INTERFERENCE

- **Today Ch. 35 Interference**
  - **Review of the general idea**
  - **Two slits**
  - **Multiple slits**
  - **Intensities**

Review of Interference

Adding two waves of the same frequency:

$$E_1 = E_1^0 \sin(kx - \omega t)$$

$$E_2 = E_2^0 \sin(kx - \omega t + \phi)$$

$$E_T = E_1 + E_2 = ?$$

Answer: $\boxed{E_T = E_T^0 \sin(kx - \omega t + \phi_T)}$

Result is a wave of the same frequency. Usually we want the ***amplitude*** E_T^0 or the ***intensity*** I_T.

Review: Phase & Path Differences

One way to get a *phase difference* Δϕ between two waves is to arrange for a *path difference* ΔL.

The *general relation* between phase difference and path difference is

$$\boxed{\Delta\phi = k\Delta L = 2\pi\frac{\Delta L}{\lambda}}$$

Remember k is phase per unit length: $E = E^0 \sin(kx - \omega t)$

Review *

- **We have discussed conditions for constructive and destructive *interference* in terms of the *phase difference* $\Delta\varphi$:**
  - **Constructive: $\Delta\varphi = 0, 360°, 720°, \ldots$**
  - **Destructive: $\Delta\varphi = 180°, 540°, \ldots$**

- **We have looked at 5 different ways to arrange for interference between two light waves:**
  - **Double slit, Reflection from glass surface, Thin films, Michelson interferometer, Different index of refraction.**

- **In most cases, we achieve a phase difference by arranging to have a *path difference* ΔL:**
  - **Constructive: $\Delta L = \lambda, 2\lambda, 3\lambda, \ldots$**
  - **Destructive: $\Delta L = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots$**

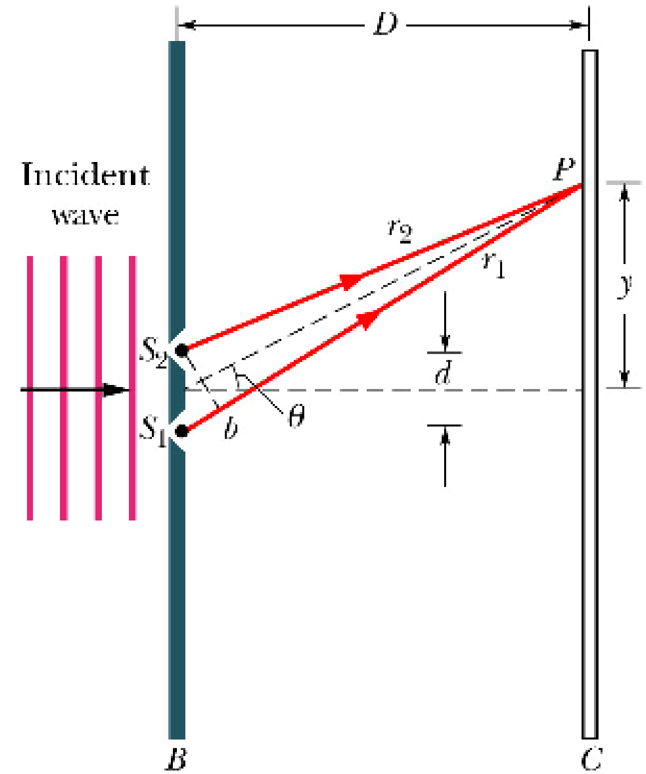
Review: Double Slit

For point P at angle θ triangle shows $$\Delta L = d\sin\theta$$

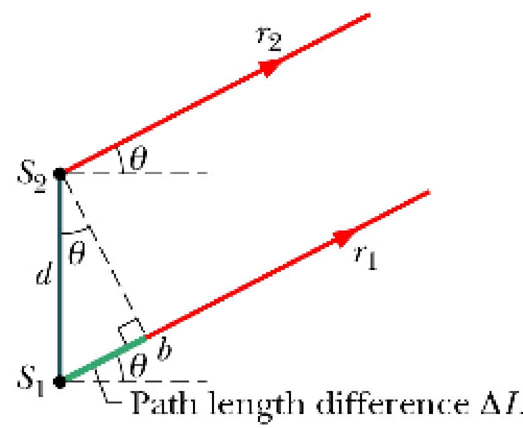
For constructive interference we need $$\Delta L = m\lambda$$

where m=0,1,2,… is any integer.

