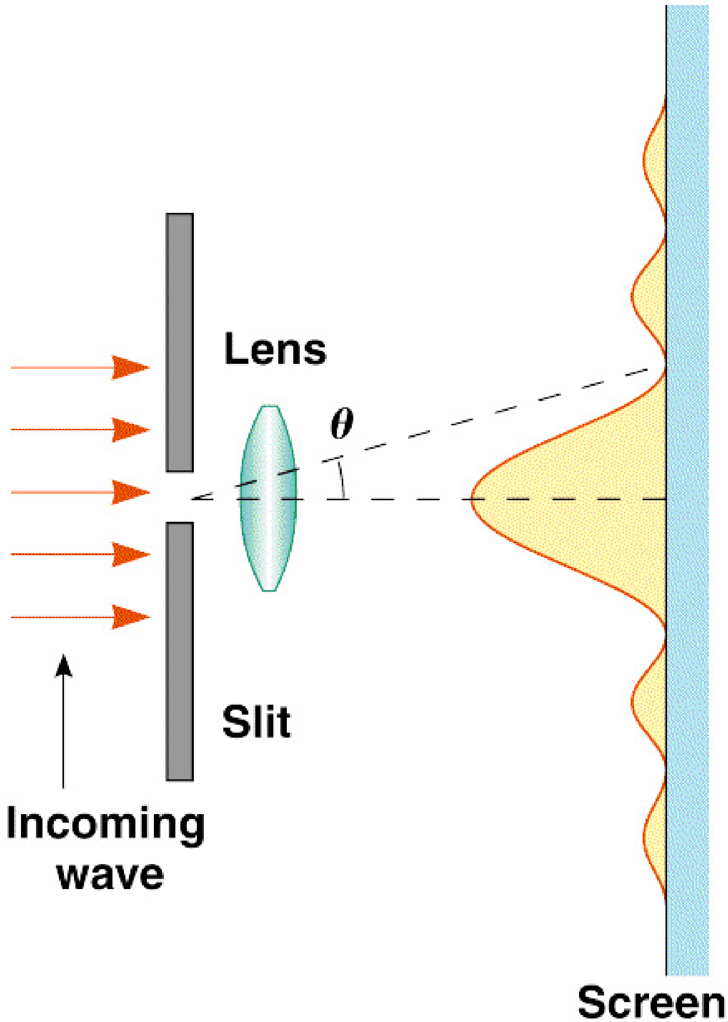
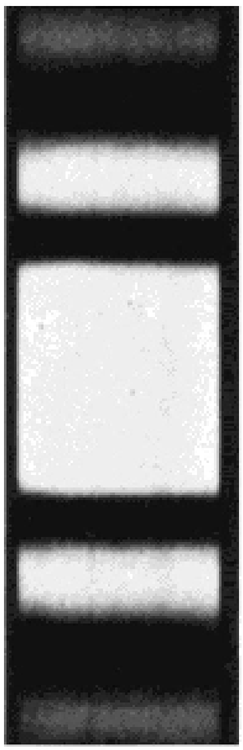
Diffraction

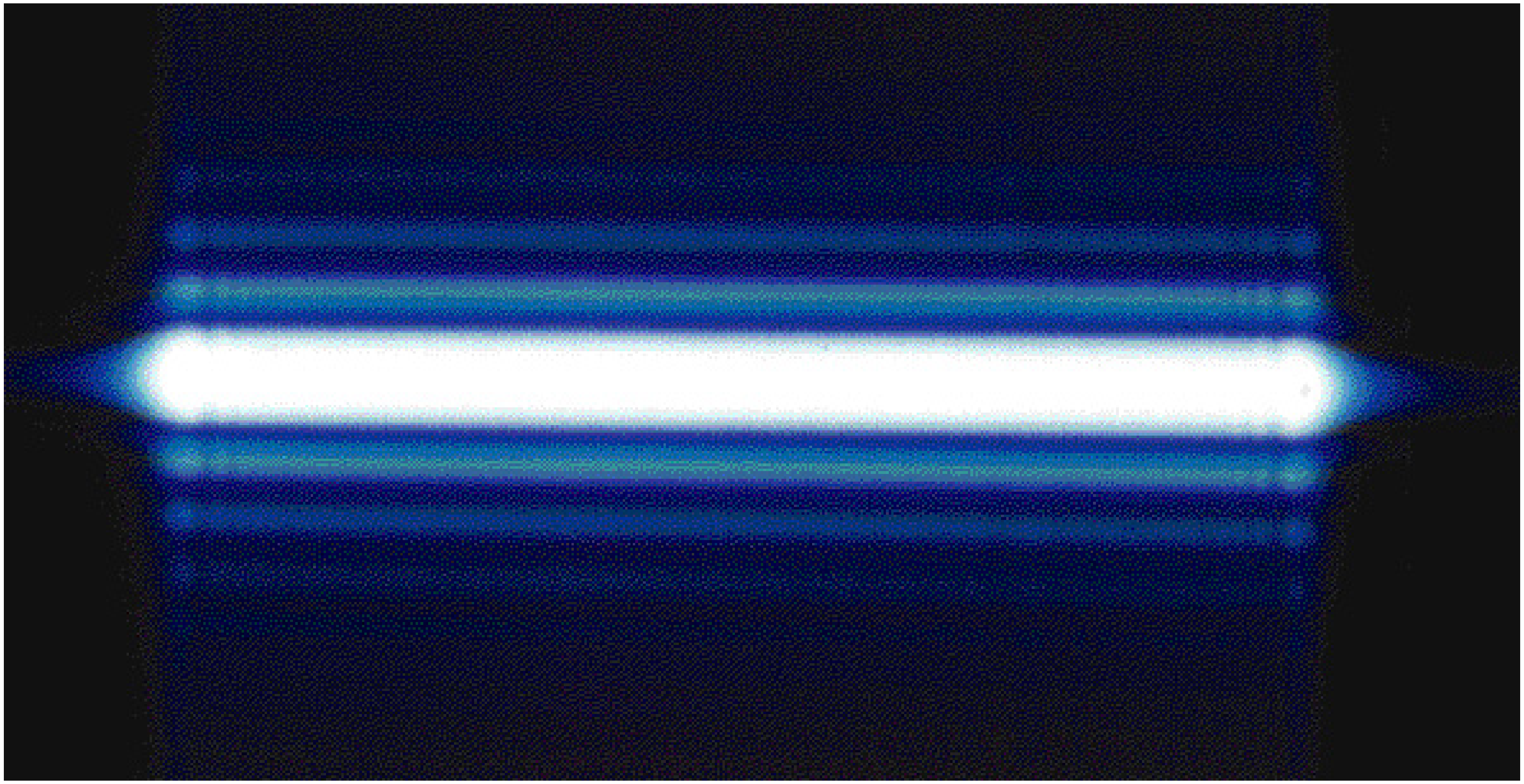
- **Today**
  - Single-slit diffraction
  - Diffraction by a circular aperture
  - Use of phasors in diffraction
  - Double-slit diffraction

Diffraction by a single slit





Single slit: Pattern on screen

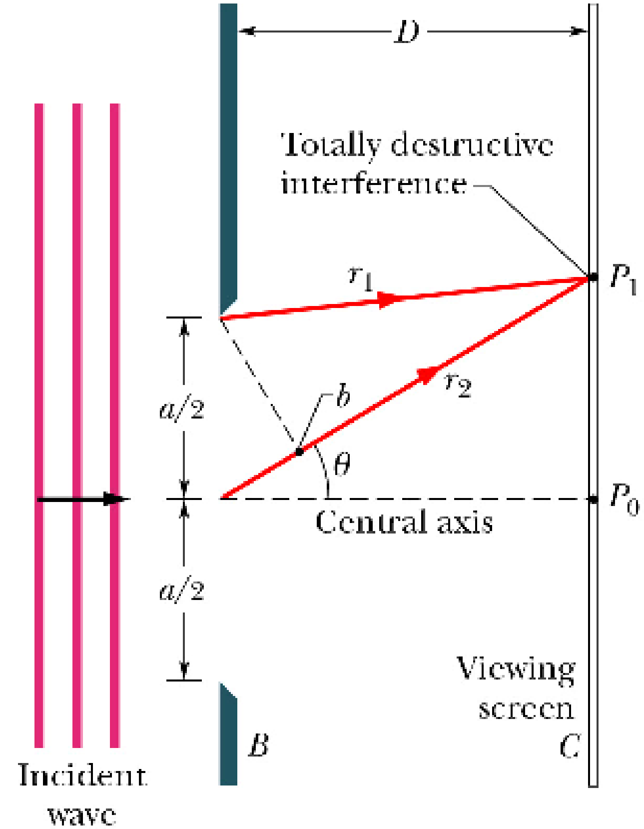


Bright and dark fringes appear behind a single <u>very thin</u> slit.

As the slit is made <u>narrower</u> the pattern of fringes becomes <u>wider.</u>

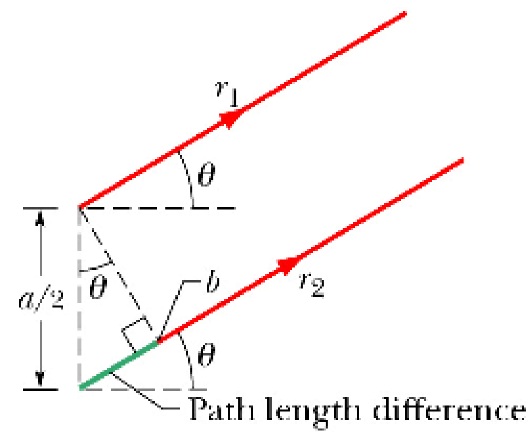
Single Slit: finding the minima

Divide the single slit into many tiny regions. If the *top ray* and the *middle ray* interfere *destructively*, then every pair of rays will also.



(a)

So each pair cancels each other and we have total cancellation!



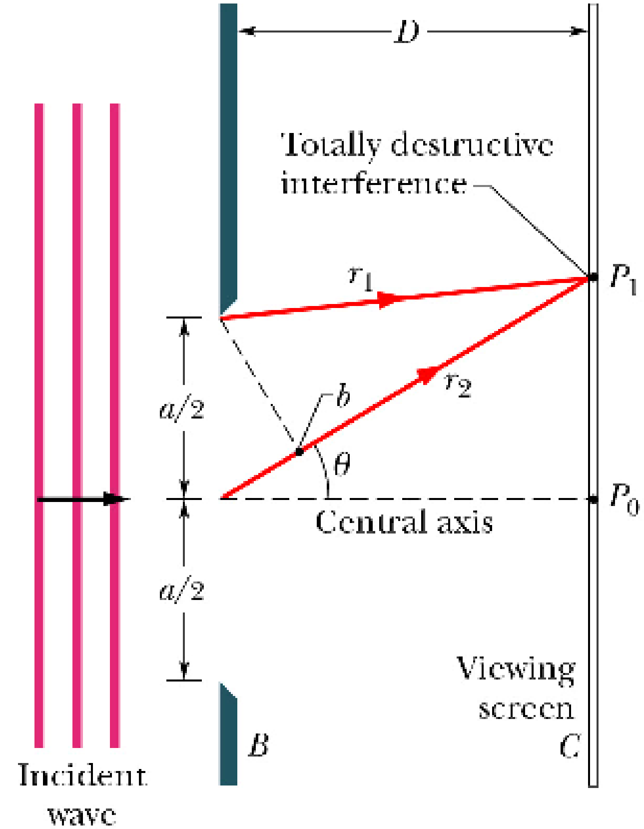
(b)

Single Slit:
the first minimum

So we get ***destructive*** interference if:

$$(a/2)\sin\theta = (\lambda/2)$$

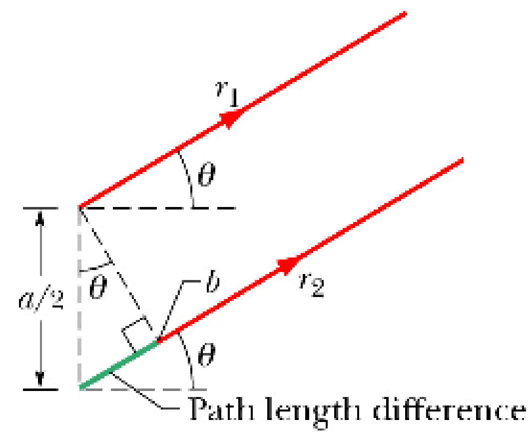
$$\boxed{a\sin\theta = \lambda}$$



(a)

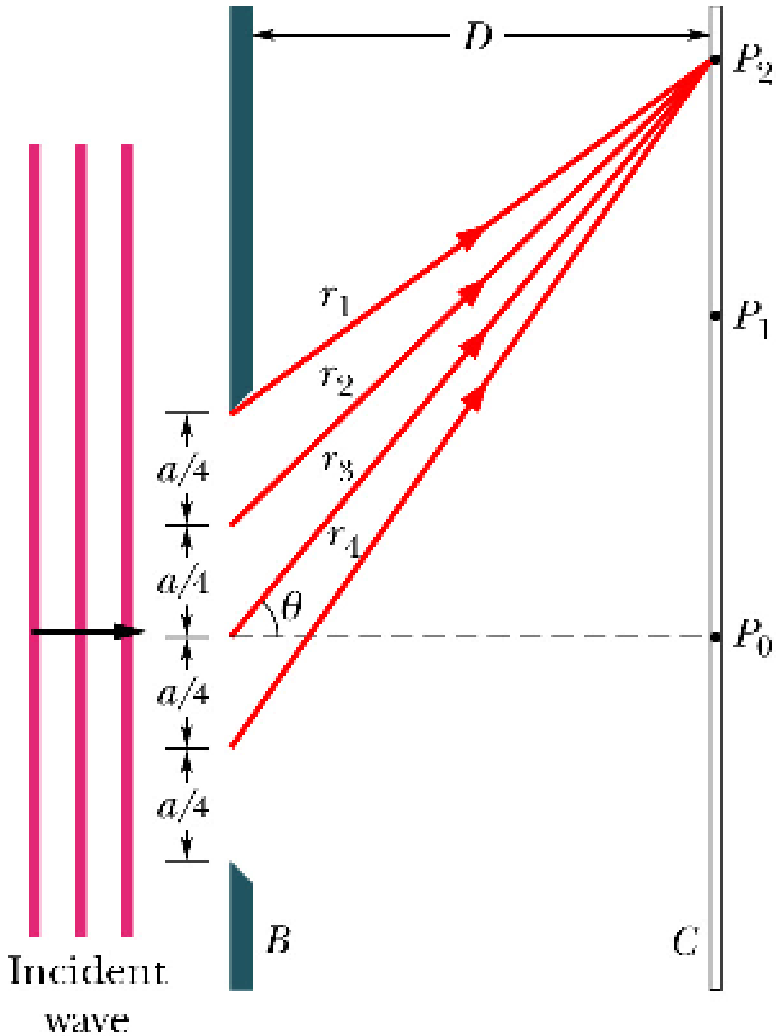
Note this mustn't be confused for the condition for *constructive* interference in the two-slit case!

Also note as slit width gets smaller, angle gets larger, and vice versa.



