

Magnetic Fields

- Ch.28: The magnetic field: Lorentz Force Law
- Ch.29: Electromagnetism: Ampere's Law

HOMWORK

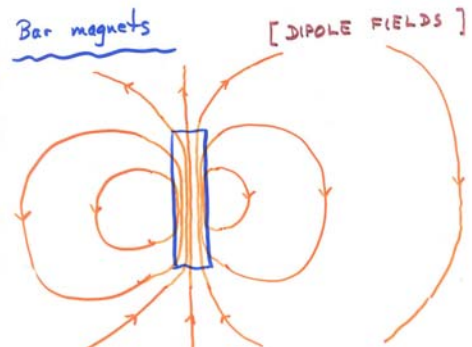
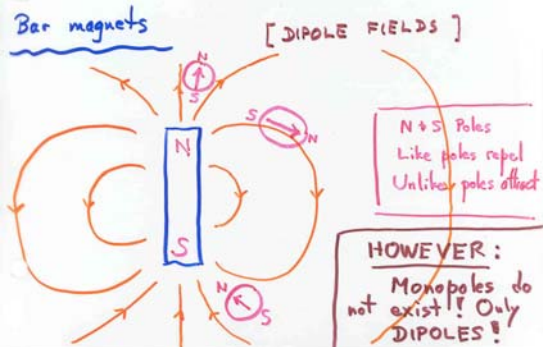
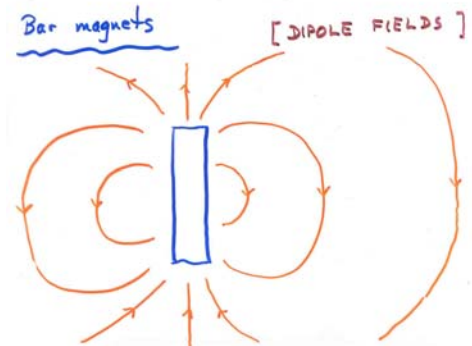
- Read Chapters 28 and 29
- Do Chapter 28 Questions 1, 7
- Do Chapter 28 Problems 3, 15, 33, 47

Today

- The Magnetic Field B .
 - Field lines
 - Direction: compass needle
- Gauss's Law for B .
- The Lorentz Force.
- Force on current-carrying wire.
- Motion of charged particles in uniform B field.
- Vector cross product and right-hand rule!

The Magnetic Field

- Another vector field \vec{B} (\vec{F}).
- Lines of \vec{B} :
 - Direction indicated by compass needle.
 - Never begin or end.
 - Density indicates field strength.



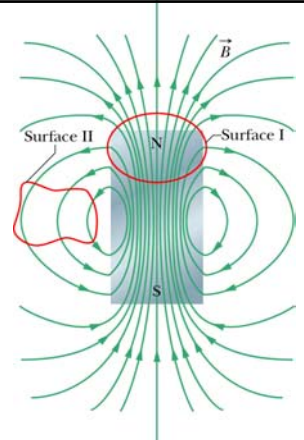
Gauss's Law for Magnetism

- Outward *electric* flux = enclosed charge
- Outward *magnetic* flux = *zero*.
- “There are no magnetic monopoles.”
- This is Maxwell Equation #2:

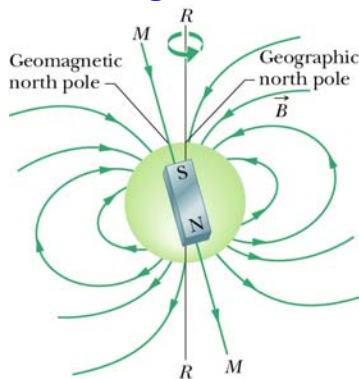
$$\oint \vec{B} \cdot d\vec{A} = 0$$

Bar magnet

Possible closed Gaussian surfaces shown in red. Zero net outward flux in both cases.



Magnetic Field of Earth



Approximately a dipole field.

