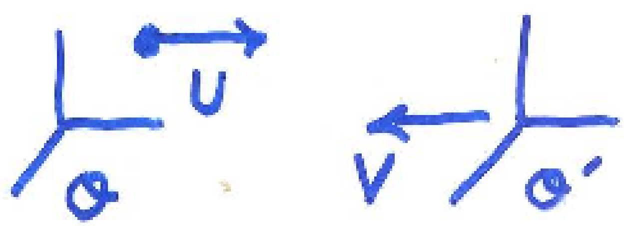
- An inertial reference frame is a coordinate system (x,y,z,t) which is at rest or moving with constant speed in a straight line.
- An inertial observer *O* is a physicist using an inertial reference frame.
- Suppose *O* and *O'* are inertial observers, with *O'* moving at velocity v relative to *O*.
- If *O* and *O'* observe the same events, which measurements to they agree upon?

"Classical" theory (Newton, Galileo) states that all inertial observers will agree on the following:

- The distance between two events (dx)
- The time between two events (dt)
- A free body moves with constant velocity
- The equations of motion (Newton's Laws)

Inertial observers will disagree about velocities, but the relation is "obvious":

$$U' = U + V$$



Special Relativity (Einstein, 1905)

states that all inertial observers will agree on the following:

- A free body moves with constant velocity
- All the laws of physics
- The velocity of light in vacuum (c)
- The "spacetime interval" between events (ds

But they will disagree about:

- The distance between two events (dx)
- The time between two events (dt)
- The velocity of an object

Furthermore the simple "obvious" addition of velocities law must be changed to a more complicated form:

$$u' = \frac{u + v}{1 + uv/c^2}$$

Notice if u,v << c we get familiar u' = u + v

Notice if u=c we get u' = (c+v)/(1+v/c) = c (!)

Where did Einstein get this?

- He saw that there seemed to be a contradiction between his two favorite physicists: Galileo and Maxwell.

- Galileo: the laws of physics are the same for all inertial observers.

- Maxwell: the speed of light is determined by electrodynamics to be $c = 3.0 \times 10^8 \ m/s$.

He worried about this for *10 years*: from age 16 to 26 (1895-1905).

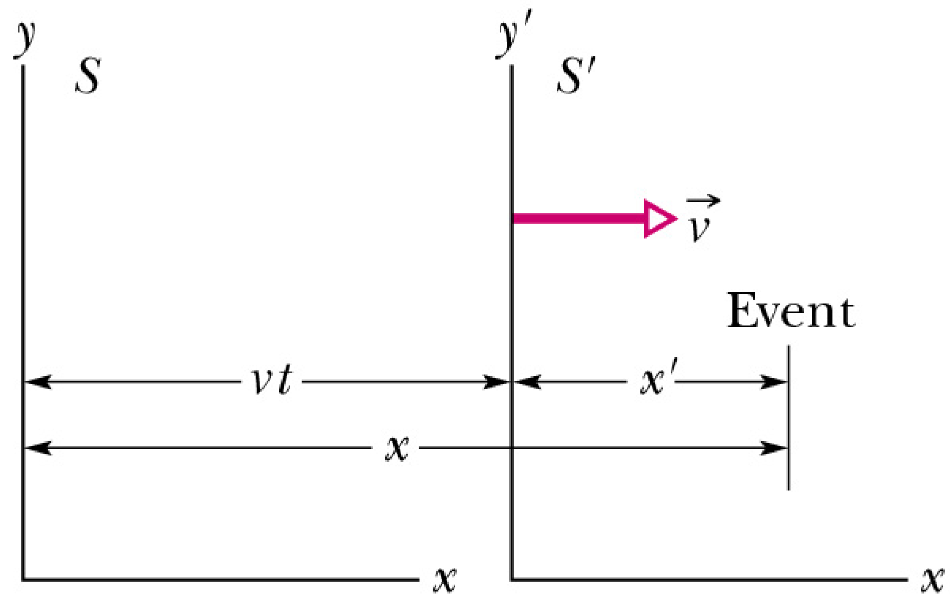
How did he solve the problem?

He found that the old "self-evident" laws for transformations between inertial frames must be replaced by new ones.

Galileo:

$$x' = x - vt$$

$$t' = t$$



Einstein: use the ***Lorentz transformation:***

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - vx/c^2)$$

where $\boxed{\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}}}$

Everything Follows

- Lorentz transformation equations
- Doppler shift for light
- Addition of velocities
- Length contraction
- Time dilation (twin paradox)
- Equivalence of mass and energy ($E=mc^2$)
- Correct equations for kinetic energy
- Nothing can move faster than c

Time Dilation

Comoving Frame:

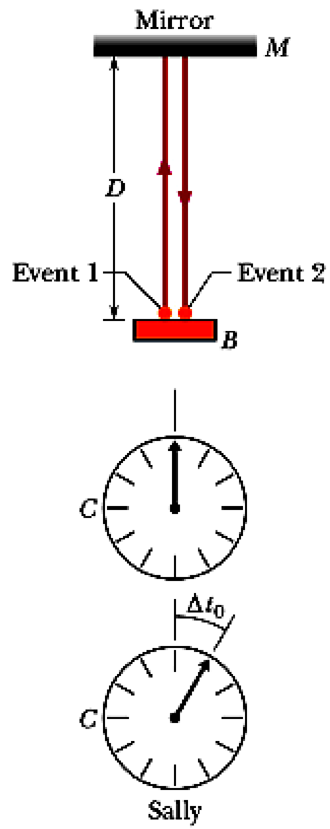
$$\Delta t_0 = 2D / c$$

Lab Frame:

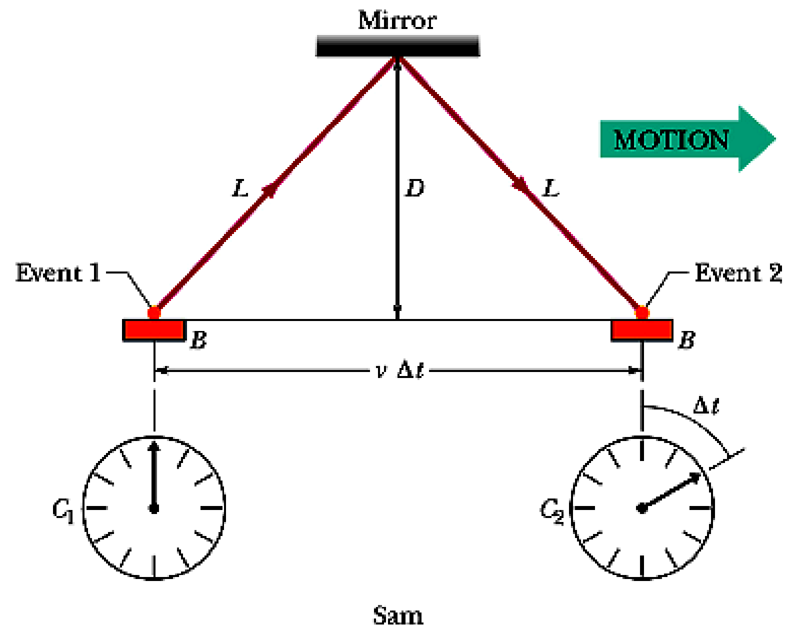
$$\Delta t = 2L / c$$

Pythagoras:

$$D^2 = L^2 - \left(v\Delta t / 2\right)^2$$



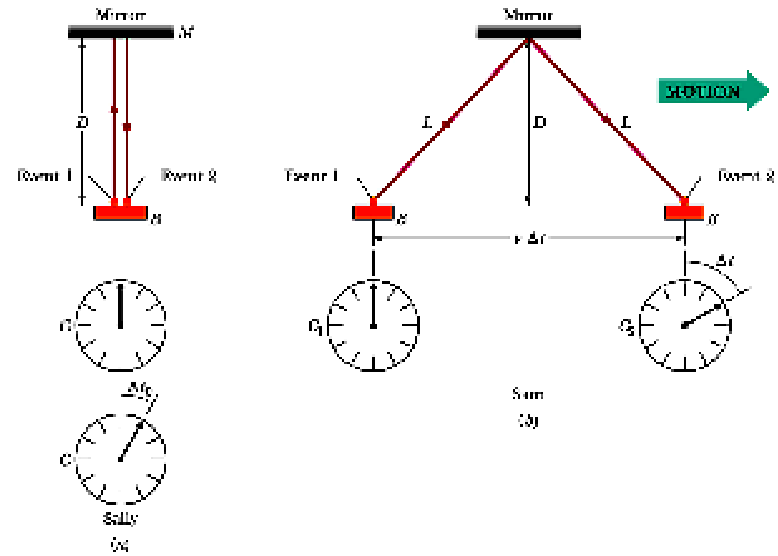
(a)



Moving train: flashbulb, mirror, detector, clock. On ground: two clocks.