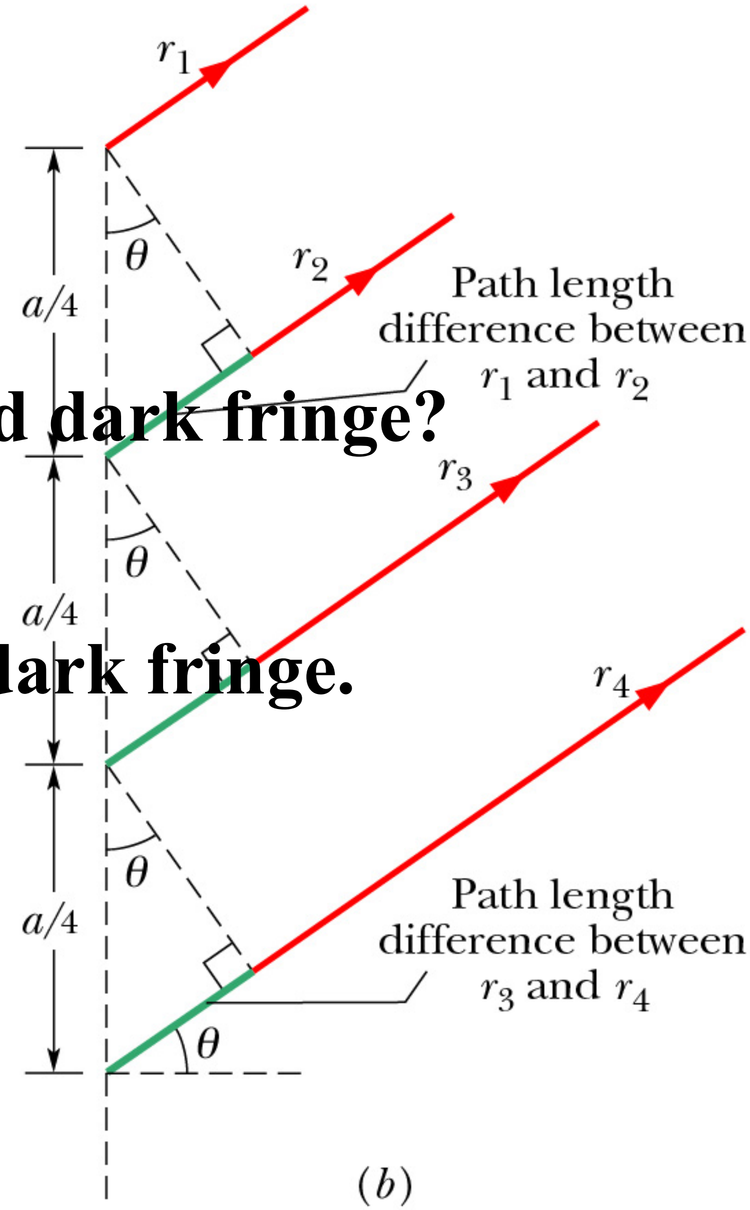
(b)

Dark Fringes in Diffraction



Second dark fringe?

First dark fringe.



Dark fringes

First is at $\underline{a \sin\theta = \lambda}$

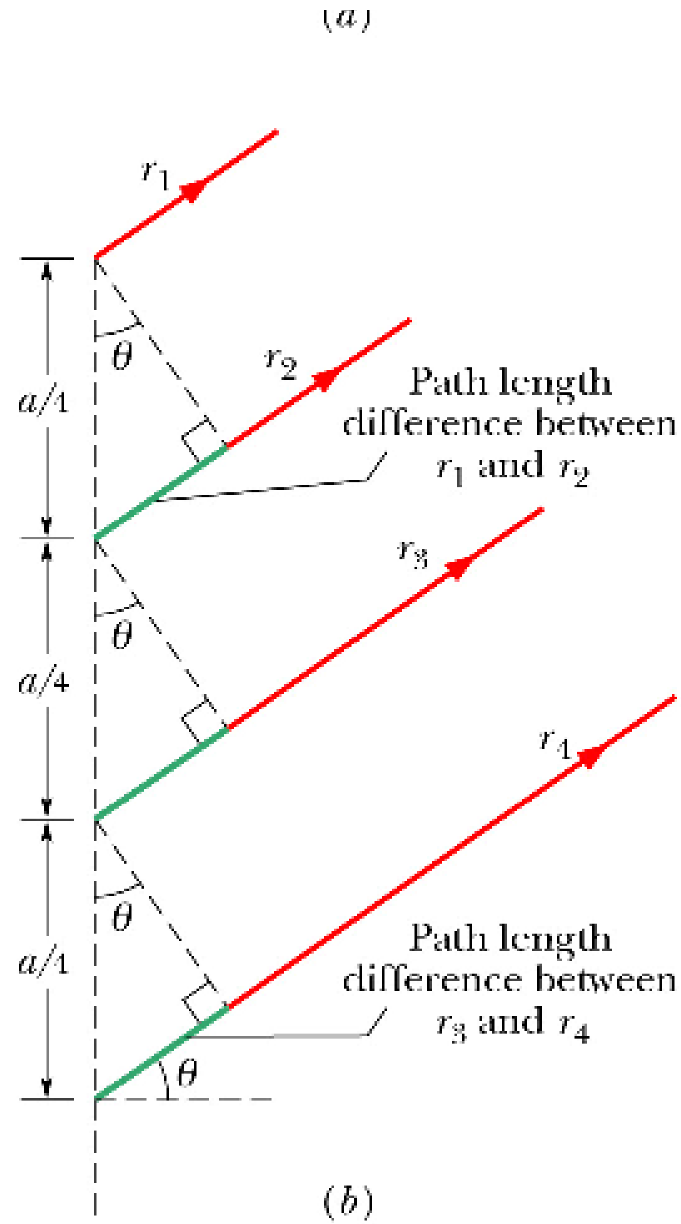
As we move up on the screen the next dark fringe occurs when the first ray interferes destructively with the one from *one-fourth* the way down the slit.

$$(a/4)\sin\theta = \lambda/2$$

$$\underline{a \sin\theta = 2\lambda}$$

We can continue to *one-sixth, one-eighth etc.* to get all the dark fringes at

$$\boxed{a \sin\theta = m\lambda}$$



Again note as slit gets narrower, pattern gets wider.

Summary of single-slit diffraction

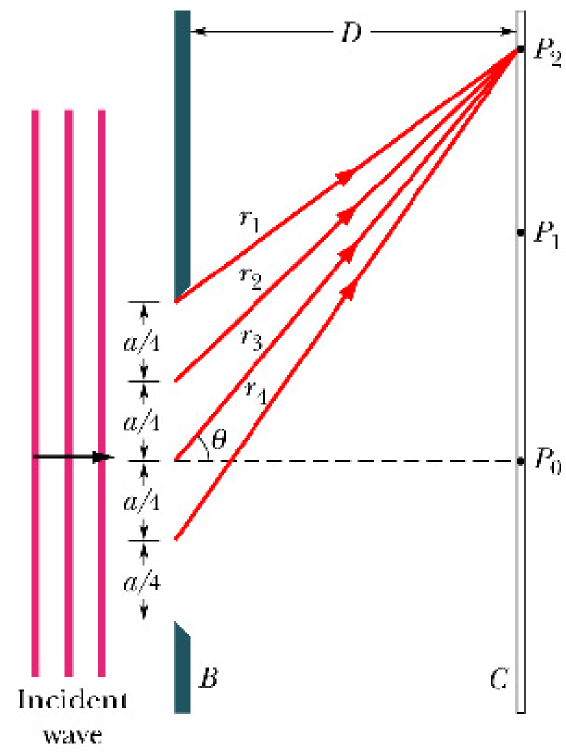
- **Given light of wavelength λ passing through a slit of width a.**
- **There are dark fringes (diffraction minima) at angles θ given by $a \sin \theta = m\lambda$ where m is an integer.**
- **Note this exactly the condition for *constructive interference* between the rays from the top and bottom of the slit.**
- **Also note the pattern gets wider as the slit gets narrower.**
- **The bright fringes are roughly half-way between the dark fringes. (Not exactly but close enough.)**

Example: Problem 37-2

Light of wavelength 441 nm is incident on a narrow slit. On a screen 2 meters away, the distance between the ***second diffraction minimum*** and the central maximum is 1.5 cm.

(a) Calculate the angle of diffraction θ of the second minimum.

(b) Find the width of the slit.

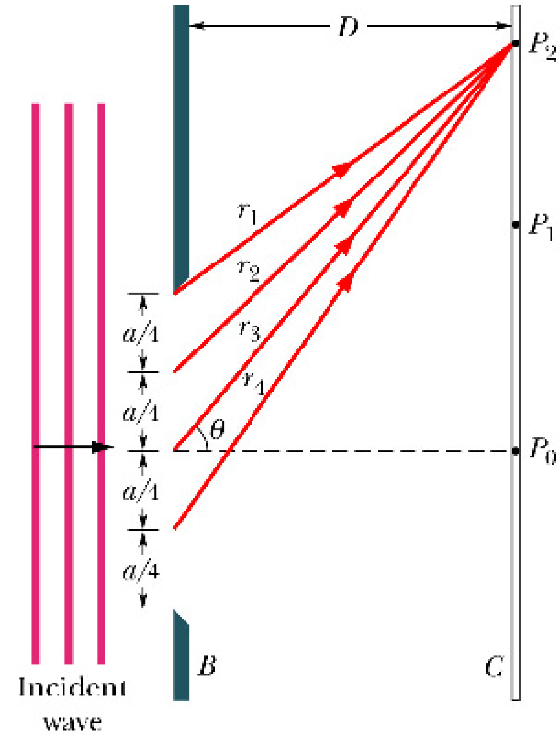


Example: (cont'd)

(a) Calculate the angle of diffraction θ of the second minimum.

$$\theta \cong \tan\theta = \frac{y}{D} = \frac{1.5\times10^{-2}}{2}$$

$$= 7.5\times10^{-3}\ rad$$



$$a\sin\theta = m\lambda$$

(b) Find the width of the slit.

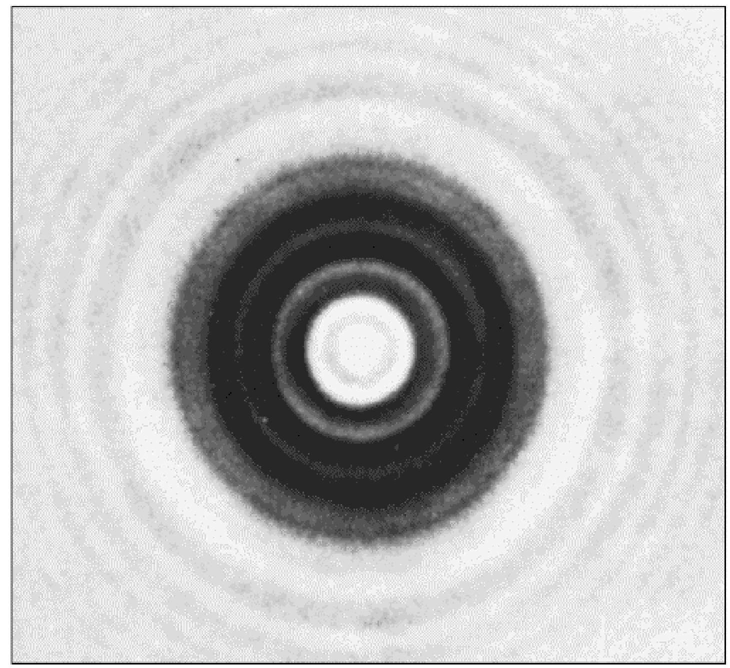
$$\theta \cong \sin\theta = m\frac{\lambda}{a}$$

$$a = m\frac{\lambda}{\theta} = 2\frac{441\times10^{-9}\ m}{7.5\times10^{-3}} = 117.6\ \mu m$$

Diffraction by a circular aperture

Consider a round hole of diameter *d*. Same idea as a long slit – only the geometry is different.

Serway, College Physics, 5/e
Text Figure 25.12



Harcourt Brace & Company

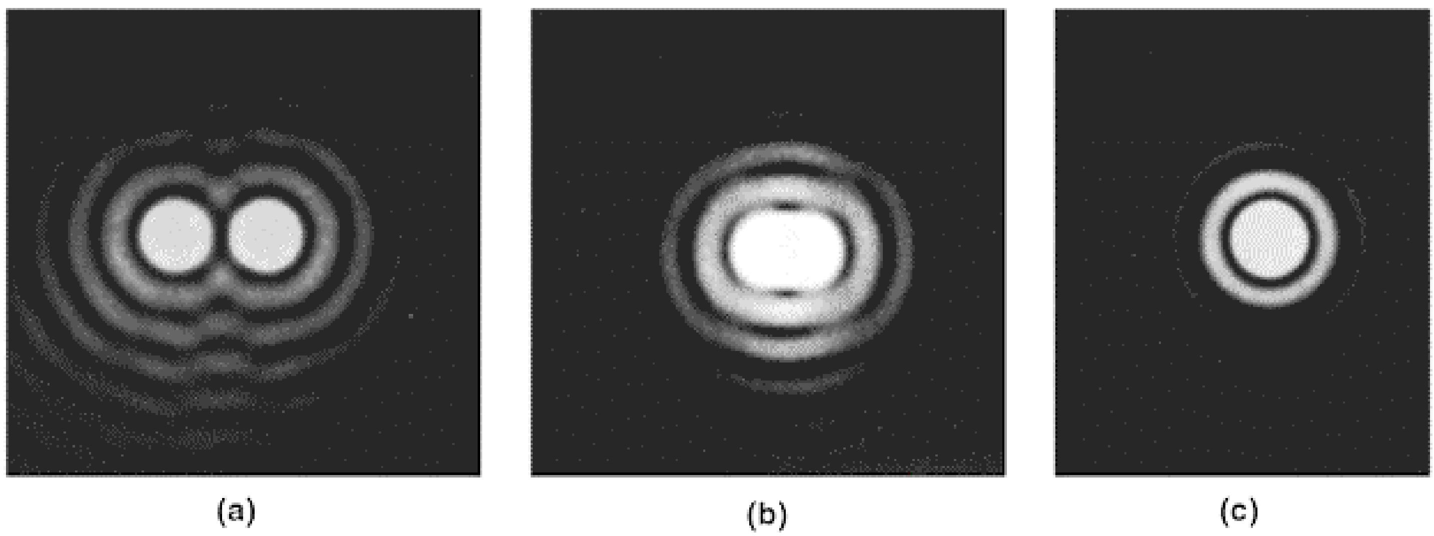
Angle for *first minimum*:

Long slit: $$a \sin\theta = \lambda$$

Circular hole: $$\boxed{d \sin\theta = 1.22\,\lambda}$$

The Rayleigh Criterion

Using a circular instrument (telescope, human eye), when can we just resolve two distant objects?

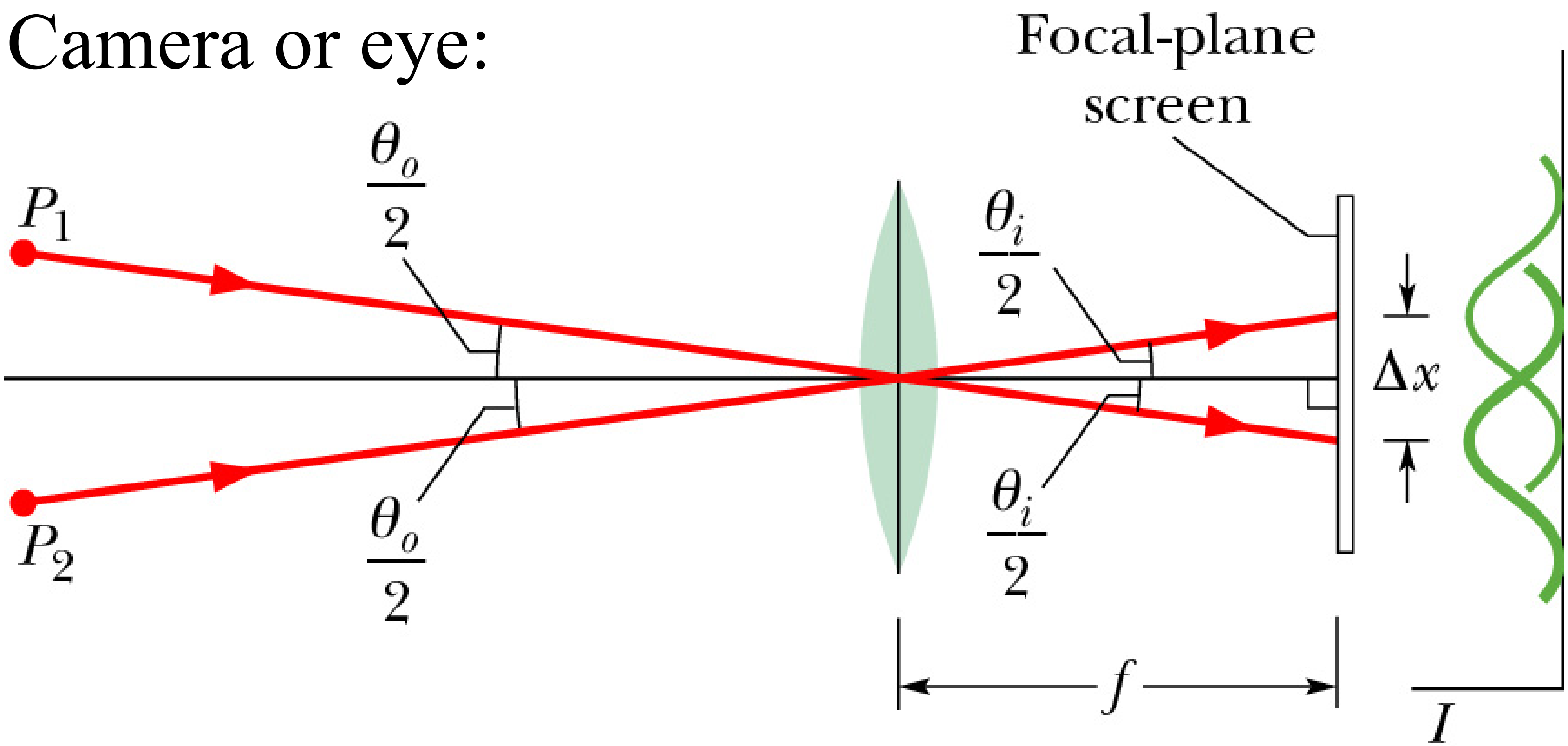


When images are separated by distance to first minimum:

$$\theta_R = 1.22\,\lambda / d$$

Sample Problem 36-4

Camera or eye:

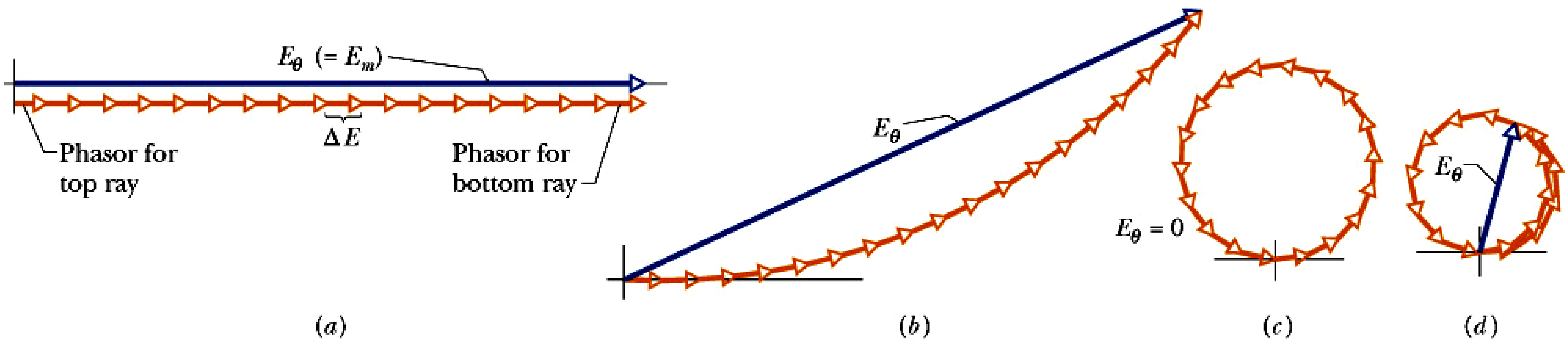


Smallest resolvable angle to two distant objects:

$$\theta_R = 1.22\,\lambda / d$$

Single slit phasor diagram

Divide slit into many tiny slits. Use a tiny phasor for each. Add them together graphically.



See we get *destructive* interference if first and last phasors interfere *constructively!*

So the condition for the *m*th dark fringe is:

$$\boxed{a \sin\theta = m\lambda}$$

Multiple-Slit Diffraction

Now we can finally put together our interference and diffraction results to see what really happens with two or more slits.

RESULT: We get the two-slit (or multiple-slit) pattern as in chapter 35, but modified by the single-slit intensity as an ***envelope.***

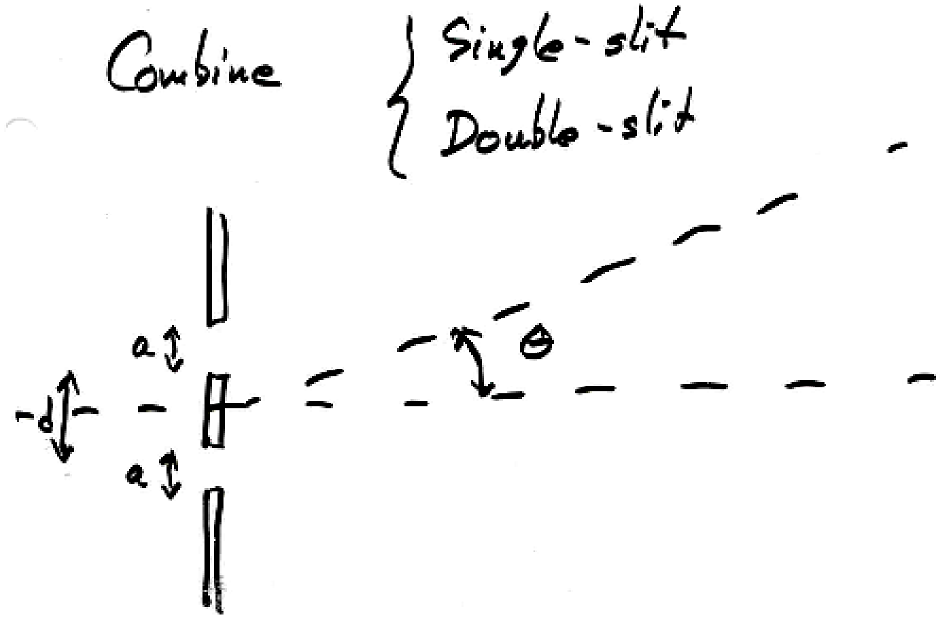
Instead of all peaks being of the same height, they get weaker at larger angles.

Double-Slit Diffraction

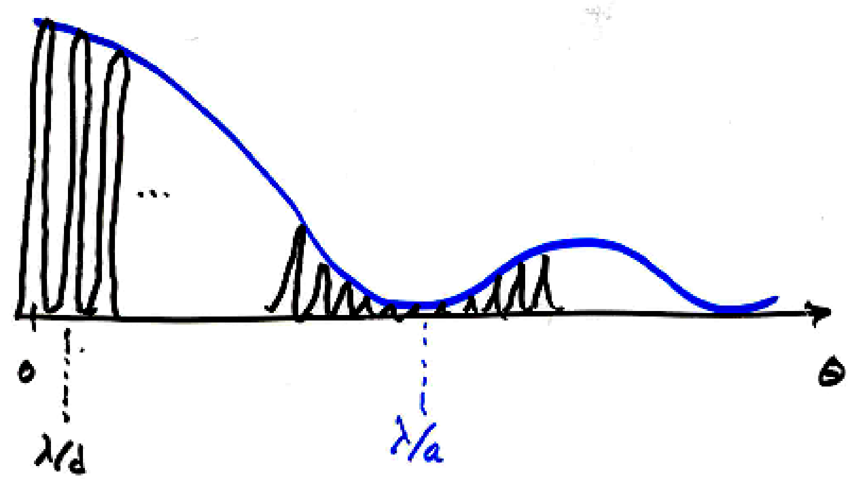
a = slit width

d = slit separation

θ = angle on screen



Bright fringes due to 2-slit interference: $\theta = m\lambda / d$



Zero due to diffraction:

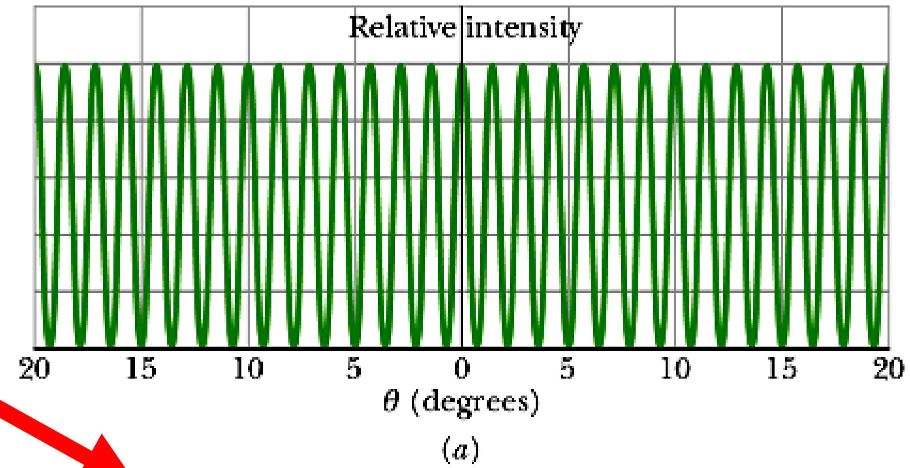
$$\theta = \lambda / a, 2\lambda / a, \cdots$$

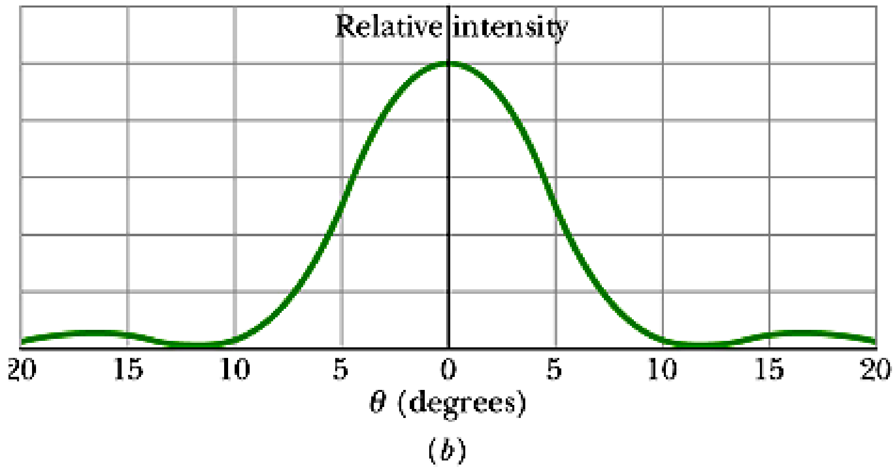
Double-slit diffraction

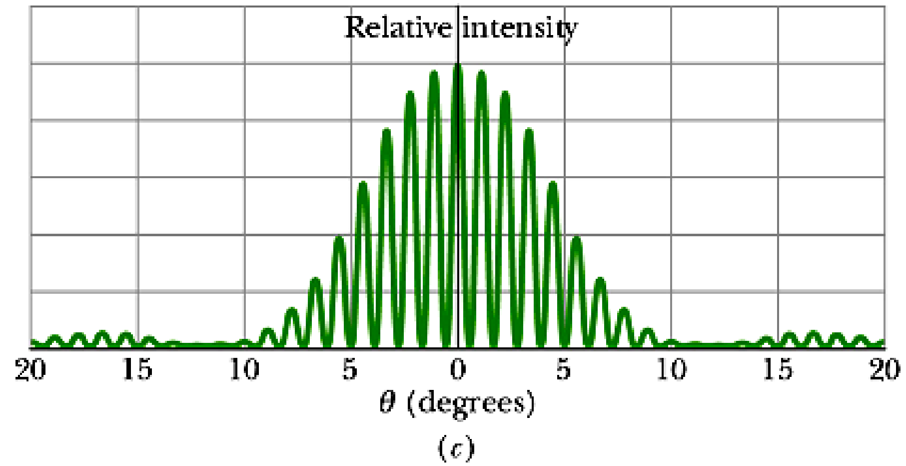
2 slits of zero width

1 slit of width a = 5λ

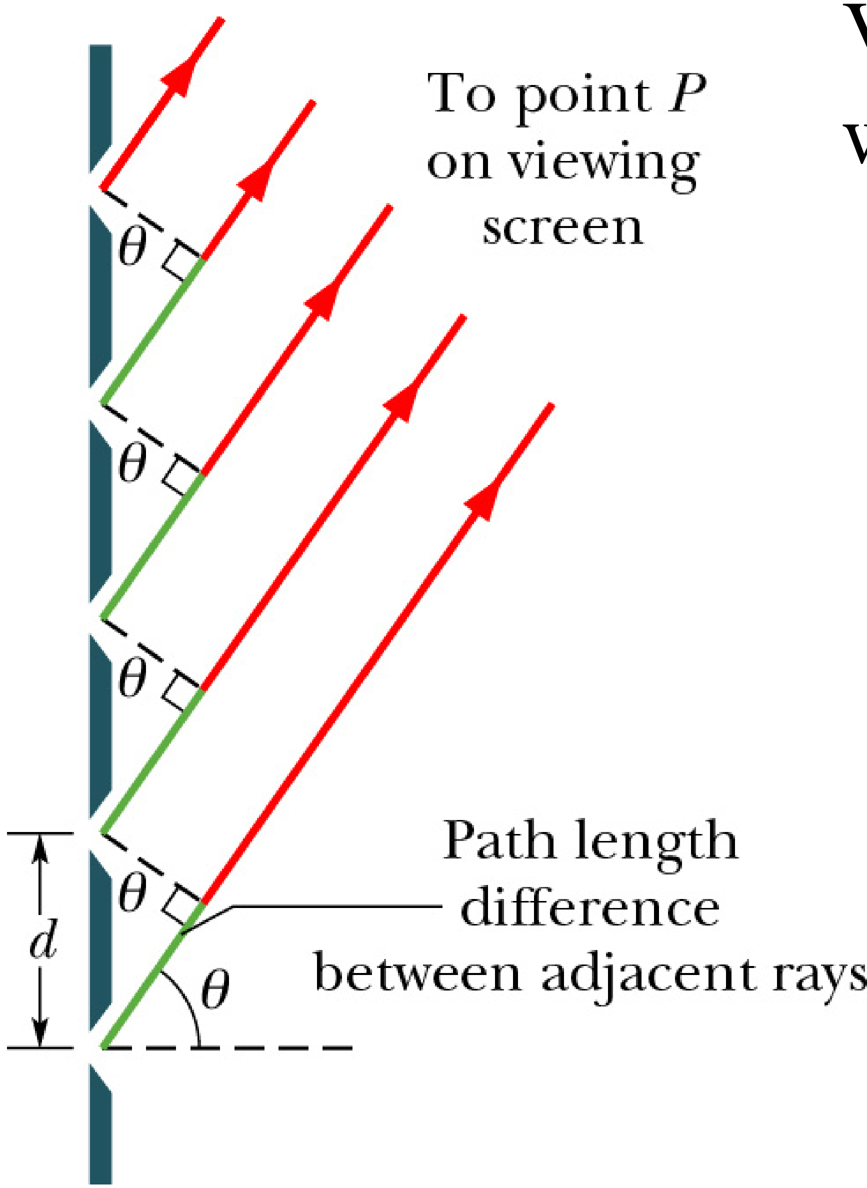
2 slits of width a = 5λ







Diffraction grating: Many Slits



Very sharp maximum when ***all rays*** are in phase.

$$d \sin\theta = m\lambda$$

m **= "order" = 1, 2, 3, …**

First-order maximum:

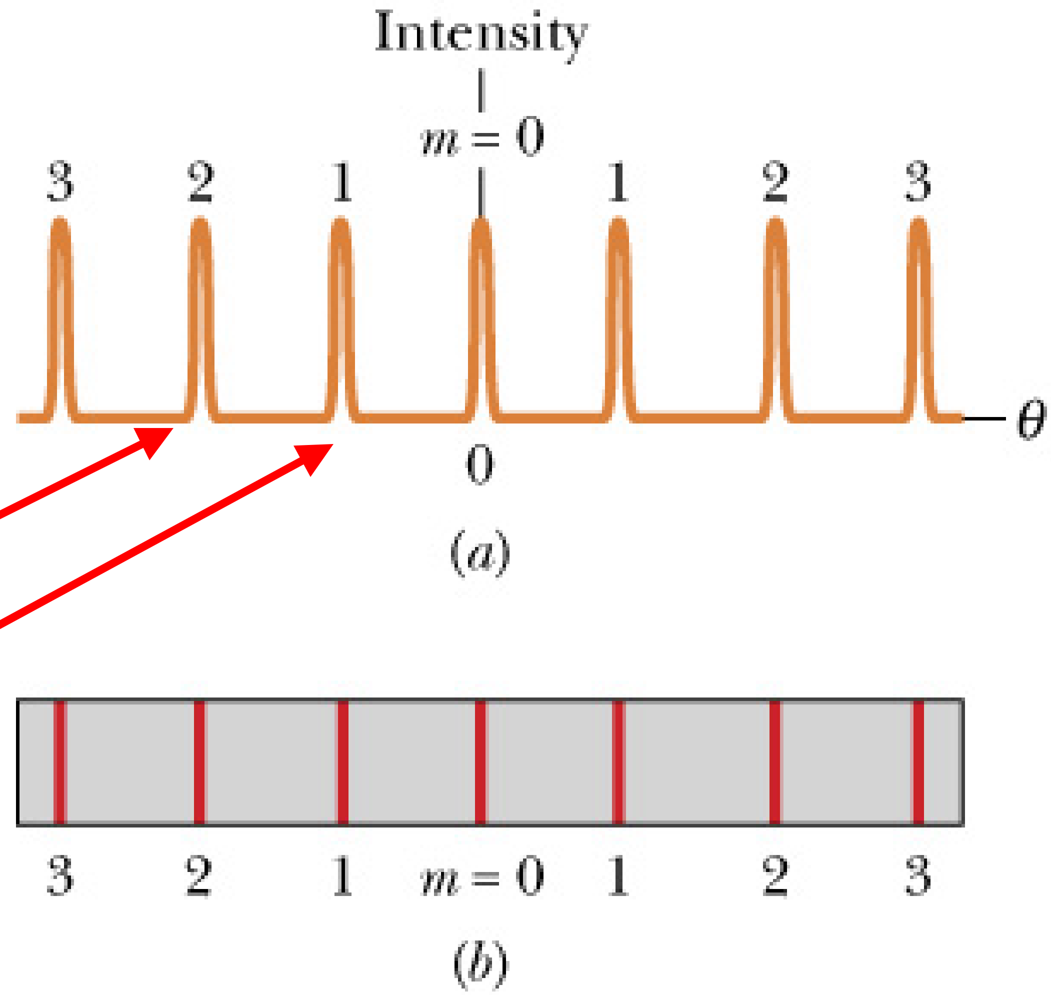
$$\sin\theta = \lambda / d$$

Second-order maximum:

$$\sin\theta = 2\lambda / d$$

Diffraction grating bright lines

$$d \sin\theta = m\lambda$$

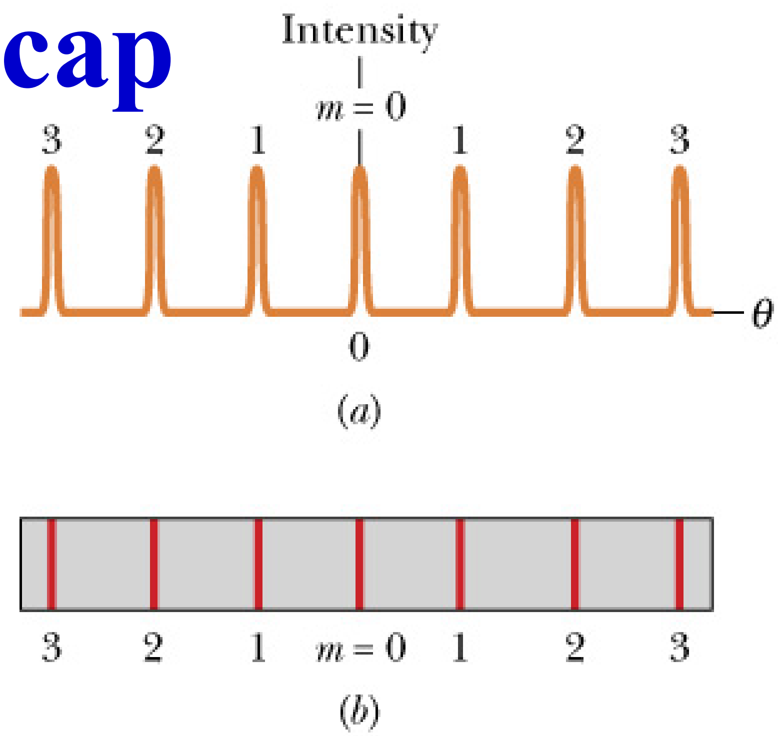


Second-order maximum:

First-order maximum:

Spectroscopy: separate lines of different wavelengths.

Diffraction grating recap



Position of lines is determined by _separation_ of rulings.

$$d \sin\theta = m\lambda$$

Sharpness of lines is determined by _number_ of rulings.

$$\Delta\theta = \lambda / Nd$$

Resolving power is determined by _number_ of rulings and _order_ of line.

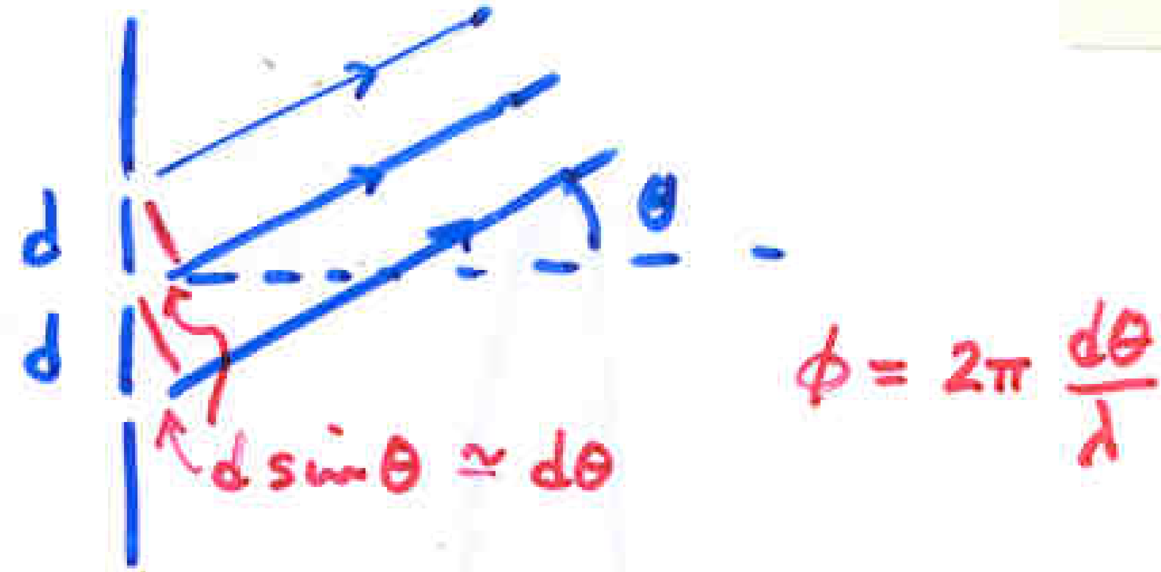
$$R = mN$$

Interference with 3 Slits

Path difference between rays from adjacent slits:

$$\Delta L = d \sin\theta$$

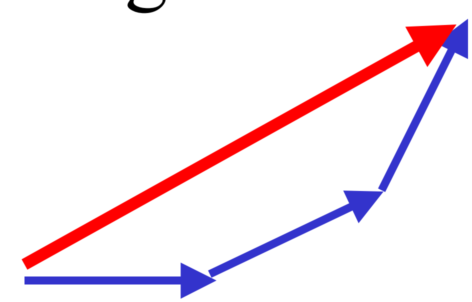
$$\theta \approx \Delta L / d$$



Phase difference between rays from adjacent slits:

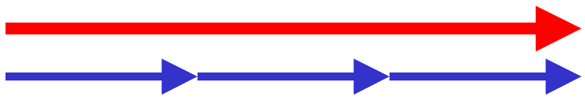
$$\phi = 2\pi \frac{\Delta L}{\lambda} = 2\pi \frac{d\theta}{\lambda}$$

Get intensity from phasor diagram:



3 Slits Continued

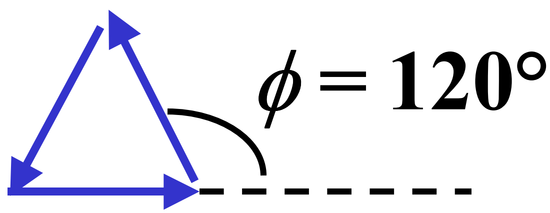
1. Central maximum:



$$\phi = 0 \qquad \theta = 0$$

$$E_T = 3E_0 \qquad I_T = 9I_0$$

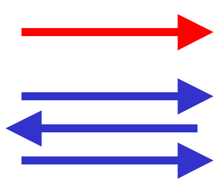
2. First minimum:



$$\phi = \frac{2\pi}{3} \qquad d\sin\theta = \frac{\lambda}{3}$$

$$E_T = 0 \qquad I_T = 0$$

3. Next maximum:



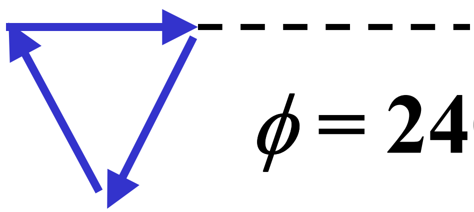
$\phi = 180°$

$$\phi = \pi \qquad d\sin\theta = \lambda/2$$

$$E_T = E_0 \qquad I_T = I_0$$

3 Slits Continued

4. Next minimum:

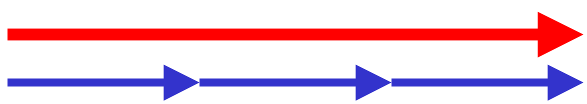


$\phi = 240°$

$$\phi = \frac{2}{3}2\pi \qquad d\sin\theta = \frac{2}{3}\lambda$$

$$E_T = 0 \qquad I_T = 0$$

5. Next maximum:



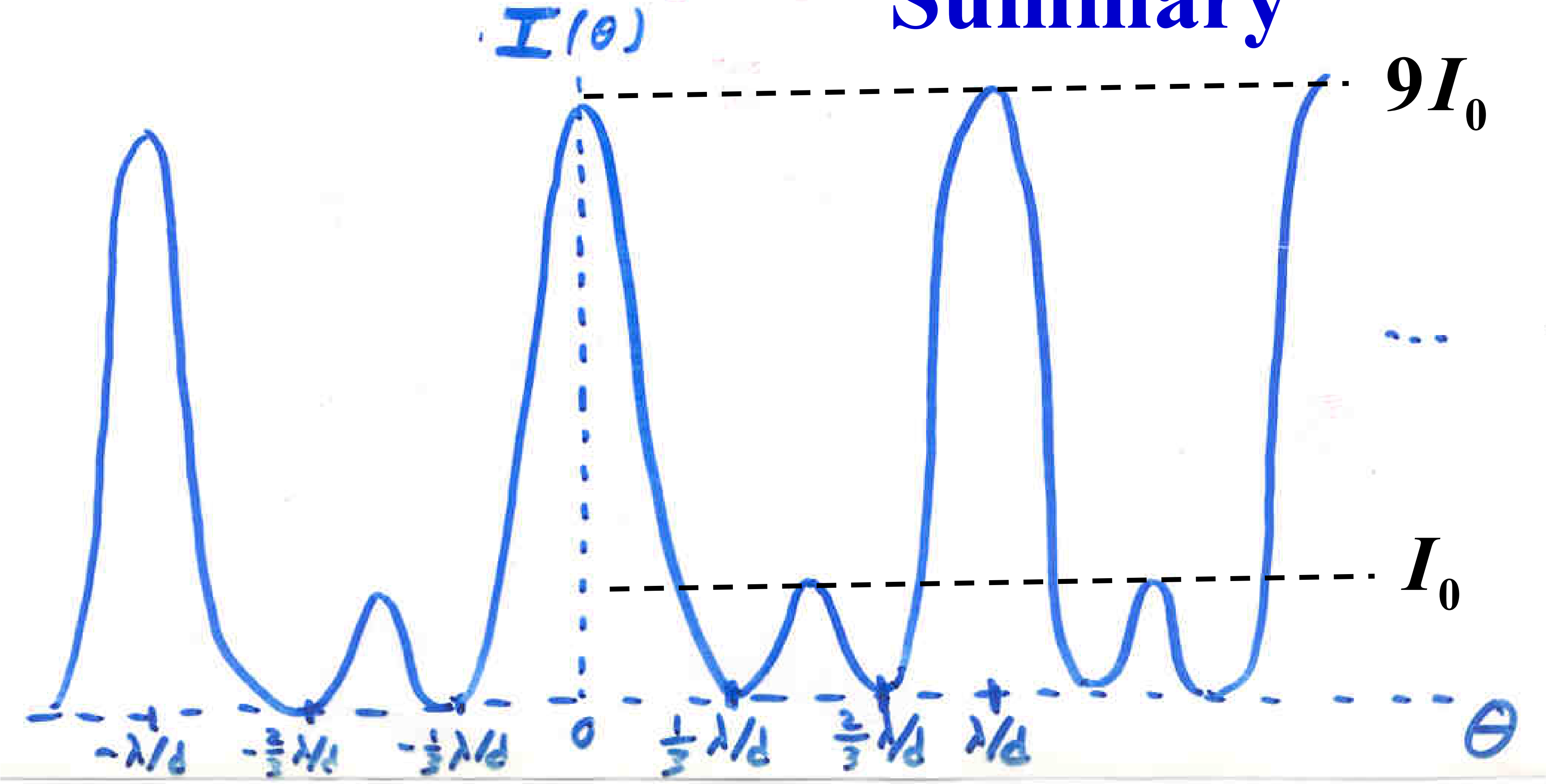
$\phi = 360°$

$$\phi = 2\pi \qquad d\sin\theta = \lambda$$

$$E_T = 3E_0 \qquad I_T = 9I_0$$

$$\theta \approx \lambda / d$$

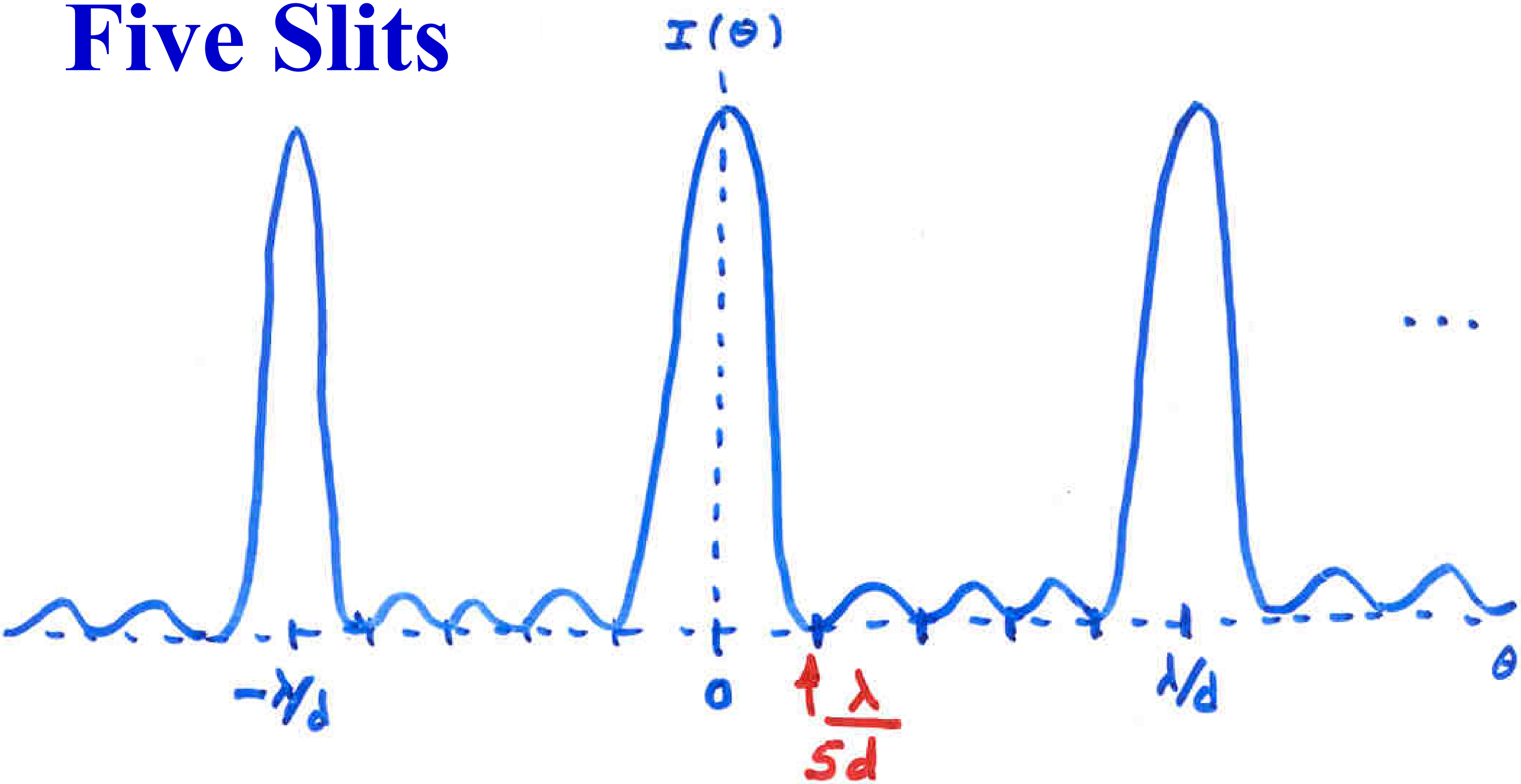
Summary



$\theta \approx \Delta L / d$ $\quad \Delta L = \quad 0 \quad \frac{\lambda}{3} \quad \frac{\lambda}{2} \quad \frac{2\lambda}{3} \quad \lambda$