The Magnetic Force

If a particle with electric charge q moves with velocity \vec{v} through a magnetic field \vec{B} , then the force by the field on the particle is

$$\vec{F} = q\vec{v} \times \vec{B}$$

Unlike the electric force $F = qE$, the magnetic force on a charged particle is **NOT** in the direction of the magnetic field. In fact, it is *perpendicular to it*.

Cross Product of Two Vectors

Given any two vectors \vec{A} and \vec{B} , and θ the angle between them, we *define* the *vector product* (cross product)

$$\vec{C} = \vec{A} \times \vec{B} :$$

(1) $C = AB \sin \theta$

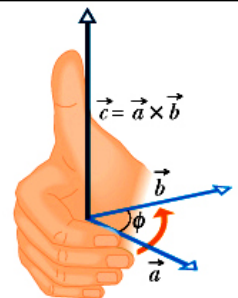
(2) \vec{C} is perpendicular to both \vec{A} and \vec{B}

(3) The direction of \vec{C} is given by the right hand rule

The Right-Hand Rule

$$\text{If } \vec{C} = \vec{A} \times \vec{B}$$

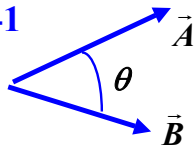
We use the right-hand rule to find the **direction** of the vector \vec{C} .



Use the fingers of your right hand to rotate \vec{a} toward \vec{b} , then your thumb points in the direction of $\vec{a} \times \vec{b}$.

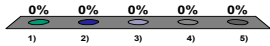
Given vectors \underline{A} and \underline{B} with angle θ between them. For fixed magnitudes A, B, for what value of θ will the value of $\underline{A} \times \underline{B}$ be a maximum?

Q.28-1



- 1) 0°
- 2) 30°
- 3) 45°
- 4) 60°
- 5) 90°

20



Q.28-1

Given any two vectors A and B , and θ the angle between them, we *define* the *magnitude* of the cross product as

$$C = AB \sin \theta$$

But $\sin \theta$ has its maximum value when $\theta = 90^\circ$

So if A and B are perpendicular the magnitude of the cross product is a maximum, and is just AB.

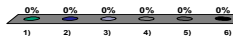
Q.28-2

Suppose I have vector \underline{A} pointing to the east and vector \underline{B} pointing to the north.

What is the direction of $\underline{A} \times \underline{B}$?

- 1) North
- 2) South
- 3) East
- 4) West
- 5) Up
- 6) Down

20

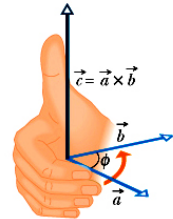


Q.28-2

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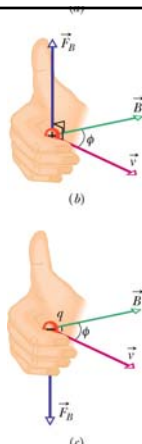
- 1) North
- 2) South
- 3) East
- 4) West
- 5) Up
- 6) Down



The Right-Hand Rule

If you use the fingers of your right hand to rotate \underline{v} toward \underline{B} , then your thumb points in the direction of $\underline{v} \times \underline{B}$.

So in the figure the force on $+q$ is upward, but the force on $-q$ is downward.



Units

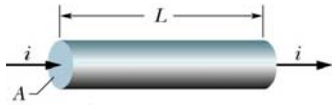
The SI unit for the magnetic field is the *tesla* (T). Since $F = qvB$, we have $B = F/qv$ and $1T = 1Ns/Cm$.

Another unit sometimes used is the *gauss* (G).

$$1T = 10^4 G$$

The field of the earth is typically about 1 G.

Force on Current-carrying Wire



For wire perpendicular to B we have

$$F = qvB$$

But $i = \left(\frac{q}{L}\right)v$

So $F = qvB = \underline{BiL}$

Force on length L

If B is at angle θ with wire: $F = BiL \sin(\theta)$

The Magnetic Force

If a particle with electric charge q moves with velocity v through a magnetic field B , then the force by the field on the particle is

$$\vec{F} = q\vec{v} \times \vec{B}$$

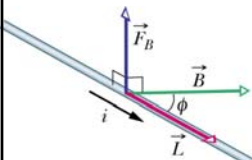
If a wire of length L carries a current i through a field B , the force by the field on the wire is

$$\vec{F} = i\vec{L} \times \vec{B}$$

Example

$$\vec{F} = i\vec{L} \times \vec{B}$$

Given wire in field with angle $\phi = 37^\circ$. $B = 0.3T$, $i = 20 \text{ mA}$. Find force per unit length on wire.