"Moving clocks run slow!"

Time Dilation



$$D^2 = L^2 - (v\Delta t/2)^2$$

$$\boldsymbol{(c\Delta t_0/2)^2 = (c\Delta t/2)^2 - (v\Delta t/2)^2}$$

$$\boldsymbol{(\Delta t_0)^2 = (\Delta t)^2 - (v\Delta t/c)^2}$$

So:

$$(\Delta t_0)^2 = (\Delta t)^2\left\{1 - (v/c)^2\right\}$$

$$\Delta t = \Delta t_0 / \sqrt{1-(v/c)^2} = \underline{\gamma\Delta t_0}$$

$$\boxed{\boldsymbol{\Delta t \geq \Delta t_0}}$$

Time Dilation

$$\Delta t = \gamma \Delta t_0 \geq \Delta t_0$$

Time measured in lab (Δt) is greater than proper time Δt_0 (measured by co-moving observer).

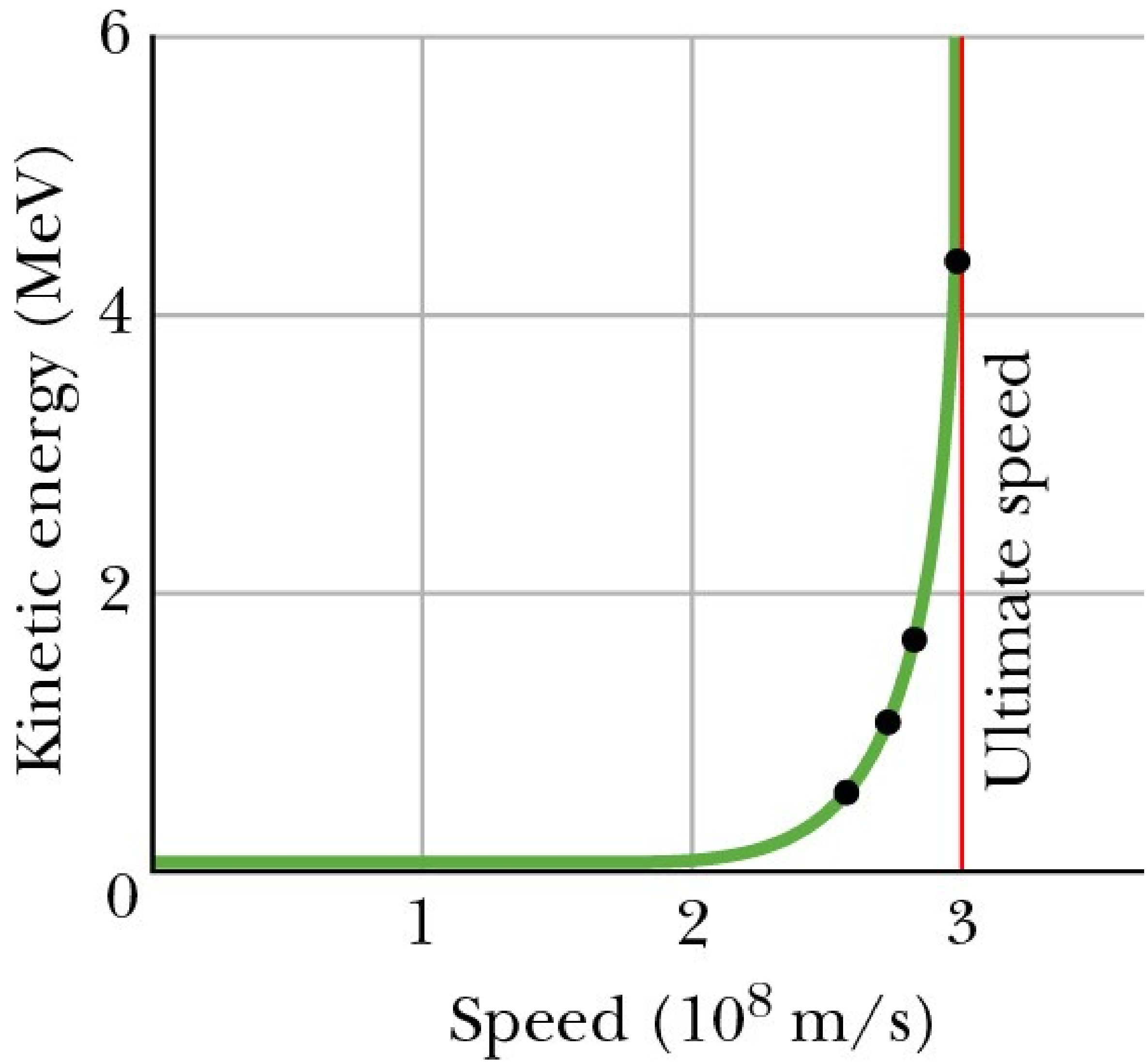
“Moving clocks run slow”.

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \geq 1$$

The Gamma Factor

$$E = \gamma\, mc^2$$

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$



$$c = 3 \times 10^8\ m/s = 300\ km/ms$$

Uses of gamma

- **Time dilation:** $\Delta t = \gamma\, \Delta t_0$
- **Length contraction:** $\Delta x = \Delta x_0 / \gamma$
- **Energy:** $E = \gamma\, mc^2$

Kinetic Energy of a Fast Particle

General relation for ***total*** energy: $$E = \gamma mc^2$$

Rest energy, v=0: $$E = mc^2$$

Kinetic energy: $$K = E - mc^2 = (\gamma - 1)mc^2$$

Momentum: $$p = \gamma mv$$

Relation between momentum and energy: $$E^2 = (mc^2)^2 + (pc)^2$$

Recap

- Relativistic equations involving the "gamma factor" such as $\boldsymbol{E = \gamma m c^2}$ are **essential** for any description of high-speed motion.
- All these equations follow directly from:
  - ***The principle of relativity.***
  - ***The invariance of the speed of light.***
- But the most powerful and elegant ideas involve the ***four-dimensional spacetime continuum.***

The spacetime continuum

Another way of expressing laws of special relativity which is often simpler than using the Lorentz transformation equations.

- Instead of thinking of space and time separately, think of a ***four-dimensional spacetime*.** The "points" in this spacetime are really ***events.***
- Then the "distance" between events is called the ***spacetime interval.***
- Now relativity follows from the fundamental assumption that the ***spacetime interval*** is ***invariant:*** **the same for all inertial observers.**

Doppler shift for light

Frequency shift for motion along the line of sight:

$$f = f_0 \sqrt{\frac{1 \mp v/c}{1 \pm v/c}}$$

Approximation for v << c:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

For motion ***transverse*** to line of sight:

$$f = f_0 \sqrt{1 - (v/c)^2} = f_0 / \gamma$$

Relativity

- **Relativistic Mechanics**
  - Review: Basics of special relativity
  - Review: The gamma factor
  - Kinetic energy and rest energy
  - Examples

Summary of Special Relativity basics

- The laws of physics are the same for all inertial observers (inertial reference frames).
- The speed of light in vacuum is a universal constant, independent of the motion of source and observer.
- The space and time intervals between two events are different for different observers.
- The equations of Newtonian mechanics (Phys. I) are only “non-relativistic” approximations, valid for speeds small compared to speed of light.

Lorentz transformation

Einstein found that the old "self-evident" laws for transformations between inertial frames must be replaced by new ones.