$$\boxed{\theta_{max} \approx m\lambda / d}$$

Five Slits

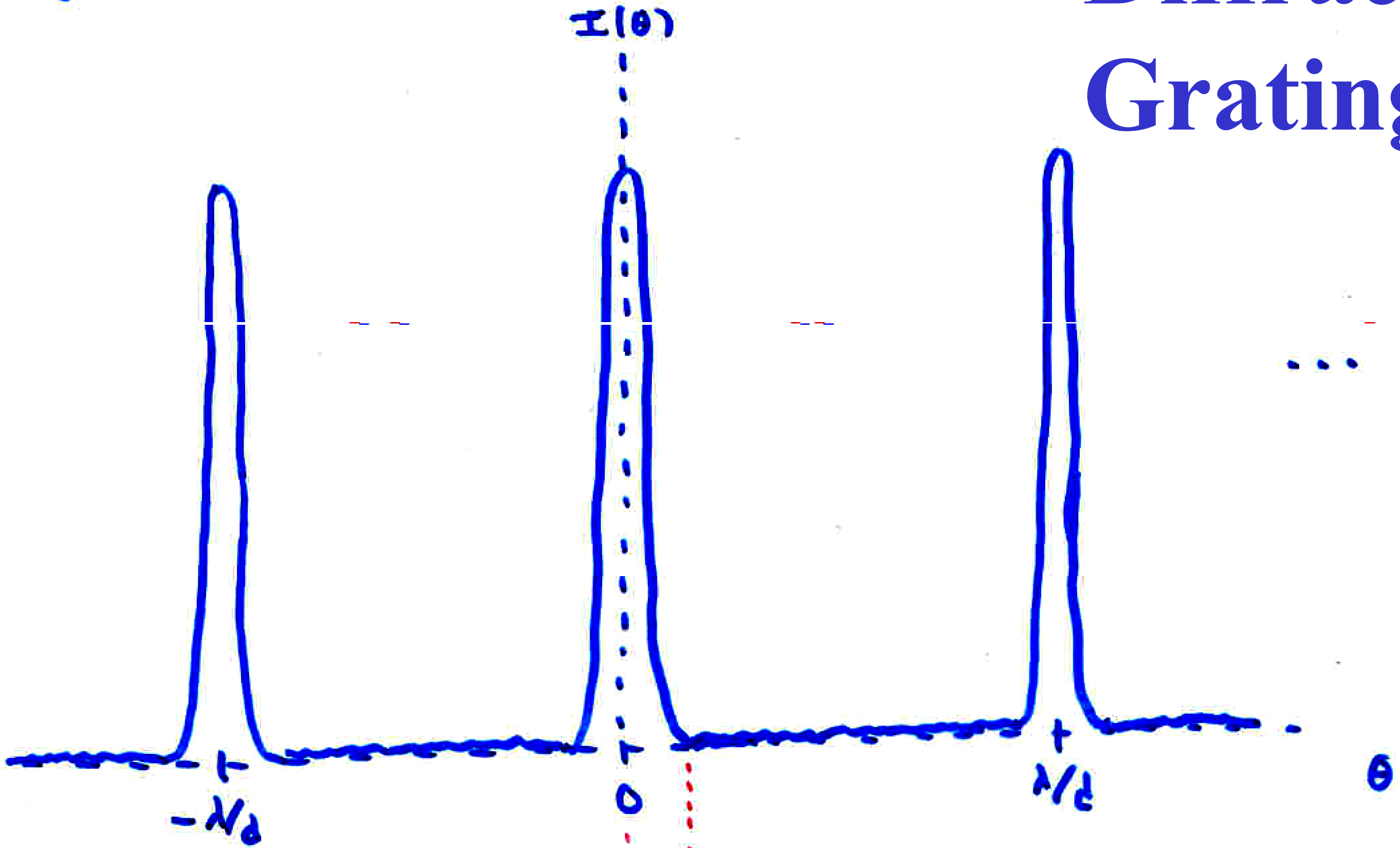


Note we still have $\theta_{max} = m\lambda / d$
But more slits makes the peaks *sharper*.

For ***<u>many</u>*** slits, we get a diffraction grating.

Diffraction Grating

N (Many) Slits (Diffraction Grating)

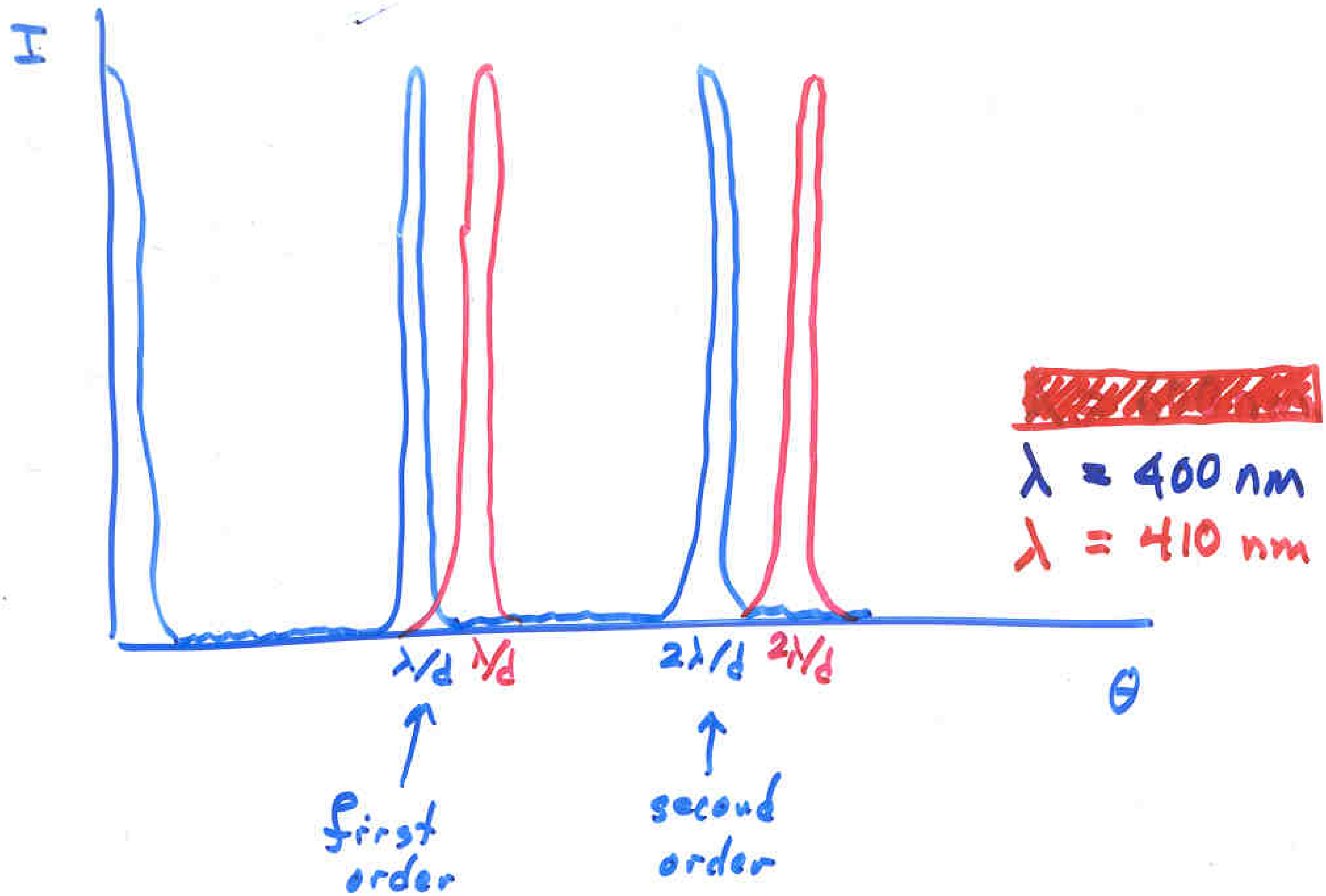


"Half-width of central peak"; really location of first minimum

$\rightarrow \Delta\theta \leftarrow$

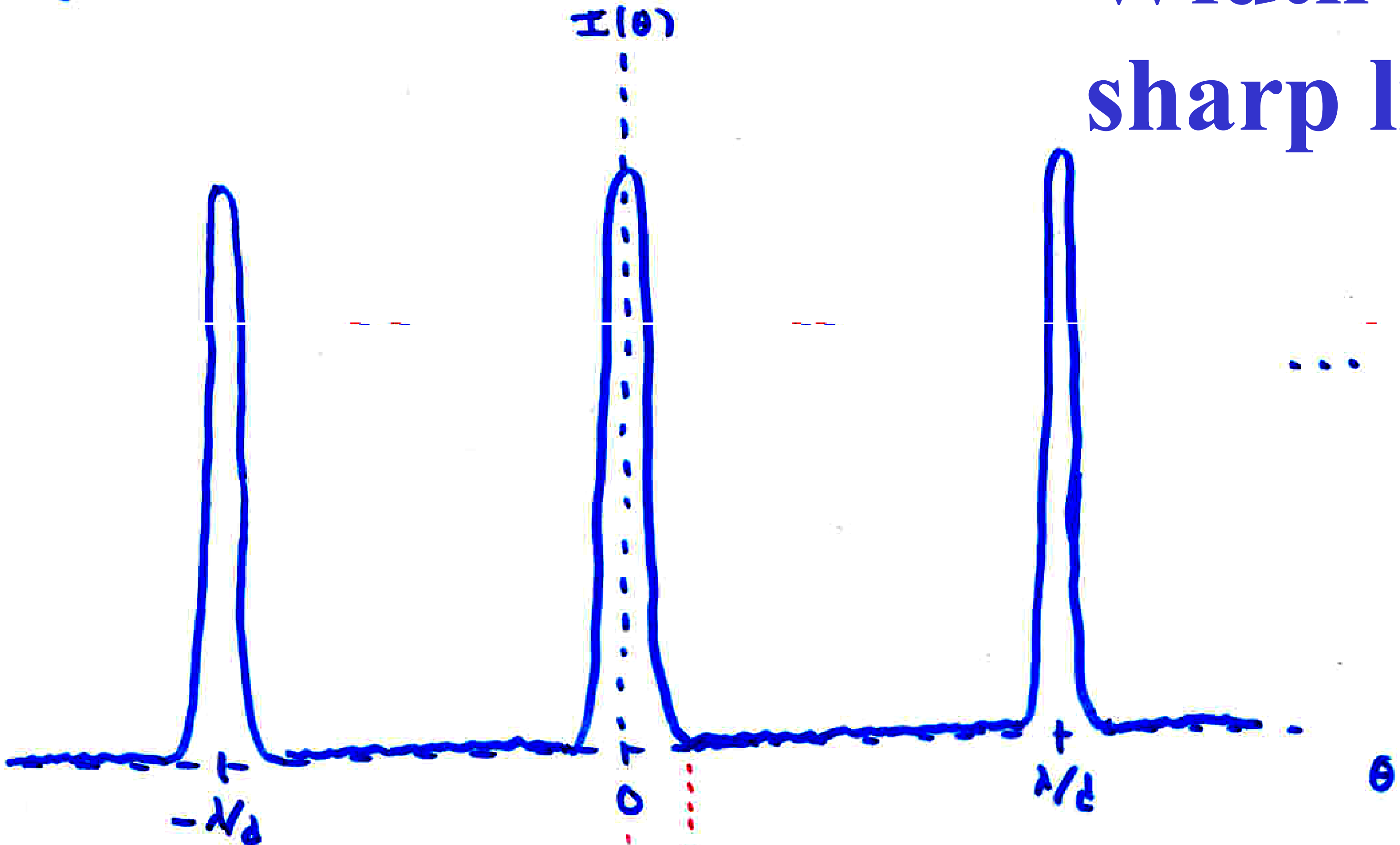
is $\Delta\theta = \frac{\lambda}{Nd}$. Note peaks get sharper as N gets larger.

Wavelength Resolution of a Grating



Width of sharp lines

N (Many) Slits (Diffraction Grating)



"Half-width of central peak"; really location of first minimum

$\Delta\theta$

is $\boxed{\Delta\theta = \frac{\lambda}{Nd}}$. Note peaks get sharper as N gets larger.

Resolving Power

Positions of maximums: $$d\sin\theta = m\lambda$$

Small angles: $$\theta = m\lambda / d$$

$$\Delta\theta = m\Delta\lambda / d$$

Widths of sharp maximums: $$\Delta\theta = \lambda / Nd$$

Wavelengths just resolved: $$m\Delta\lambda / d = \lambda / Nd$$

$$m\Delta\lambda = \lambda / N$$

Resolving power definition:

$$R = \frac{\lambda}{\Delta\lambda}$$

So we get: $$R = mN$$

Example: Yellow sodium vapor lines

Problem 36-50

The strong yellow lines in the sodium spectrum are at wavelengths 589.0 nm and 589.6 nm.

How many rulings are needed in a diffraction grating to resolve these lines in second order?

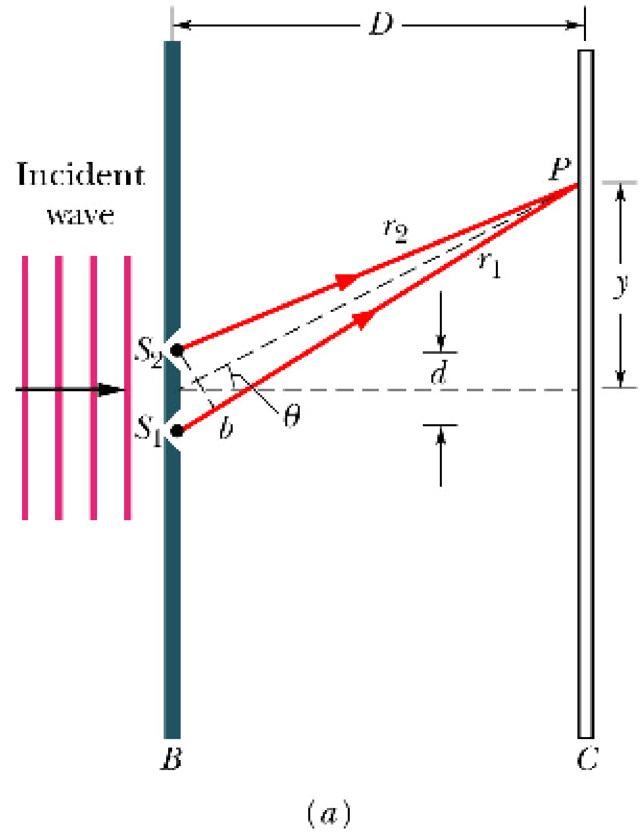
We need $$R = \frac{\lambda}{\Delta\lambda} = \frac{589\ nm}{0.6\ nm} = 982$$

But $R = mN$ **so** $$N = \frac{R}{m} = \frac{982}{2} = \underline{491}$$

Diffraction II

- Today
  - Single-slit diffraction review
  - Multiple slit diffraction review
  - Xray diffraction
  - Diffraction intensities

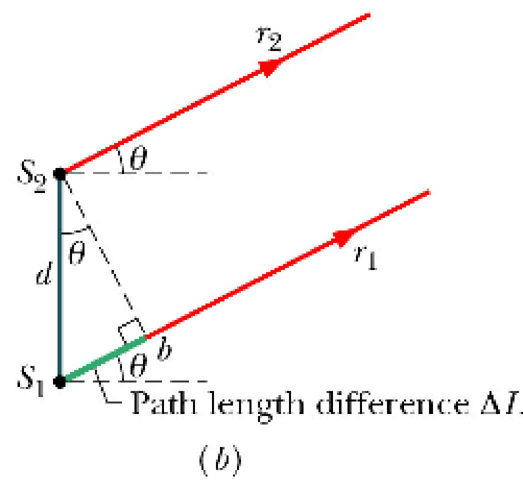
Review: Double Slit Path Differences



For point P at angle θ triangle shows $\Delta L = d\sin\theta$

For constructive interference we need $\Delta L = m\lambda$

where m=0,1,2,… is any integer.