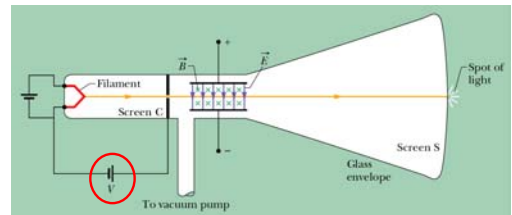


(1) Direction of force:
By right-hand rule, force is upward as shown.

(2) Magnitude of force:
 $F = i|\vec{L} \times \vec{B}| = iLB \sin \phi$

$$F / L = iB \sin \phi = .02 \times .3 \times .6 = 3.6 \times 10^{-3} \text{ N / m}$$

Cathode Ray Tube

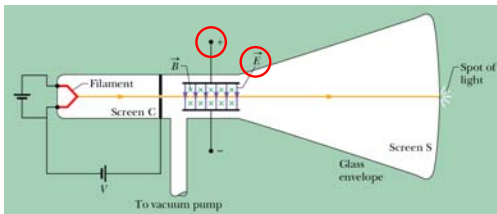


Electron gun: potential V gives electron energy in eV.

$$K = \frac{1}{2}mv^2 = qV$$

So if $V = 500$ volts, electron energy is $K = 500 \text{ eV}$.

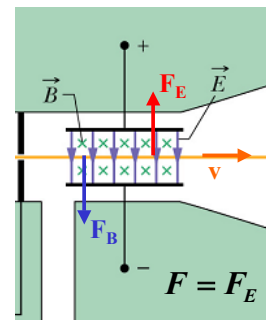
Crossed Fields



Crossed E and B fields:

If $qE = qvB$ then $F = 0$

To deflect beam upward, increase E .



Forces on electron.

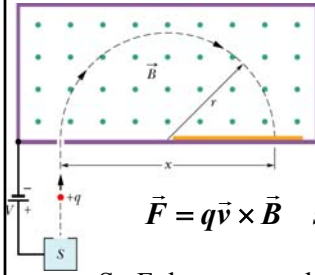
$$F_B = evB$$

$$F_E = eE$$

$$F = F_E - F_B = e(E - vB)$$

If $v = \frac{E}{B}$ then $F = 0$

Charge in Uniform B Field



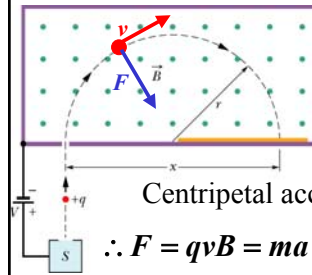
Positive ion enters field with energy

$$K = \frac{1}{2}mv^2 = qV$$

$$\vec{F} = q\vec{v} \times \vec{B} \quad \text{so} \quad \vec{F} \cdot \vec{v} = 0$$

So F does no work so K remains constant and so as the ion moves through field its *speed* remains constant .

Charge in Uniform B Field



B is *out of* screen so F is *toward center* so ion moves in circle with constant speed.

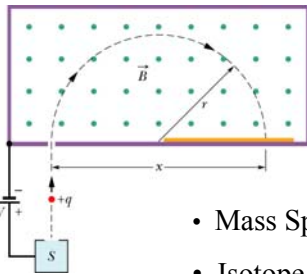
Centripetal acceleration is $a = v^2 / r$

$$\therefore F = qvB = ma = mv^2 / r$$

So solving for r gives the radius of curvature of the path:

$$r = \frac{mv}{qB}$$

Charge in Uniform B Field



Applications

- Mass Spectrometer
- Isotope Separator
- Particle Accelerator

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