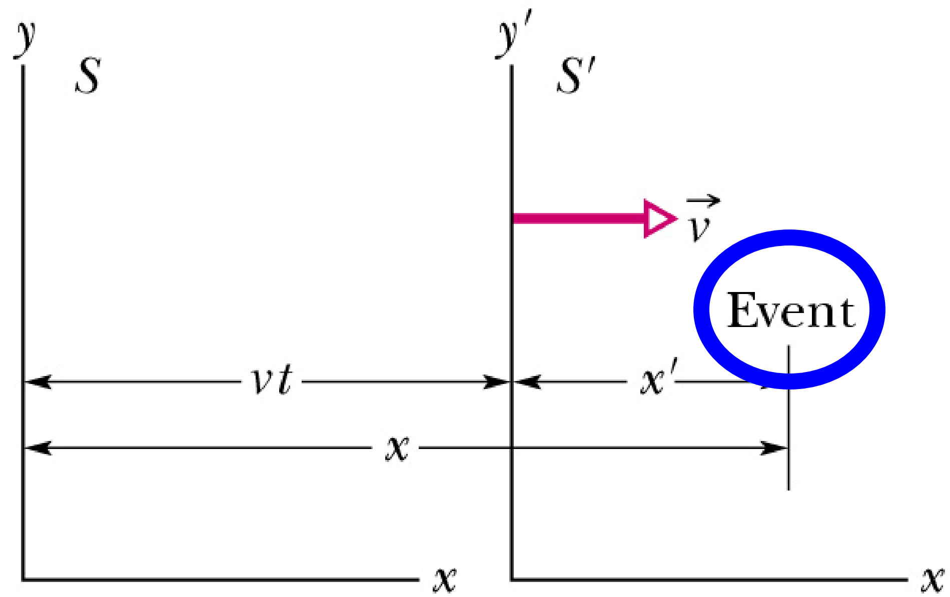
Galileo: $$x' = x - vt$$

$$t' = t$$

Einstein: use the ***Lorentz transformation:***

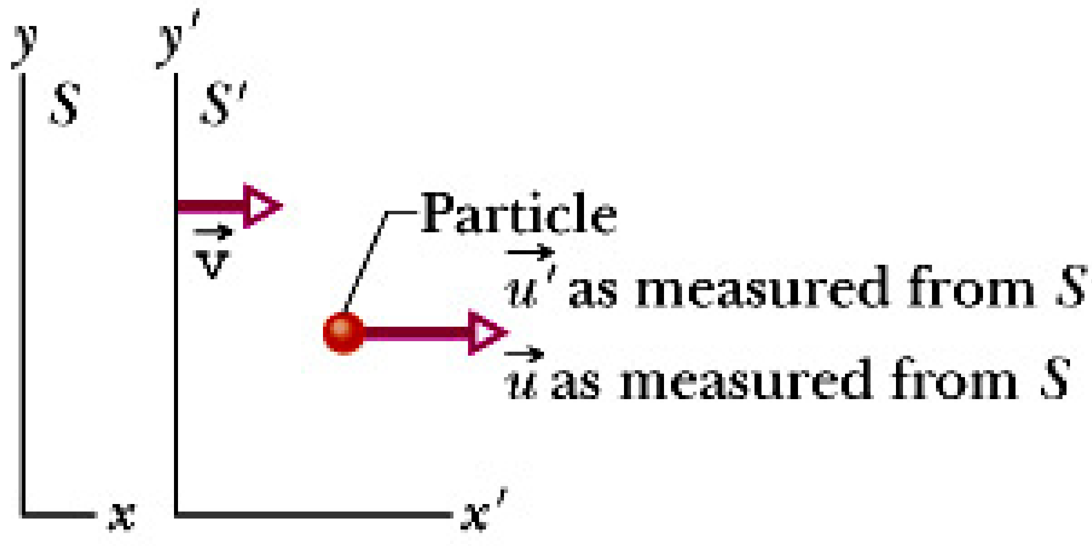
$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - vx/c^2)$$



where $$\boxed{\gamma = \frac{1}{\sqrt{1-(v/c)^2}}}$$

Adding velocities



If *S'* moves with speed *v* relative to *S*, and a particle moves with speed *u'* relative to *S'*, then what is its speed *u* relative to *S*?

"Obvious" answer:

$$\cancel{u = u' + v}$$

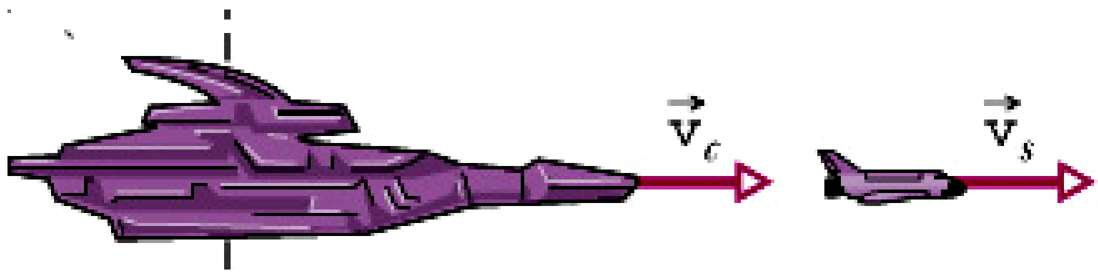
Correct answer:

$$u = (u' + v)/(1 + u'v/c^2)$$

This gives desired result that if $u'=c$, then $u=c$ also, independent of the value of v!

Example

An enemy spaceship approaches the earth at a speed of 0.5c. It fires a torpedo at us, which has a speed of 0.5c relative to the spaceship. What is the torpedo's speed relative to the earth when it hits us?

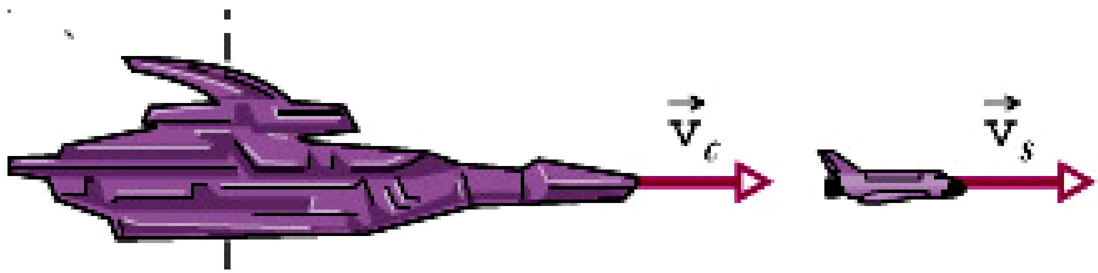


$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{.5c + .5c}{1 + .5 \times .5}$$

$$= \frac{c}{1.25} = \frac{4}{5}c = 0.8c$$

Q.37-1

An enemy spaceship approaches the earth at a speed of 0.5c. It fires an X-ray pulse at us, which has a speed of 1.0c relative to the spaceship. What is the speed of the X-ray pulse relative to the earth when it hits us?



(1) 0 (2) 0.5c (3) 0.8c (4) 1.0c (5) 1.5c

A spaceship approaches the earth at a speed of 0.5c. It fires an X-ray pulse at us, which has a speed of 1.0c relative to the spaceship. What is the speed of the X-ray pulse relative to the earth when it hits us?

(1) 0

(2) 0.5c

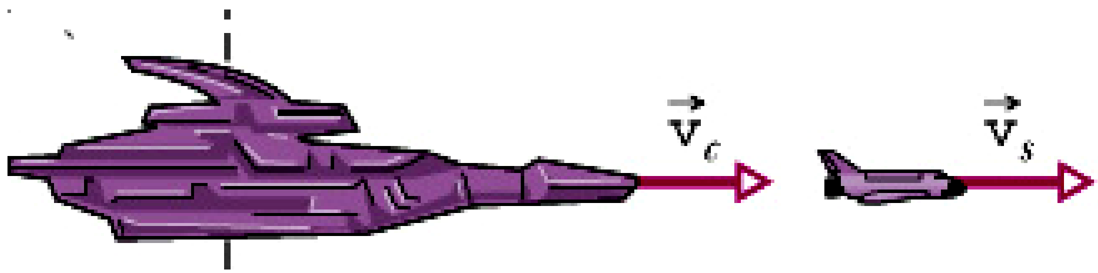
(3) 0.8c

(4) 1.0c

(5) 1.5c

Q.37-1

An enemy spaceship approaches the earth at a speed of 0.5c. It fires an X-ray pulse at us, which has a speed of 1.0c relative to the spaceship. What is the speed of the X-ray pulse relative to the earth when it hits us?



Electromagnetic waves in vacuum always travel at speed *c* independent of motion of source or observer!

(1) 0 (2) 0.5c (3) 0.8c (4) 1.0c (5) 1.5c

Q.37-1 (Alternative solution)

$$u = \frac{v + u'}{1 + u'v/c^2}$$

$$= \frac{.5c + c}{1 + .5 \times 1} = \frac{1.5c}{1.5} = c$$

Kinetic Energy of a Fast Particle

General relation
for ***total*** energy:

$$E = \gamma mc^2$$

Rest energy, ***v=0:***

$$E = mc^2$$

Kinetic energy: $K = E - mc^2 = (\gamma - 1)mc^2$

Momentum: $p = \gamma mv$

Relation between momentum and energy:

$$E^2 = (mc^2)^2 + (pc)^2$$

Q.37-2

An electron is moving with a velocity $v = 0.94c$, which means that it has $\gamma = 3$.

What is its kinetic energy?

(Recall that the electron rest energy mc^2 is about 0.5 MeV.)

(1) 0.17 MeV (2) 0.5 MeV (3) 1.0 MeV (4) 1.5 MeV

Q.37-2

An electron is moving with a velocity v = 0.94c, which means that it has $\gamma = 3$.

What is its kinetic energy?

(Recall that mc^2 is about 0.5 MeV.)