So the bright fringes are at angles given by $\boxed{d\sin\theta = m\lambda}$

Double-slit interference fringes

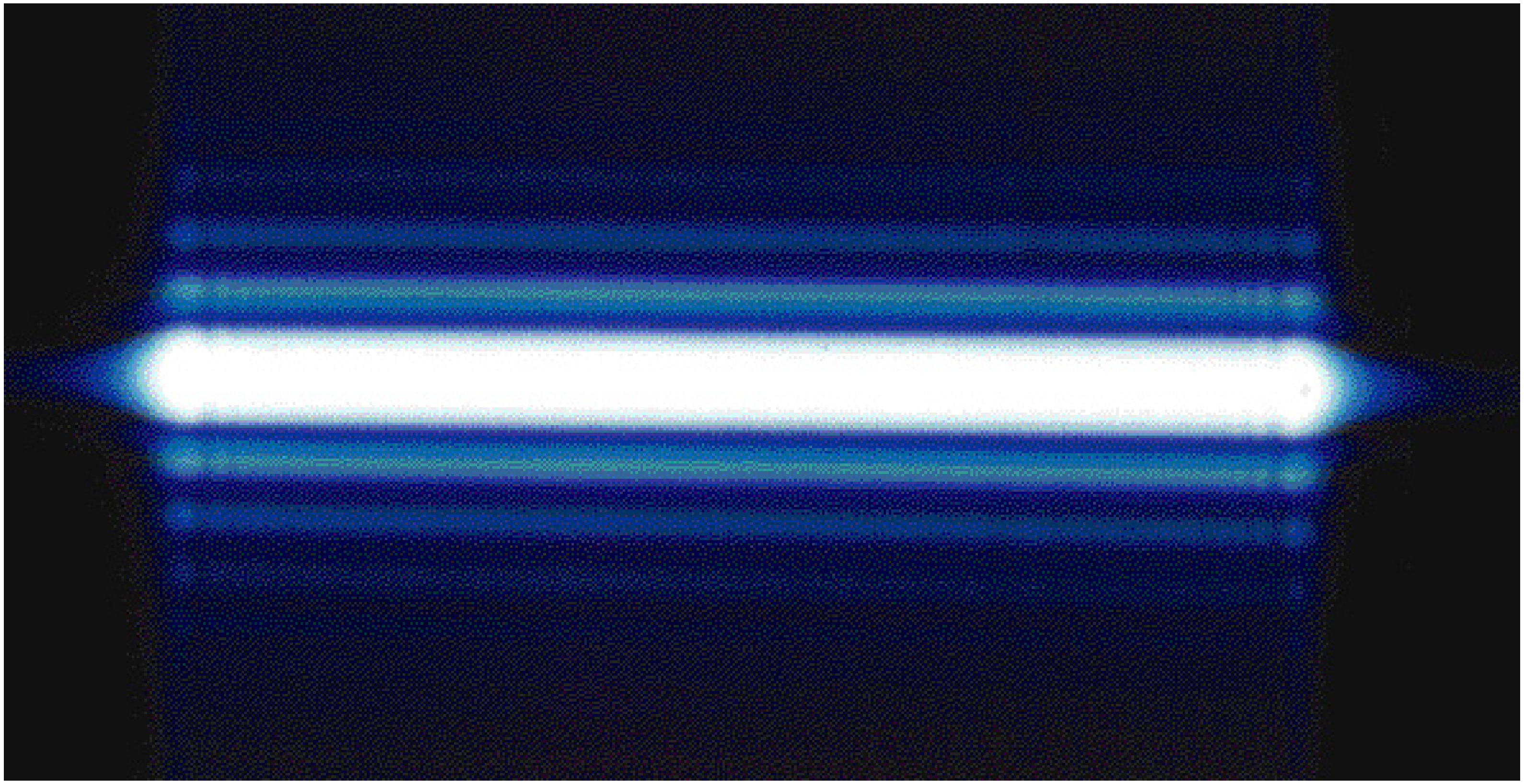
So the *bright fringes* are at angles given by

$$d \sin\theta = m\lambda$$

And the *dark fringes* are at angles given by

$$d \sin\theta = (m + \frac{1}{2})\lambda$$

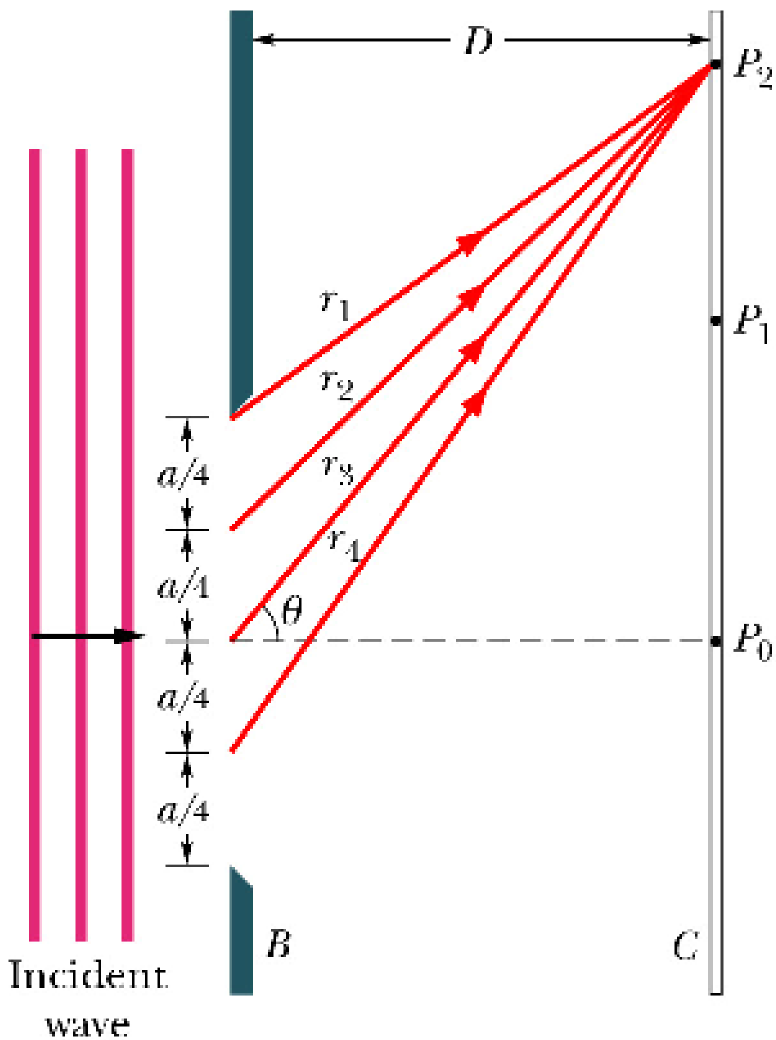
Single slit: Pattern on screen

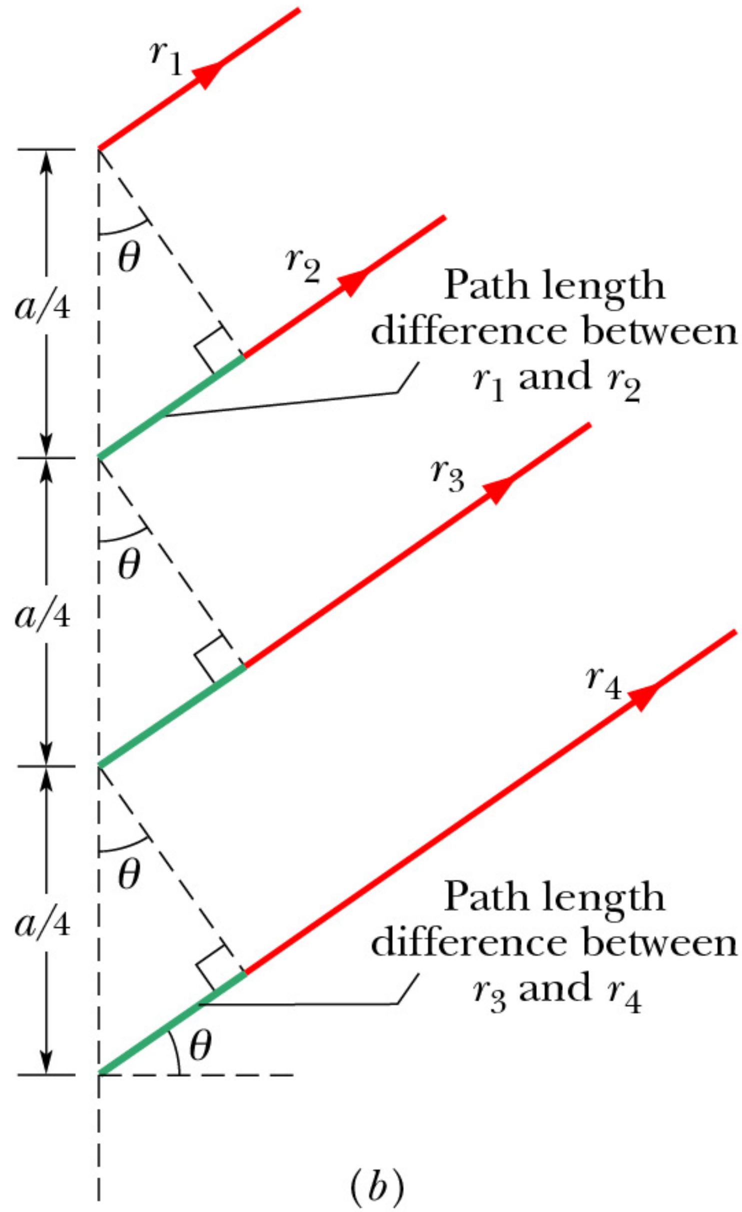


Bright and dark fringes appear behind a single very thin slit.

As the slit is made narrower the pattern of fringes becomes wider.

Dark Fringes in Diffraction





(b)

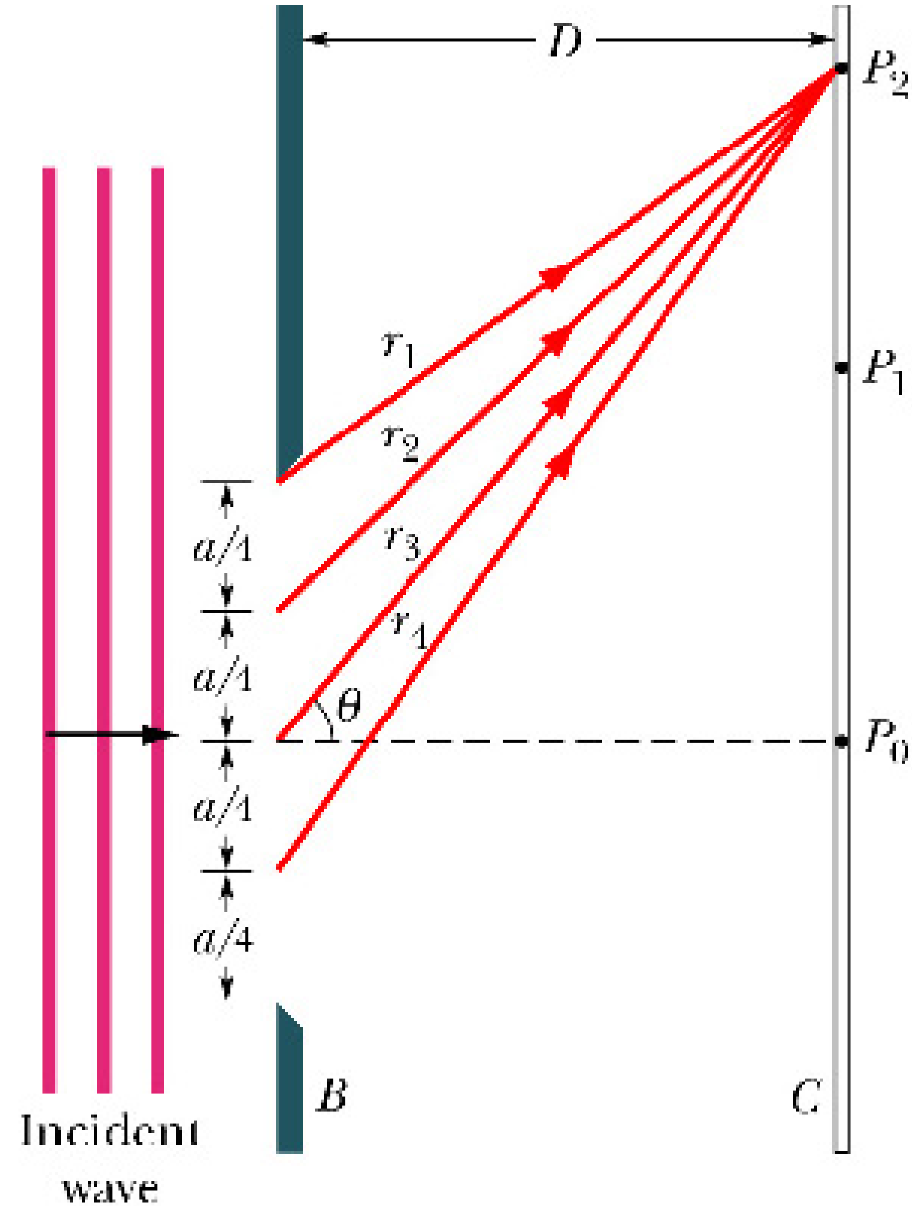
Single Slit dark fringes

Destructive interference:

$$\left(a/2m\right)\sin\theta = \left(\lambda/2\right)$$

$$\boxed{a\sin\theta = m\lambda}$$

Don't confuse this with the condition for *constructive* interference for *two* slits!



In fact, note that there is a *dark fringe* when the rays from the top and bottom interfere *constructively!*

Summary of single-slit diffraction

- **Given light of wavelength λ passing through a slit of width a.**
- **There are dark fringes (diffraction minima) at angles θ given by $a \sin \theta = m\lambda$ where m is an integer.**
- **Note this exactly the condition for *constructive interference* between the rays from the top and bottom of the slit.**
- **Also note the pattern gets wider as the slit gets narrower.**
- **The bright fringes are roughly half-way between the dark fringes. (Not exactly but close enough.)**

Example: Problem 36-6

The distance between the first and fifth minima of a single-slit diffraction pattern is 0.35 mm with the screen 40 cm away from the slit, with light of wavelength 550 nm.

Find the slit width.

$$\sin\theta \approx \tan\theta = y / D$$

$$a\sin\theta = m\lambda \qquad \sin\theta = m\lambda / a$$

$$\sin\theta_5 - \sin\theta_1 = \frac{5\lambda}{a} - \frac{1\lambda}{a} = \frac{4\lambda}{a}$$

$$y_5 - y_1 = D\sin\theta_5 - D\sin\theta_1 = \frac{4\lambda D}{a}$$

$$a = \frac{4\lambda D}{y_5 - y_1} = \frac{4 \times 550 \times 10^{-9} \times 0.4}{3.5 \times 10^{-4}}$$

$$= 2.5 \times 10^{-3}\ m = 2.5\ mm$$

Q.36-1

A slit of width 50 μm is used with monochromatic light to form a diffraction pattern. The distance between dark fringes on a distant screen is 4 mm. If the slit width is increased to 100 μm, what will be the new distance between dark fringes?

Give your answer in mm. (In the range 0-9.)

Q.36-1

A slit of width 50 µm is used with monochromatic light to form a diffraction pattern. The distance between dark fringes on a distant screen is 4 mm. If the slit width is increased to 100 µm, what will be the new distance between dark fringes?

Pattern size is inversely proportional to slit size: 2 times slit width means (1/2) times the distance between fringes. Answer: $\boxed{\text{2 mm.}}$

Multiple-Slit Diffraction

Now we can finally put together our interference and diffraction results to see what really happens with two or more slits.