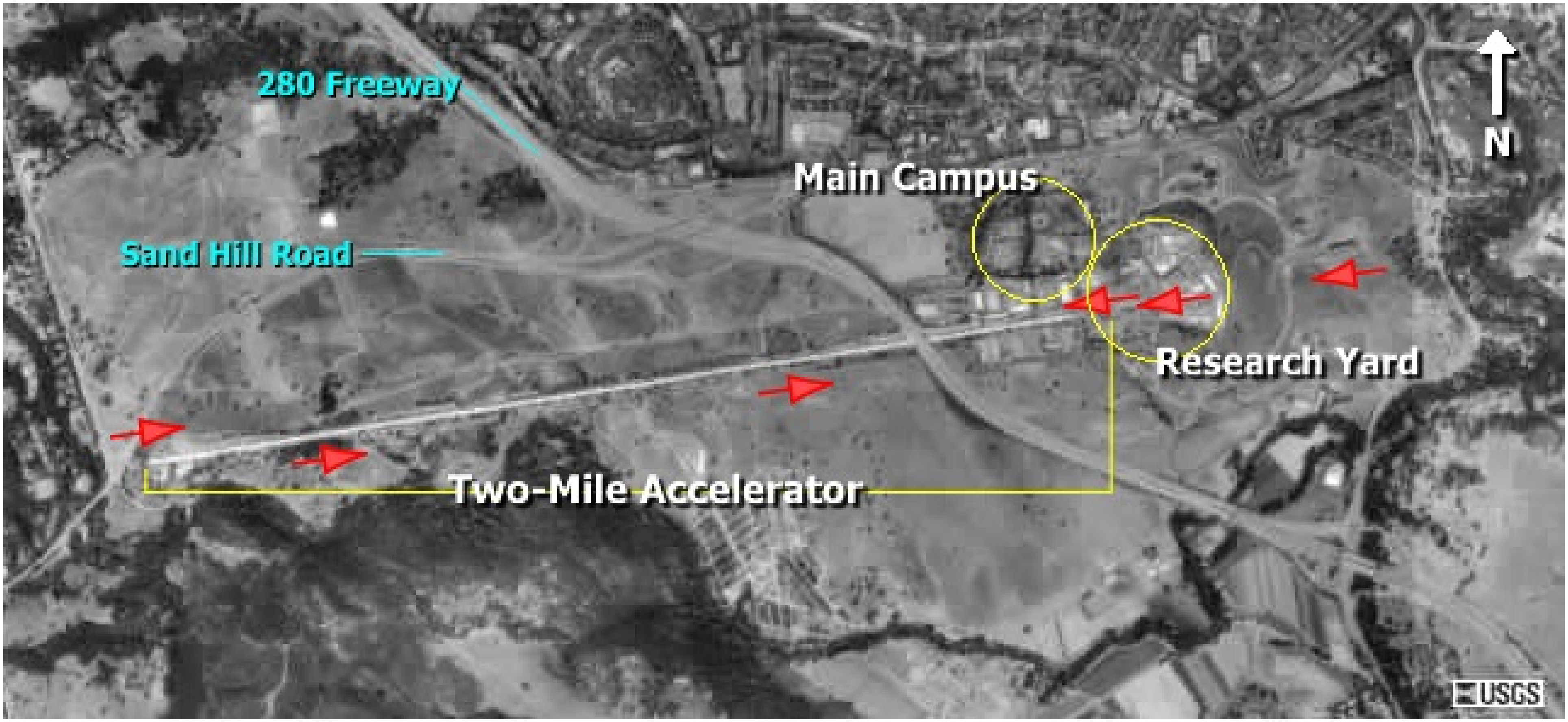
Total energy is $E = \gamma mc^2 = 3 \times 0.5\,MeV$

Kinetic energy is

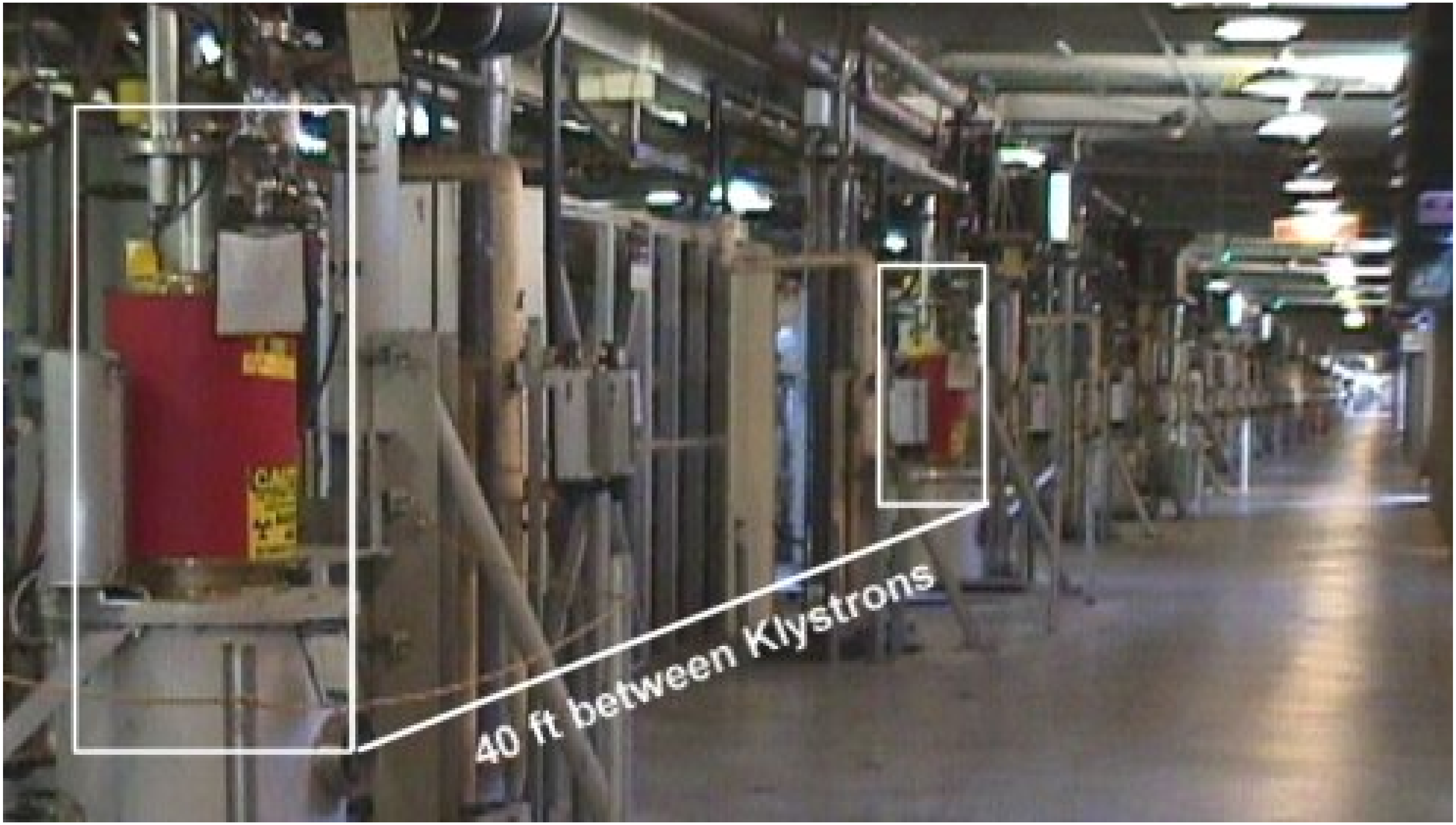
$$K = E - mc^2 = 1.5\,MeV - 0.5\,MeV = 1.0\,MeV$$

(1) 0.17 MeV (2) 0.5 MeV $\boxed{\text{(3) 1.0 MeV}}$ (4) 1.5 MeV

SLAC: Stanford Linear Accelerator



Accelerates electrons for *2 miles*: $E = \gamma\, mc^2 = 20\ GeV$


40 ft between Klystrons

Example: SLAC electron beam

- The Stanford Linear Accelerator (SLAC) accelerates a beam of electrons for two miles.
- The final kinetic energy is 20 GeV.
- ***<u>What is the electrons' final velocity?</u>***

Step 1: find the γ factor

We know $E = \gamma\, mc^2$ so given ***E*,*m*** solve for γ.

$$1\, eV = (1V) \times (e) = 1.6 \times 10^{-19}\ J$$

$$20\ GeV = 20 \times 10^{9} \times 1.6 \times 10^{-19} = 3.2 \times 10^{-9}\ J$$

$$m = 9.1 \times 10^{-31}\ kg$$

So $$\gamma = \frac{E}{mc^2} = \frac{3.2 \times 10^{-9}}{9.1 \times 10^{-31} \times 9.0 \times 10^{16}} = 3.9 \times 10^{4}$$

Step 1 (cont'd)

Actually nobody would do it that way.

Don't use mass in *kg* and energy in *J*.

$$E = 20\ GeV = 2\times10^4\ MeV \quad and \quad mc^2 = 0.51\ MeV$$

$$\gamma = \frac{E}{mc^2} = \frac{2\times10^4}{0.51} = 3.9\times10^4$$

Solution

Now that we have γ we can solve for v.

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \quad \text{so} \quad \begin{aligned} 1-(v/c)^2 &= 1/\gamma^2 \\ v/c &= \sqrt{1-1/\gamma^2} \end{aligned}$$

$$v/c = \sqrt{1-1/(3.9\times10^4)^2} = \sqrt{1-6.6\times10^{-10}}$$

$$= \sqrt{0.999999934} = \underline{0.999999967}$$

Example: "Newtonian" SLAC

- SLAC gives an electron an energy of 20 GeV by providing a constant force acting over a distance of about 2 miles, or about 3 km.
- What work is done during one meter of flight?
- Using ***Newtonian*** kinetic energy formula, how long would the acceleration tube need to be to bring the electrons to the speed of light?

Newtonian SLAC Solution

Work = Force × Distance

Acceleration for 3 km: $\mathbf{20\ \mathit{GeV} = 3.2 \times 10^{-9}\ \mathit{J}}$

Acceleration for 1 m: $$\mathbf{\frac{3.2 \times 10^{-9}}{3000} \cong 1 \times 10^{-12}\ \mathit{J}}$$

Required to reach ***v*=*c*** using $\boldsymbol{W = K = \frac{1}{2} m v^2}$

$$\boldsymbol{W = \frac{1}{2} m c^2 = 0.5 \times 9.1 \times 10^{-31} \times (3 \times 10^{8})^2 \cong 4 \times 10^{-14}\ J}$$

Distance needed is: $$\boldsymbol{\frac{4 \times 10^{-14}}{1 \times 10^{-12}} \cong 4 \times 10^{-2}\ m = \underline{4\ cm}}$$

Error in Newtonian Solution

Newtonian answer for the distance needed is:

$$4 \times 10^{-2}\ m = 4\ cm$$

Distance needed in real life is:

$$3\ km$$

Newtonian error factor is:

$$\frac{3\ km}{4\ cm} = \frac{3 \times 10^{3}\ m}{4 \times 10^{-2}\ m} = .75 \times 10^{5} = \underline{75000}$$

Review: Kinetic Energy

General relation for ***total*** energy: $$\boldsymbol{E = \gamma mc^2}$$

***Rest energy**, v=0:* $$\boldsymbol{E = mc^2}$$

Kinetic energy: $$\boldsymbol{K = E - mc^2 = (\gamma - 1)mc^2}$$

Momentum: $$\boldsymbol{p = \gamma mv}$$

Relation between momentum and energy: $$\boldsymbol{E^2 = (mc^2)^2 + (pc)^2}$$

Exact vs non-relativistic calculations

Last time we saw that for the Stanford LINAC, a nonrelativistic calculation was terribly wrong (3 cm vs 2 miles).

That was a case where $K >> mc^2$. (ER case)

Now let's look at the case $K << mc^2$. (NR case)

Example: He beam from THIA

K = 300keV   v = ?

$$mc^2 = 4 \times 1\, GeV = 4 \times 10^9\ eV \gg K$$

So non-relativistic calculation should be OK.

$$K = \frac{1}{2} m v^2 \rightarrow \quad v = \sqrt{\frac{2K}{m}}$$

$$\frac{v}{c} = \sqrt{\frac{2K}{mc^2}} = \sqrt{\frac{6 \times 10^5\ eV}{4 \times 10^9\ eV}} = \sqrt{1.5 \times 10^{-4}} = .01224745$$

$$v = .0122c = 3.67 \times 10^6\ m/s = \underline{3.67\ mm/ns}$$

Compare with exact relativistic answer

$$K = (\gamma - 1)mc^2$$

$$\gamma = 1 + \frac{K}{mc^2} = 1 + \frac{3\times 10^5\ eV}{4\times 10^9\ eV} = 1 + 7.5\times 10^{-5}$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

$$\frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{(1.000075)^2}} = \sqrt{1-0.99985} = .01224676$$

Compare with non-relativistic approximation:

$$\frac{1224745 - 1224676}{1224676} = 6\times 10^{-5} = .006\ \%\ \text{error}$$

Q.37-3

An electron ($mc^2 = 0.5$ MeV) moves with a speed $v = 0.94c$ so that $\gamma = 3$.

What is its kinetic energy?

1. **0.1 MeV**
2. **0.5 MeV**
3. **1.0 MeV**
4. **2.0 MeV**
5. **5.0 MeV**

Q.37-3

$mc^2 = 0.5$ MeV, $\gamma = 3$: K = ?

$$K = E - mc^2 = (\gamma - 1)mc^2$$

$$= (3-1)\,0.5\ MeV = 1.0\ MeV$$

1. 0.1 MeV
2. 0.5 MeV
3. $\boxed{\text{1.0 MeV}}$
4. 2.0 MeV
5. 5.0 MeV

Time Dilation

A lab observer compares two stationary clocks against a clock moving with speed v, as it passes first one then the other. Lab clocks give Δt, moving clock Δt_0.

$$\Delta t = \gamma \Delta t_0 \geq \Delta t_0$$

Time measured in lab (Δt) is greater than proper time Δt_0 measured by co-moving observer.

"Moving clocks run slow".

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \geq 1$$

Example: Problem 37-21

A clock moves along the x axis at speed v = 0.6c and reads zero as it passes the origin. What time does the clock read as it passes x = 180 m?

Lab time: $$\Delta t = \frac{x}{v} = \frac{180\ m}{.6 \times 3 \times 10^8\ m/s} = 1\ \mu s$$

Gamma factor: $$\gamma = \frac{1}{\sqrt{1-.6^2}} = \frac{1}{.8} = 1.25$$

Proper time: $$\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{1\ \mu s}{1.25} = \underline{0.8\ \mu s}$$

Q.37-4

A clock moves relative to a laboratory, at speed v such that γ=5. During the time taken for the moving clock to advance 10 ns, how much time elapses according to the lab clocks?

(1) 0.5 ns (2) 2 ns (3) 5 ns (4) 10 ns (5) 50 ns

Q.37-4

A clock moves relative to a laboratory, at speed v such that γ=5. During the time taken for the moving clock to advance 10 ns, how much time elapses according to the lab clocks?

Solution:

$$\Delta t = \gamma \Delta t = 5 \times 10\ ns = 50\ ns$$

(1) 0.5 ns (2) 2 ns (3) 5 ns (4) 10 ns $\boxed{\text{(5) 50 ns}}$

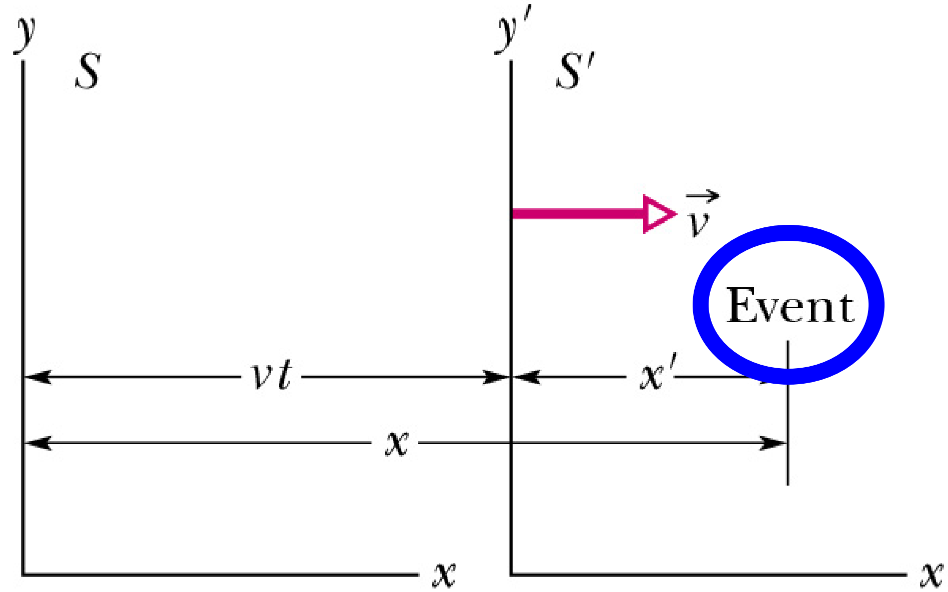
Lorentz transformation

Einstein found that the old "self-evident" laws for transformations between inertial frames must be replaced by new ones.