RESULT: We get the two-slit (or multiple-slit) pattern as in chapter 35, but modified by the single-slit intensity as an ***envelope.***

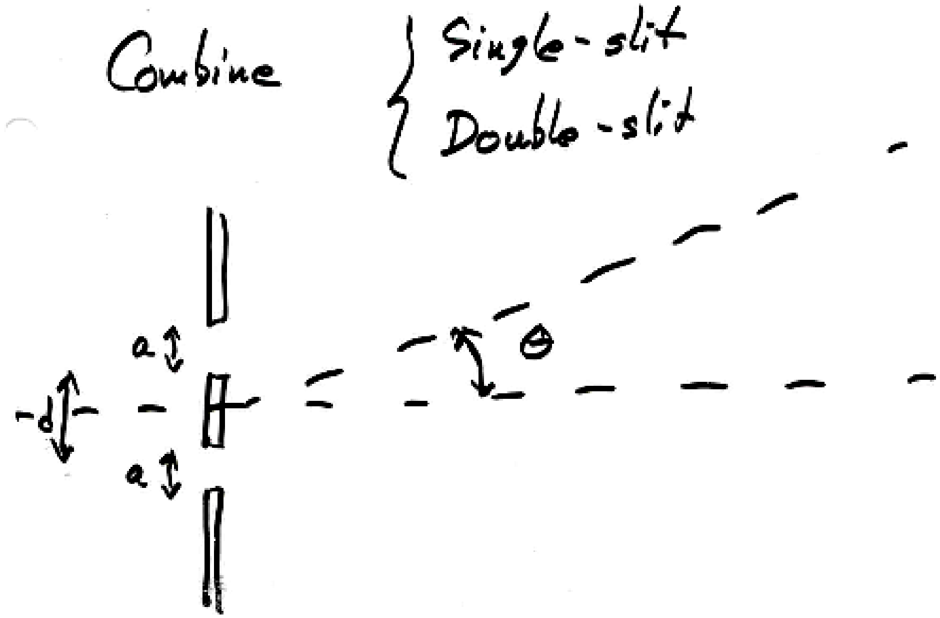
Instead of all peaks being of the same height, they get weaker at larger angles.

Double-Slit Diffraction

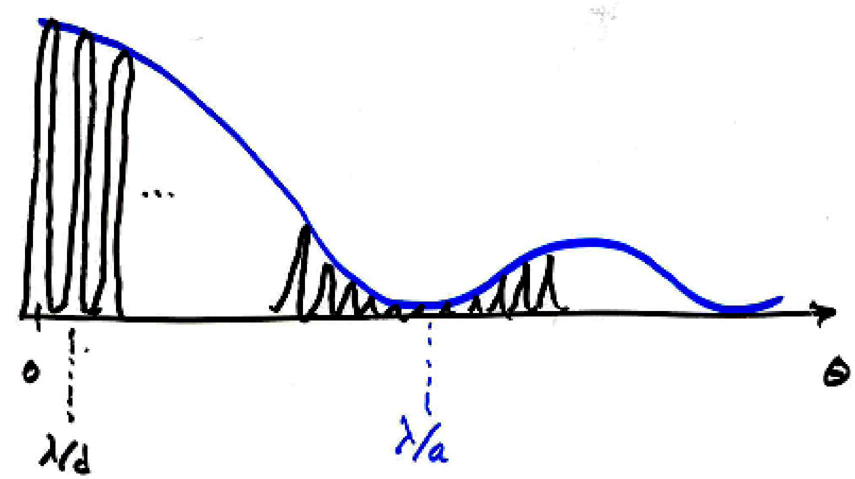
a = slit width

d = slit separation

θ = angle on screen



Bright fringes due to 2-slit interference: $\theta = m\lambda / d$



Zero due to diffraction:

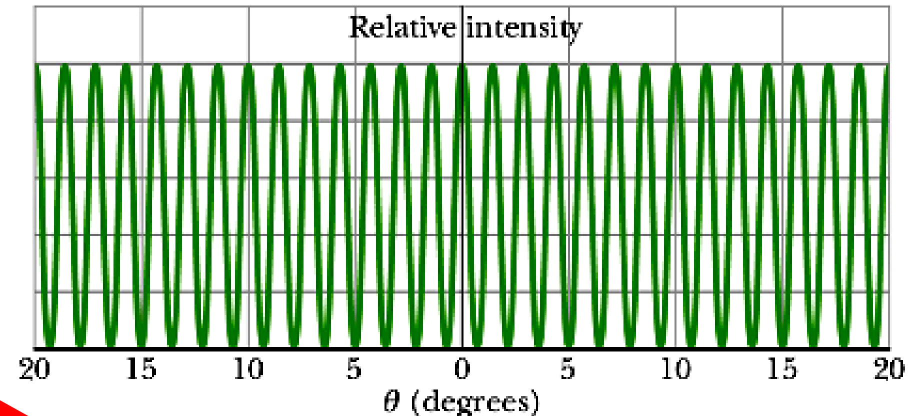
$$\theta = \lambda / a, 2\lambda / a, \cdots$$

Double-slit diffraction

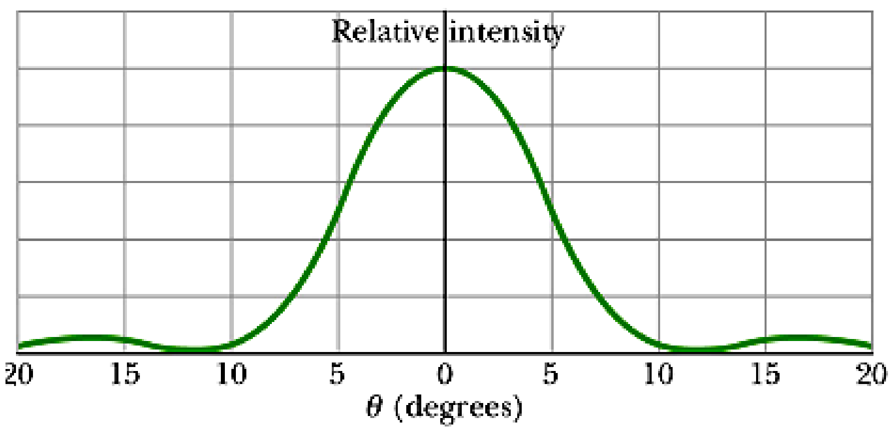
2 slits of zero width

1 slit of width a = 5λ

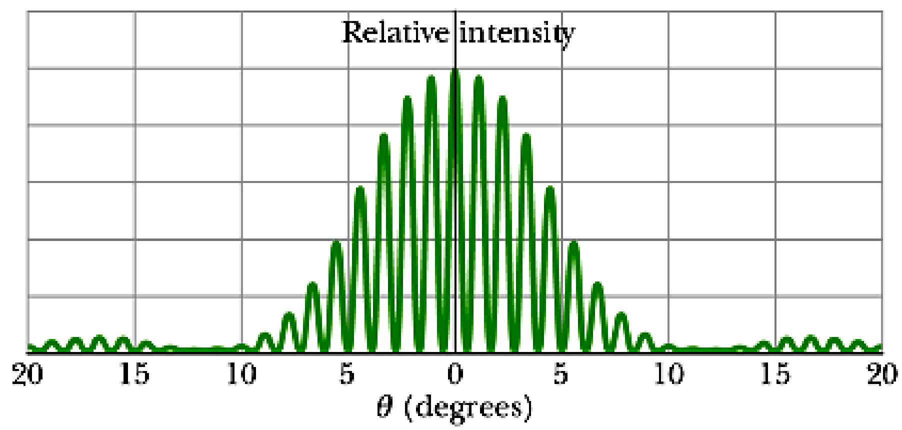
2 slits of width a = 5λ



(a)



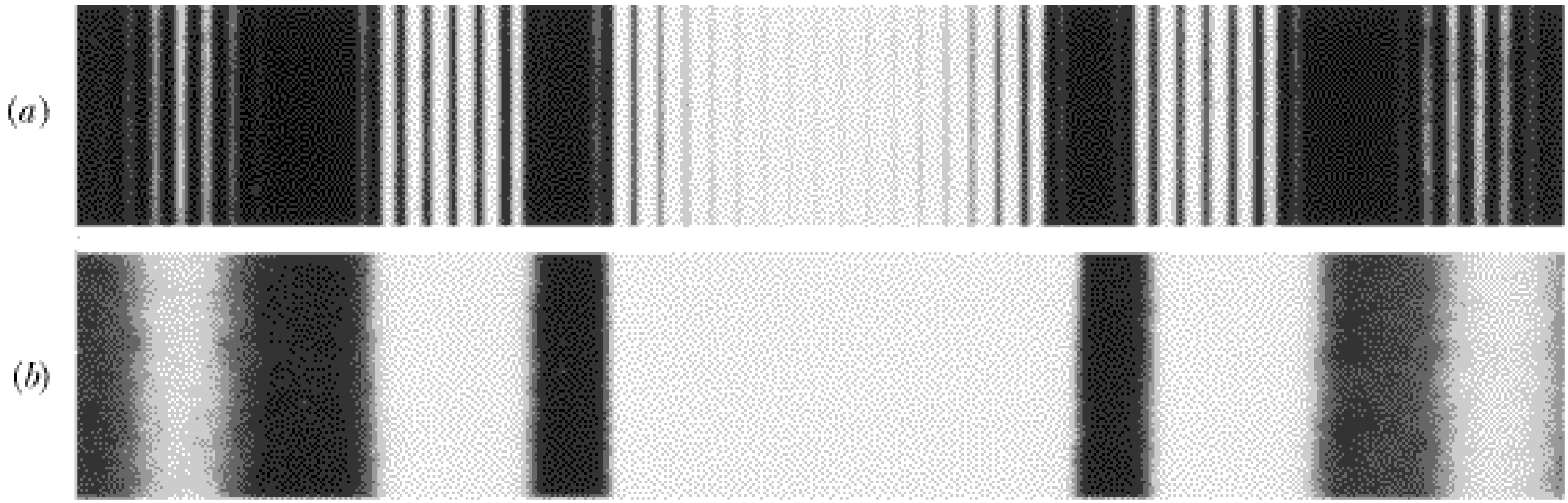
(b)



(c)

Two-slit and one-slit patterns

Actual photograph:
(a)= two slits
(b)=one list covered



(Figure 36-15 from text page 1003.)

Scaling of diffraction patterns

Notice a common feature of interference and diffraction patterns: The large-scale features of the pattern are determined by the small-scale regularities of the object, and vice-versa.

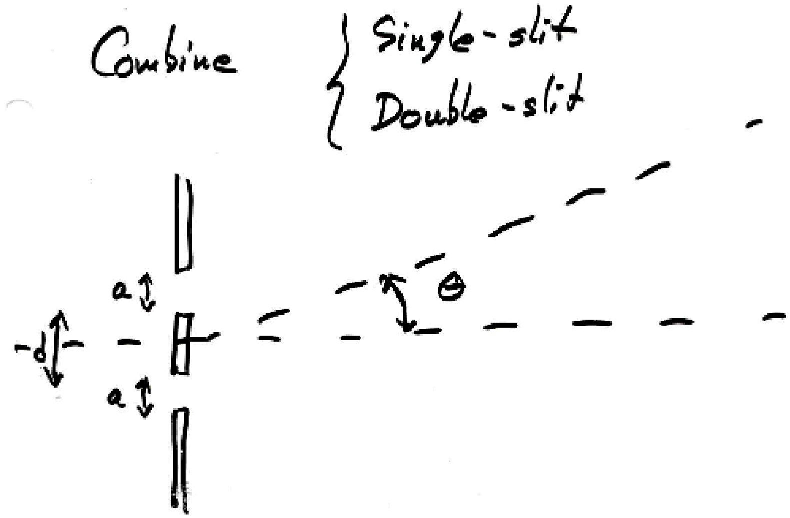
Holograms and X-ray diffraction patterns are examples.

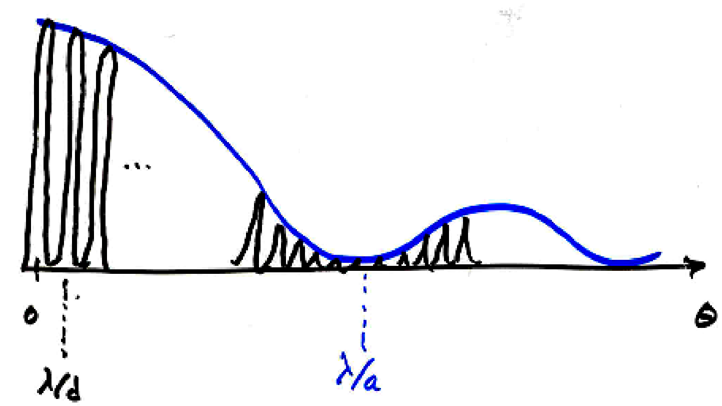
Example:Sample Problem 36-5

Two slits: d=19.44 nm, a=4.05 μm, λ=405 nm.

(a) How many bright fringes within the central peak?

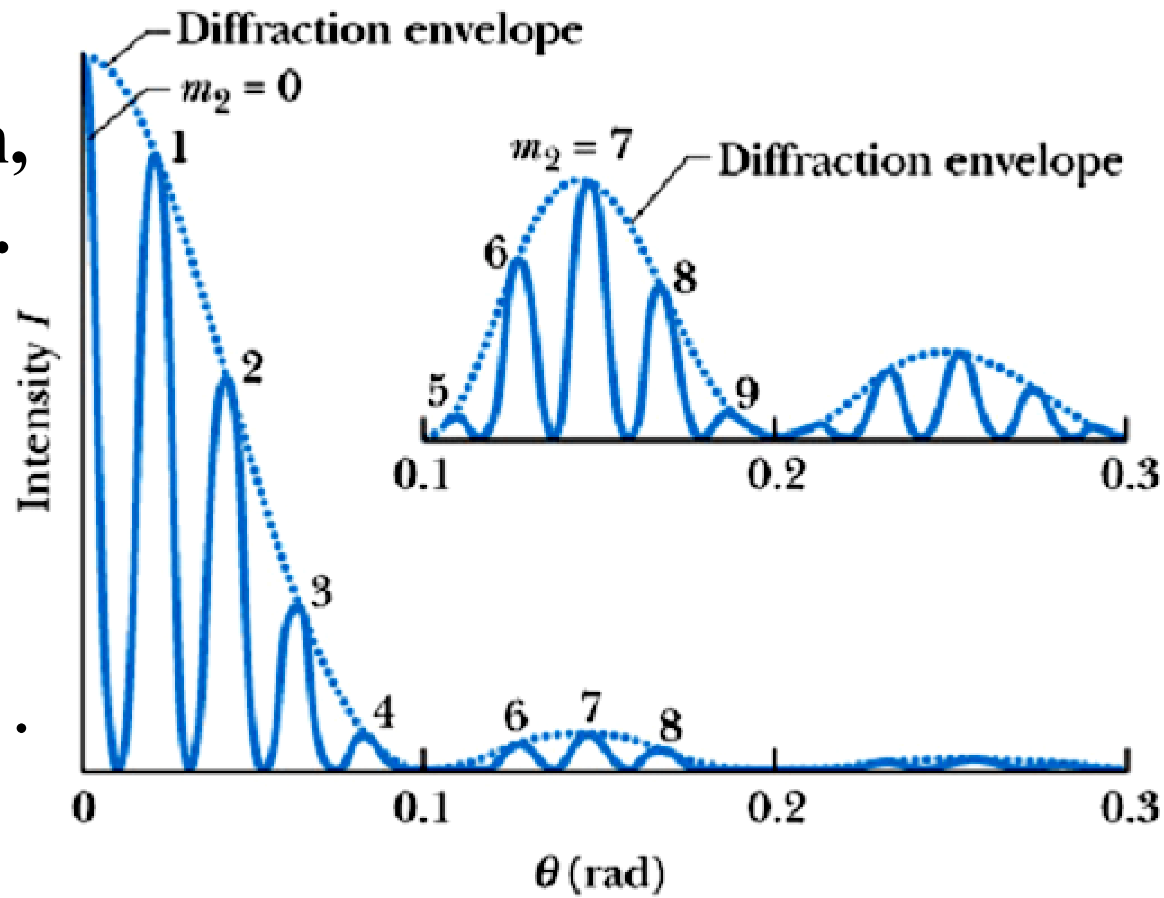
(b) How many in the first side peak?





Example:Sample Problem 36-5

Two slits: d=19.44 nm, a=4.05 µm, λ=405 nm.



Solution:

One-slit:

$$\theta = m\lambda / a$$

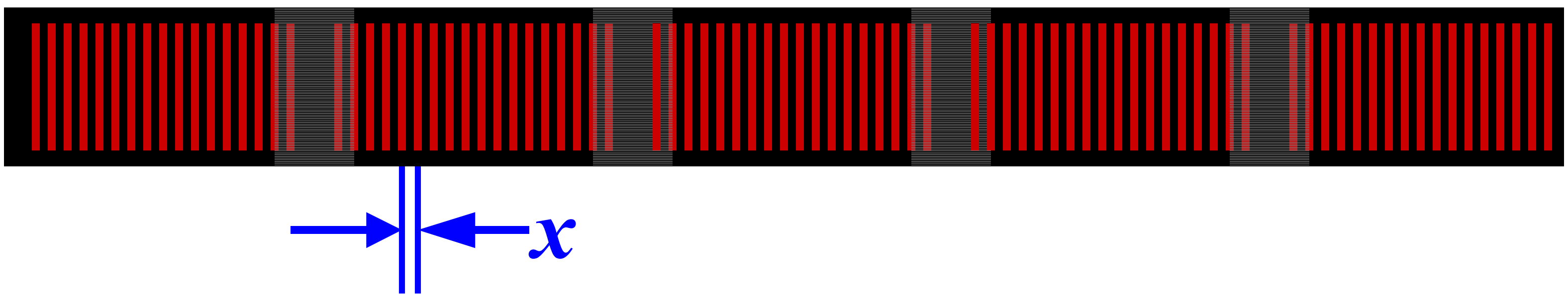
$$= .10, .20, .30, .04, \cdots$$

Two-slit:

$$\theta = m\lambda / d = .02083m$$

$$= .0208, .0416, .0625, .0833, .1042, .1250, \cdots$$

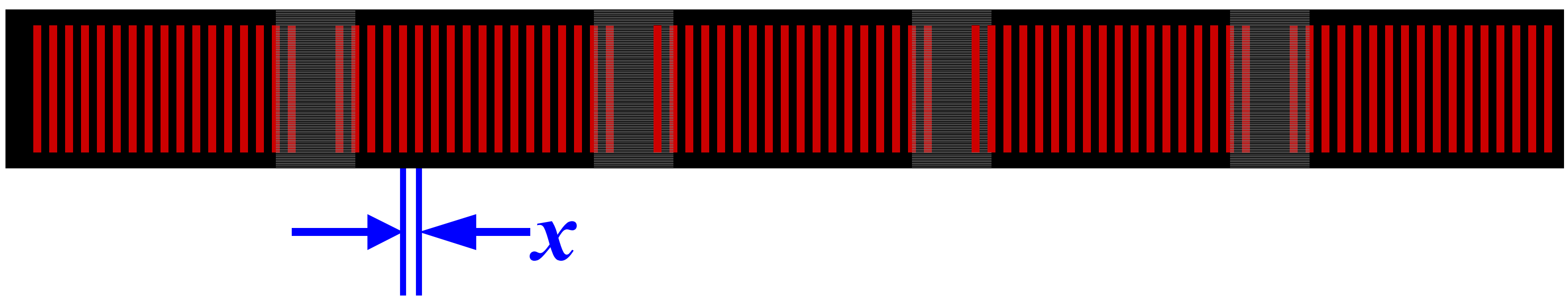
Q.36-2 When red laser light is diffracted by two slits of equal width, there are many closely spaced bands of light inside wider bands. What features of the slits determine the separation x between the closely-spaced bands?



(1) Width of the individual slits.
(2) Distance between the two slits.
(3) Ratio of distance to width.
(4) None of the above: it's more complicated.

Q.36-2

When red laser light is diffracted by two slits of equal width, there are many closely spaced bands of light inside wider bands. What features of the slits determine the separation x between the closely-spaced bands?



(1) Width of the individual slits.

(2) Distance between the two slits.

(3) Ratio of distance to width.

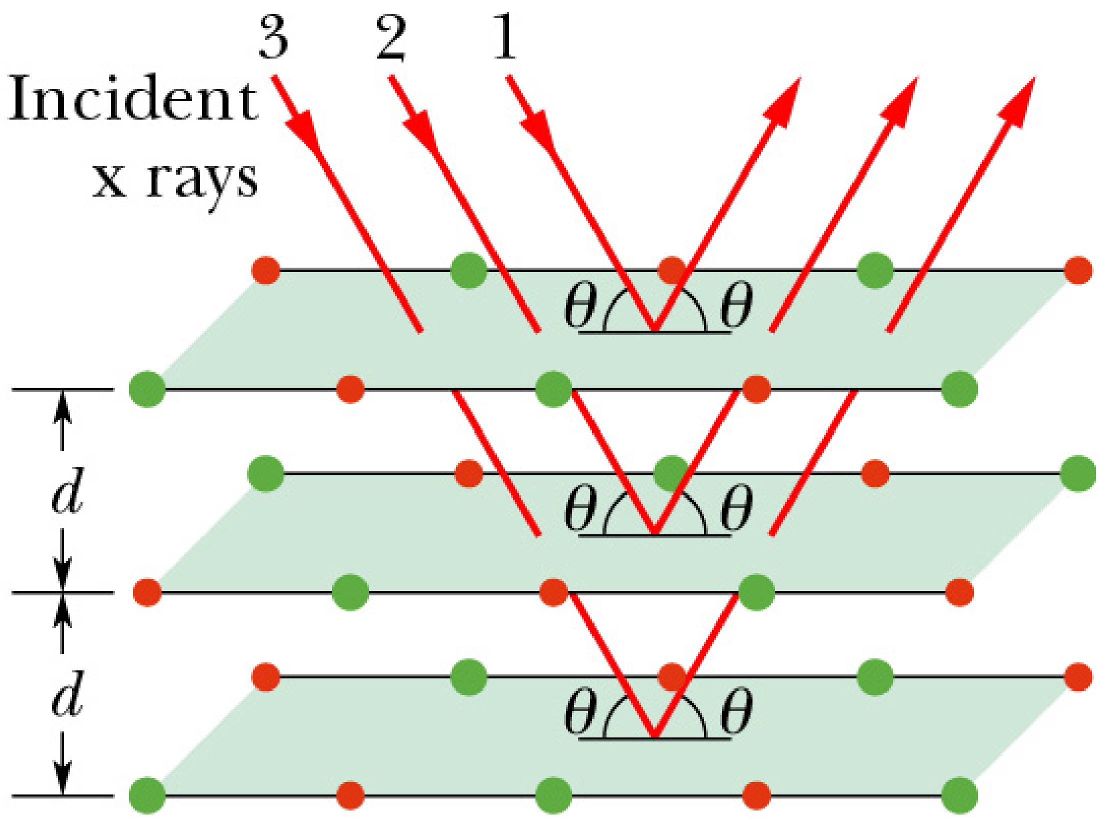
(4) None of the above: it's more complicated.

X-Rays

- **X-rays are just light waves with shorter wavelengths and higher photon energies.**
- **Since X-ray wavelengths are comparable to atomic sizes, they are perfect for studying atoms and the arrangement of atoms in crystals.**

X-Rays and Crystals

A crystal surface acts like a diffraction grating for X rays.



Bragg condition for a bright spot:

$$\boxed{2d\sin\theta = m\lambda}$$

X-ray crystallogrphy

In this way, using xrays of known wavelength we can measure the distances between atoms in a crystal and determine the crystal structure.

Single-slit Intensity

- We know where to find the dark fringes in the single-slit pattern. But can we calculate the actual intensity at a general point?

- Yes, using the phasor method.

- Book gives result on page 998: $$\frac{I_\theta}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2$$

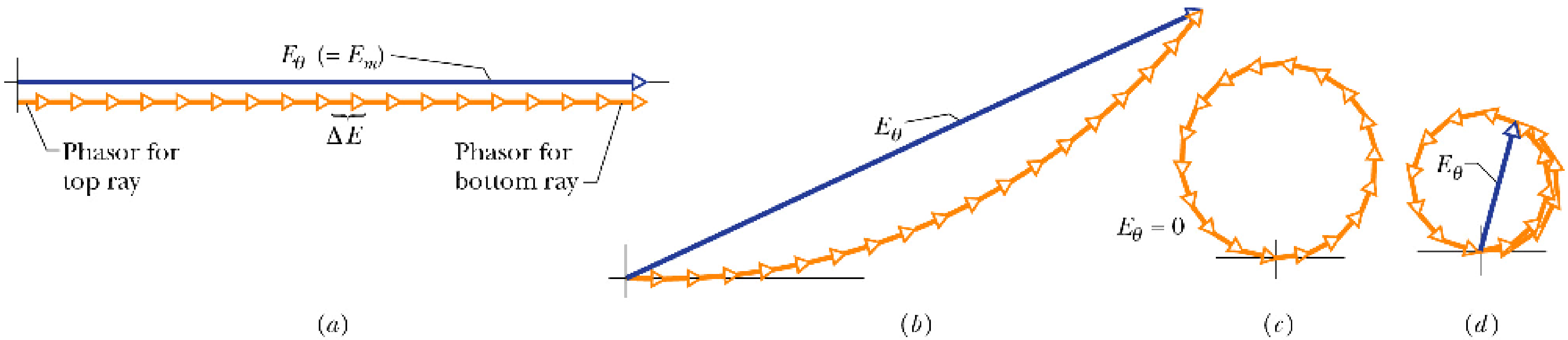
Here I_θ is the intensity at angle θ on the screen.

I_m is the intensity at the central maximum.

The angle $\alpha = \varphi/2$, and φ is the phase difference between the rays from top and bottom of slit.

Phasors for Single Slit

Break up the slit into ***many*** tiny zones, giving ***many*** rays of light, which come together on the screen.



$\Delta\phi =$ Phase difference between ***adjacent*** rays

$\phi =$ Phase difference between top and bottom rays

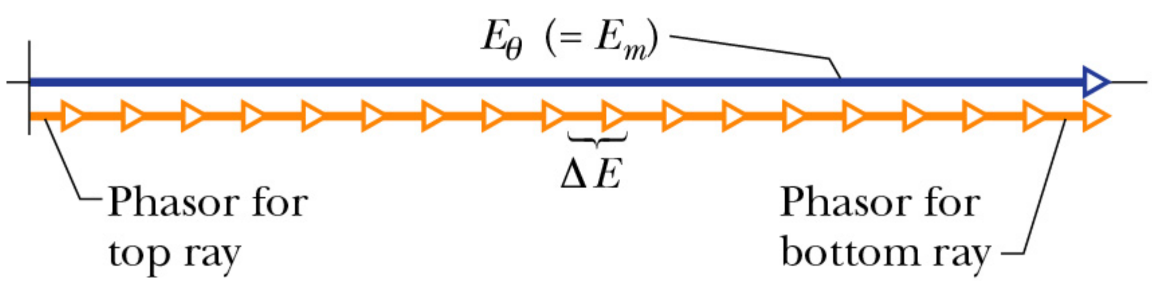
$E_m =$ **Amplitude at center = Sum of all phasors**

$E_\theta =$ **Amplitude at angle θ, get from diagram**

First Maximum and Minimum

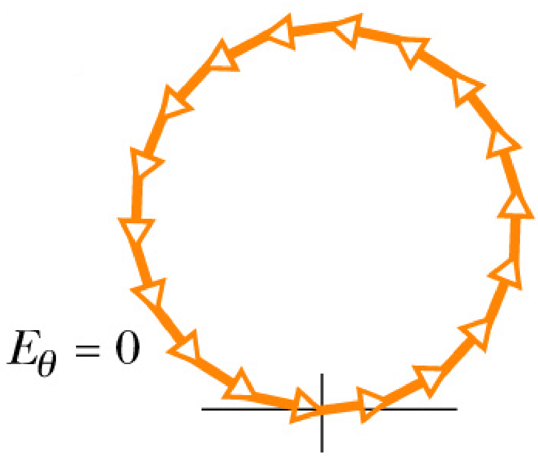
Remember of course the relation between phase difference and path difference

$$\phi = \left(\frac{2\pi}{\lambda}\right)(a\sin\theta)$$



$$\phi = 0$$

$$E_\theta = E_m$$

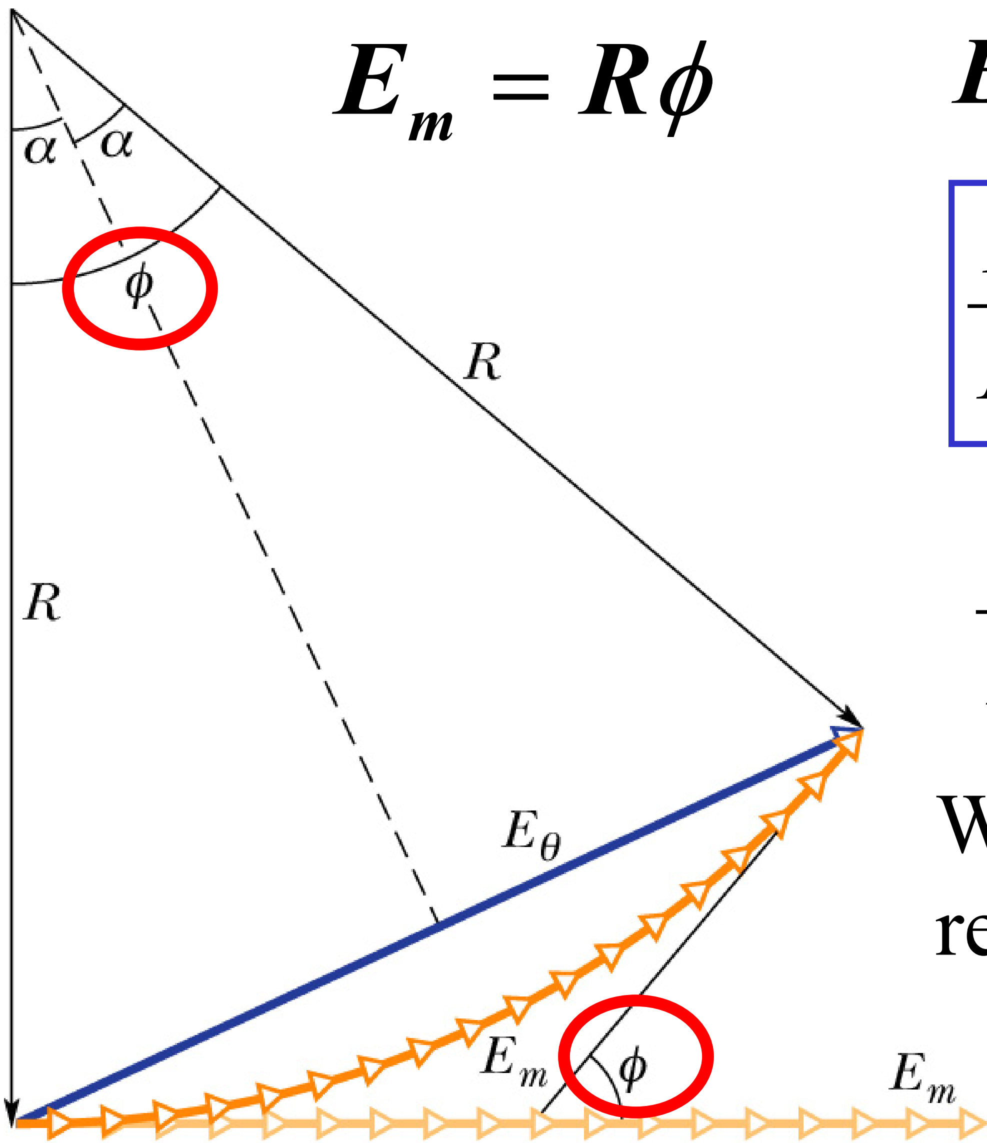


$$E_\theta = 0$$

$$\phi = m(2\pi)$$

$$a\sin\theta = m\lambda$$

Intensity for Single Slit



$$E_m = R\phi$$

$$E_\theta = 2R\sin(\phi/2)$$

$$\boxed{\frac{E_\theta}{E_m} = \frac{2\sin(\phi/2)}{\phi}}$$

$$\frac{I_\theta}{I_m} = \frac{4\sin^2(\phi/2)}{\phi^2}$$

Which gives the textbook result:

$$\frac{I_\theta}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2$$