Galileo:

$$x' = x - vt$$

$$t' = t$$



Einstein: use the

Lorentz transformation:

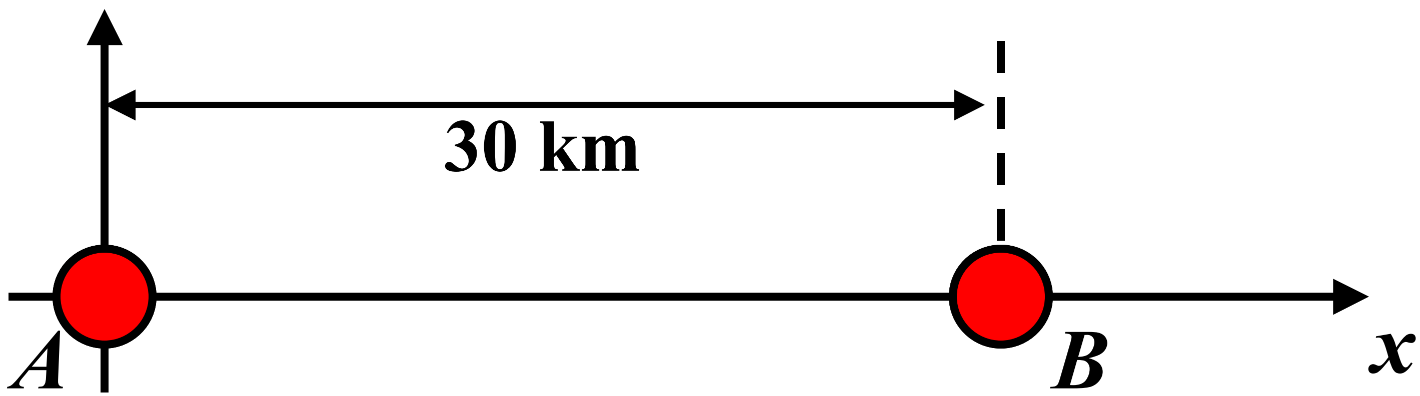
$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - vx/c^2)$$

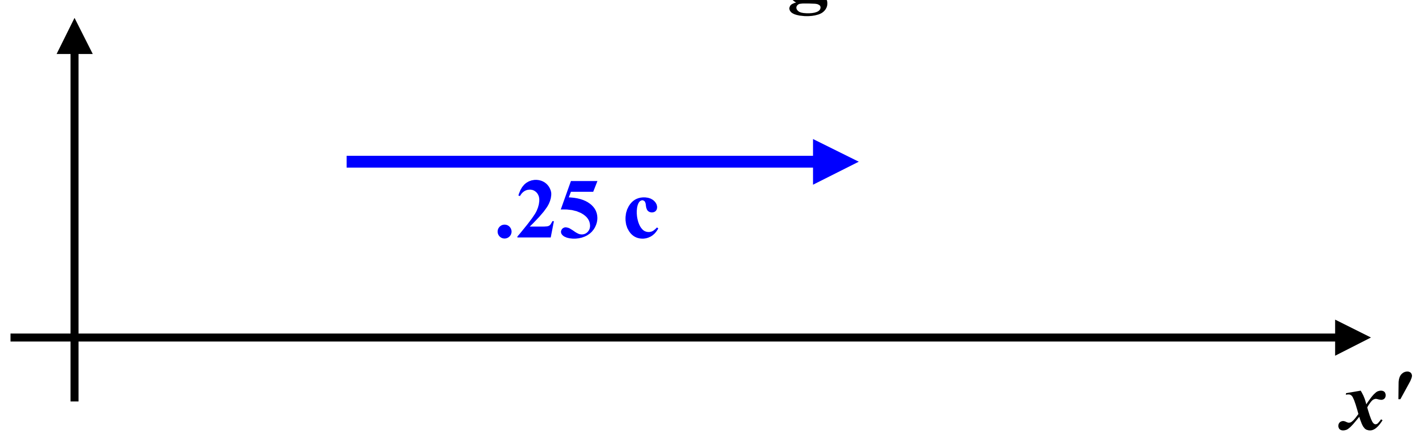
where $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}}$

Example: Problem 37-19

Two flashbulbs triggered simultaneously.



Also viewed from moving frame.



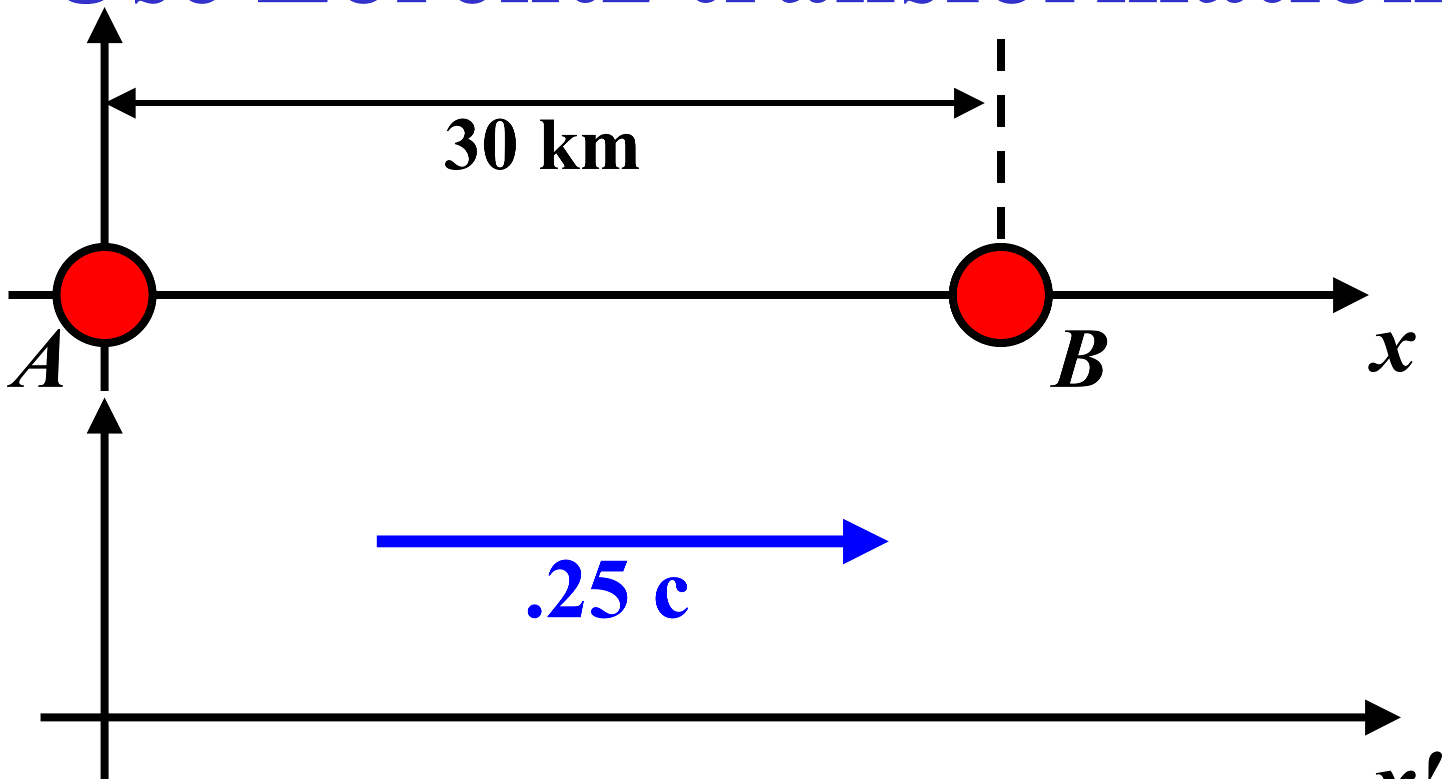
Use Lorentz transformation

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - vx / c^2)$$

For each event, knowing x,t, calculate x',t'.

Use Lorentz transformation



Coordinates in lab frame:

$$t_A = t_B = 0, \qquad x_A = 0, \qquad x_B = L = 30\ km$$

Coordinates in moving frame:

$$t'_A = 0, \qquad x'_A = 0, \qquad t'_B = ?$$

Example continued

Use Lorentz transformation equation for time of event B in moving frame.

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(.25)^2}} = 1.0328$$

$$t'_B = \gamma\left(t_B - vx_B/c^2\right) = -\gamma vL/c^2$$

$$= -\frac{1.0328 \times 0.25 \times 3 \times 10^4\ m}{3 \times 10^8\ m/s} = \underline{-25.8\ \mu s}$$

But t'_A=0. So B happens before A as seen in moving frame!

Example summarized

Times in lab frame:

$$t_A = t_B = 0$$

Times in moving frame:

$$t'_A = 0 \qquad t'_B = -25.8\ \mu s$$

Events are not simultaneous in moving frame.

B is first, then A is 25.8 microseconds later.

The spacetime interval

- **Our text doesn't stress this point, but there is another way of expressing laws of special relativity which is often simpler than using the Lorentz transformation equations or going back to the "two postulates".**

- **This is the *invariant spacetime interval.***

The spacetime continuum

Another way of expressing laws of special relativity which is often simpler than using the Lorentz transformation equations.

- **Instead of thinking of space and time separately, think of a *four-dimensional spacetime*. The "points" in this spacetime are really *events.***
- **Then the "distance" between events is called the *spacetime interval.***
- **Now relativity follows from the fundamental assumption that the *spacetime interval* is *invariant:* the same for all inertial observers.**

The spacetime interval

Given two ***events*** (x_1,t_1) and (x_2,t_2).

As seen by inertial observer ***O*** they are separated by intervals

$$\Delta x = x_2 - x_1$$

$$\Delta t = t_2 - t_1$$

As seen by another observer ***O'*** these intervals are different (***relative***).

$$\Delta x' \neq \Delta x$$

$$\Delta t' \neq \Delta t$$

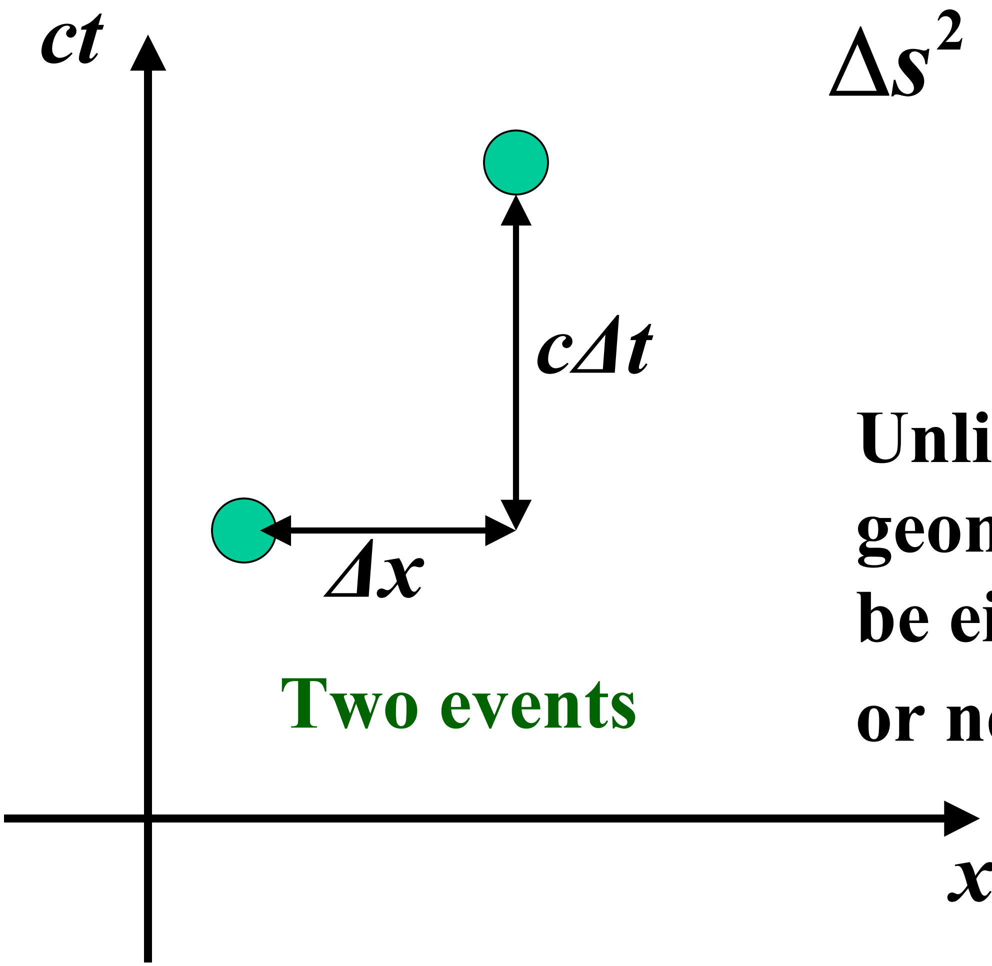
But the **space-time interval** is the same (***invariant***):

$$\boxed{\Delta s^2 = c^2 \Delta t^2 - \Delta x^2}$$

$$\Delta s' = \Delta s$$

The spacetime diagram

An alternative to the Lorentz transformation equations is the invariant spacetime interval:



$$\Delta s^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$\underline{\Delta s' = \Delta s}$$

Unlike ordinary (Euclidean) geometry, this interval can be either positive (timelike) or negative (spacelike).

Derive time dilation using Δs' = Δs.

Example

Two events occur at the same place: $\Delta x = 0, \quad \Delta s = c\Delta t$

Same events seen by observer moving with speed v:

$$\Delta x' = v\Delta t', \quad (\Delta s')^2 = c^2(\Delta t')^2 - (\Delta x')^2$$

$$(\Delta s')^2 = c^2(\Delta t')^2 - v^2(\Delta t')^2$$

$$\Delta s' = c\Delta t'\sqrt{1-(v/c)^2}$$

So now use invariance of spacetime interval:

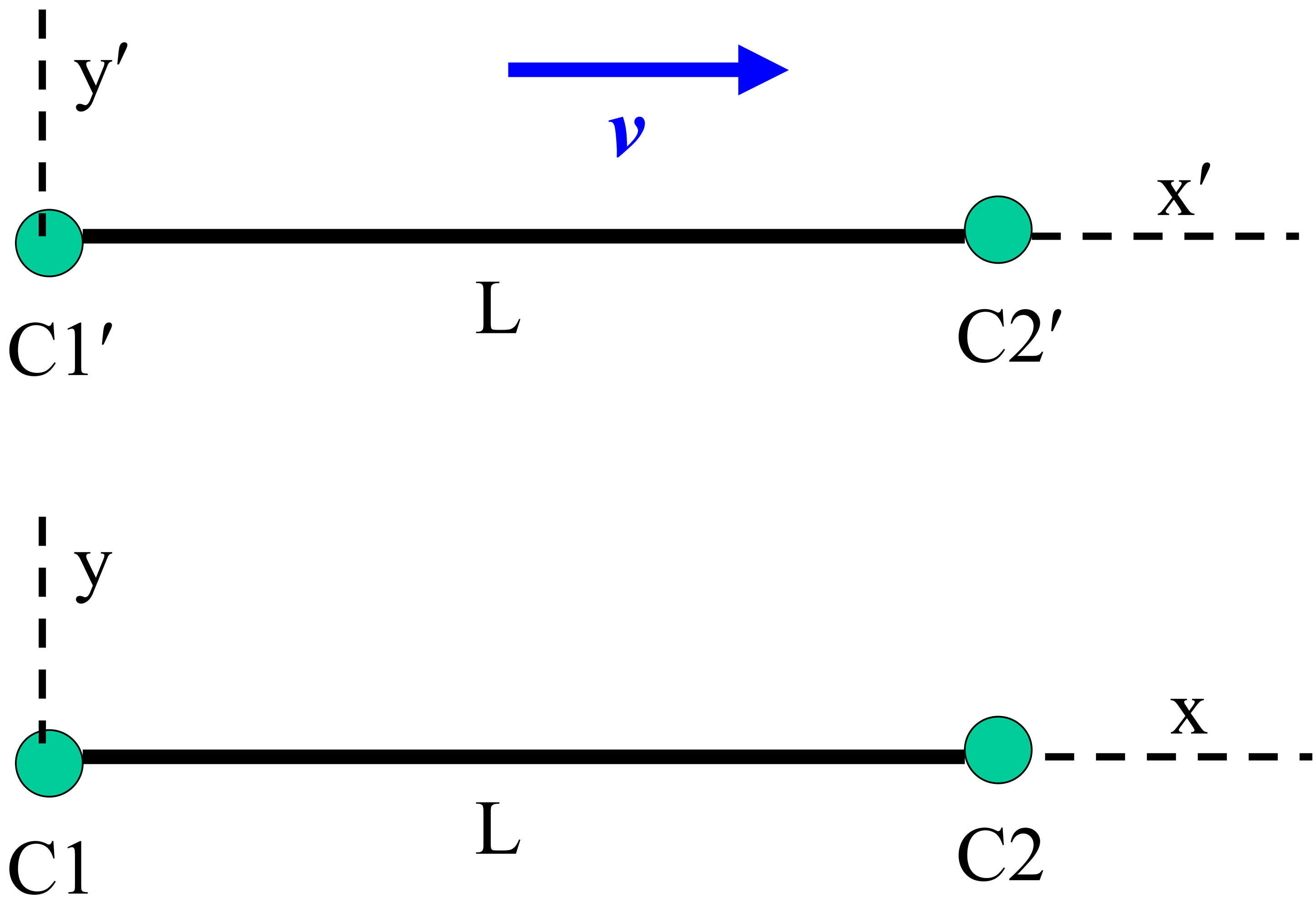
$\Delta s' = \Delta s$ gives $\Delta t' = \Delta t / \sqrt{1-(v/c)^2} = \underline{\gamma\Delta t}$

All Observers are Equivalent

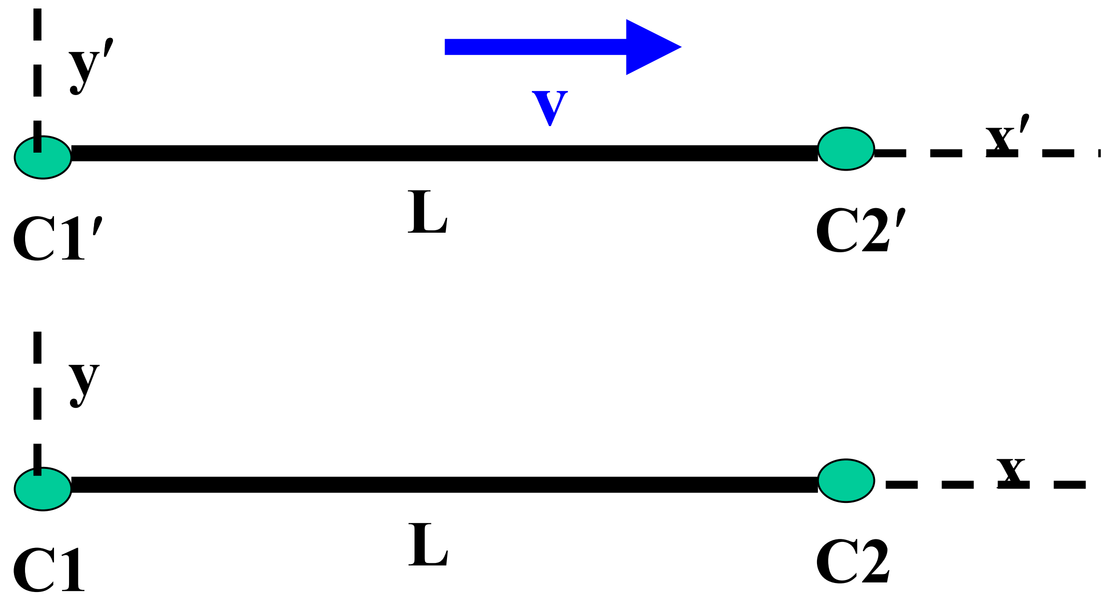
- **In a moving reference frame, lengths contract, time dilates. "Moving rods are short and moving clocks are slow!"**
- **How is this possible if all inertial observers are equivalent?**
- **If B moves with speed v relative to A, then B's clocks are slow as measured by A.**
- **But according to B, it's A that's moving, so A's clocks are slow as measured by B.**
- **Is this possible?**
- **Yes, because they also disagree about how the clocks are originally synchronized.**
- **A careful analysis shows that the effect is perfectly symmetrical: *"Each observer thinks the other is using slow clocks!"***

Simple Problem to Show Symmetry

Observers O and O' with relative speed v.



Use invariant spacetime interval



- **Four events:** **(a)** C1′ passes C1, **(b)** C1′ passes C2, **(c)** C2′ passes C1, **(d)** C2′ passes C2
- **For each *pair of events* figure out the invariant interval** $\Delta s^2 = c^2 \Delta t^2 - \Delta x^2$
- **Now apply the invariance** $\Delta s' = \Delta s$

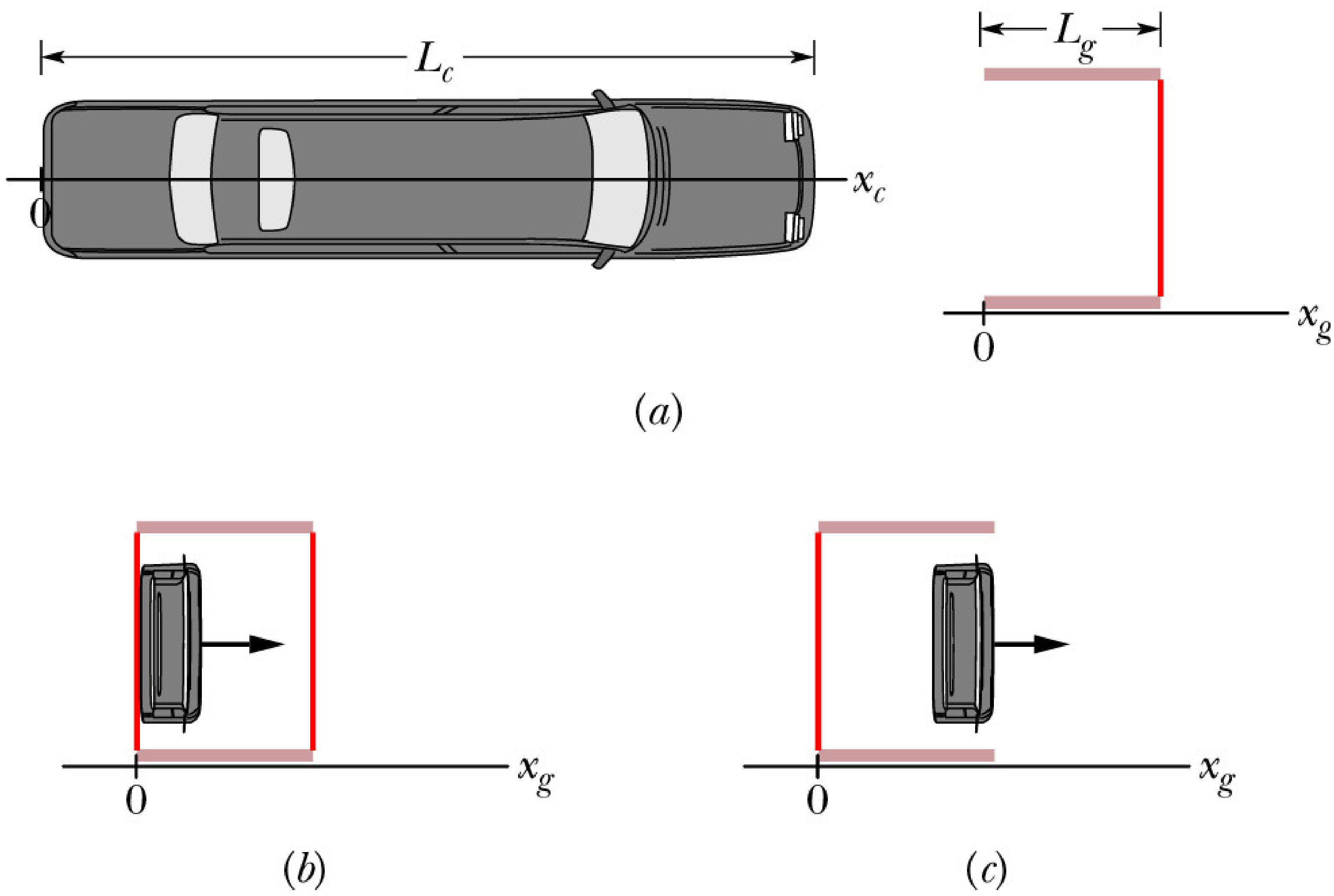
Result: Find each observer thinks the other's clocks are slow ***and improperly synchronized!***

Relativity III

- **Today:**
  - Time dilation examples
  - The Lorentz Transformation
  - Four-dimensional spacetime
  - The invariant interval
  - Examples
- **Exam Tomorrow Chapters 33-37**

Example: Problem 37-65

Can a long limo fit in a short garage temporarily?



37-65 (cont'd)

(a) Length of car according to Garageman

$$L'_c = \frac{L_c}{\gamma} = L_c\sqrt{1-\beta^2} = (30.5\,\mathrm{m})\sqrt{1-(0.9980)^2} = 1.93\ \mathrm{m}\ .$$

(b) Coordinates of event 2 according to Garageman

$$\Delta t_g = t_{g2} - t_{g1} = \frac{L_g - L'_c}{v} = \frac{6.00\,\mathrm{m} - 1.93\,\mathrm{m}}{0.9980(2.998\times 10^8\,\mathrm{m/s})} = 1.36\times 10^{-8}\ \mathrm{s}\ .$$

(c) Car spends this time inside according to G.

(d) Length of garage according to Carman

$$L'_g = \frac{L_g}{\gamma} = L_g\sqrt{1-\beta^2} = (6.00\,\mathrm{m})\sqrt{1-(0.9980)^2} = 0.379\ \mathrm{m}\ .$$

37-65 (cont'd)

(e) Time between events according to Carman

$$\Delta t_c = t_{c1} - t_{c2} = \frac{L_c - L'_g}{v} = \frac{30.5\,\mathrm{m} - 0.379\,\mathrm{m}}{0.9980(2.998\times 10^8\,\mathrm{m/s})} = 1.01\times 10^{-7}\,\mathrm{s}\,.$$

$$But\ t_{C1} = 0 \quad so\ t_{C2} = -1.01\times 10^{-7}\ s$$

So to Carman, event 2 occurs first, and the car is never entirely inside the garage.