So the bright fringes are at angles given by $$\boxed{d\sin\theta = m\lambda}$$



(a)



(b)

Bright and Dark Fringes

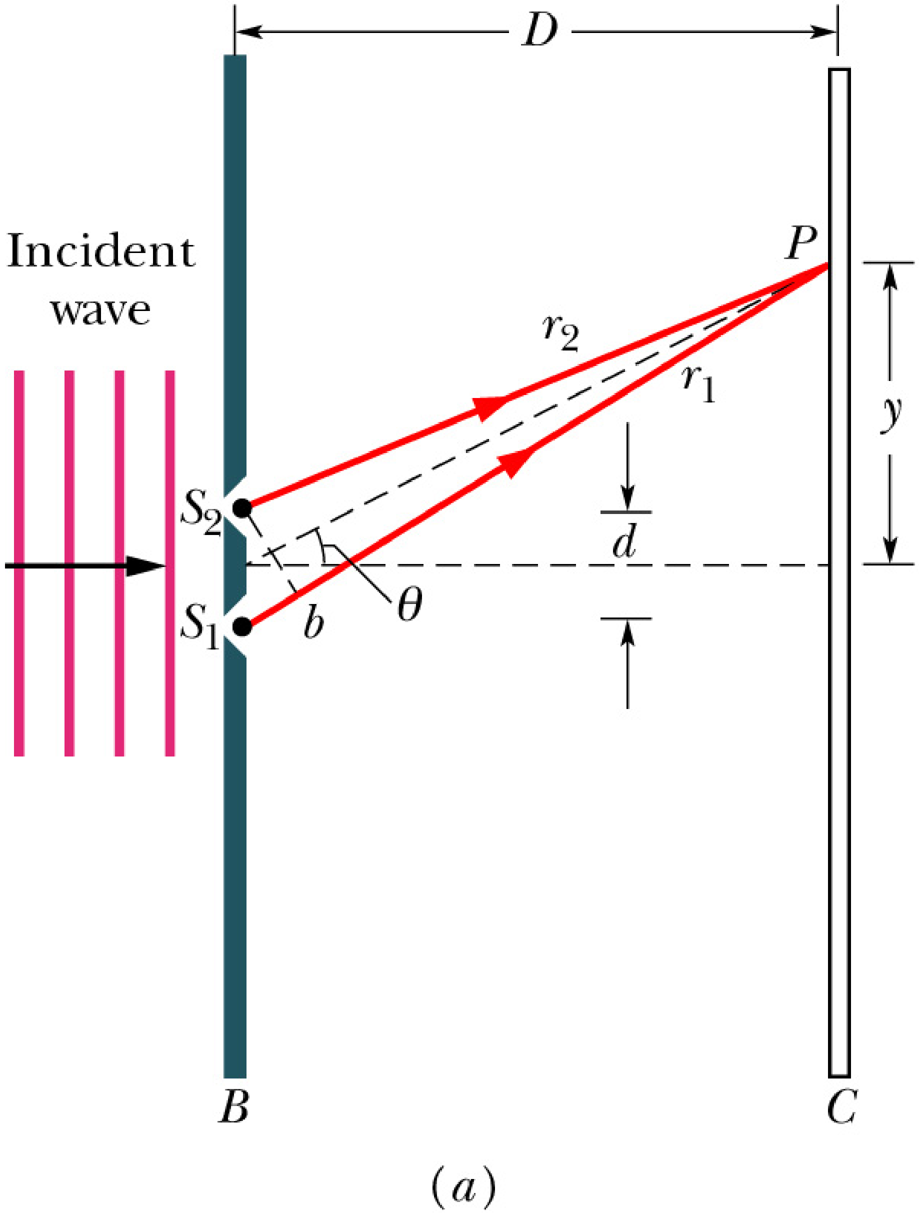
So the *bright fringes* are at angles given by

$$d\sin\theta = m\lambda$$

And the *dark fringes* are at angles given by

$$d\sin\theta = (m + \frac{1}{2})\lambda$$

Locating the Fringes



For a bright spot we need $d \sin\theta = m\lambda$.

From the figure we see $\tan\theta = y / D$.

But for small angles we have $\sin\theta \approx \tan\theta$.

So the bright lines are at

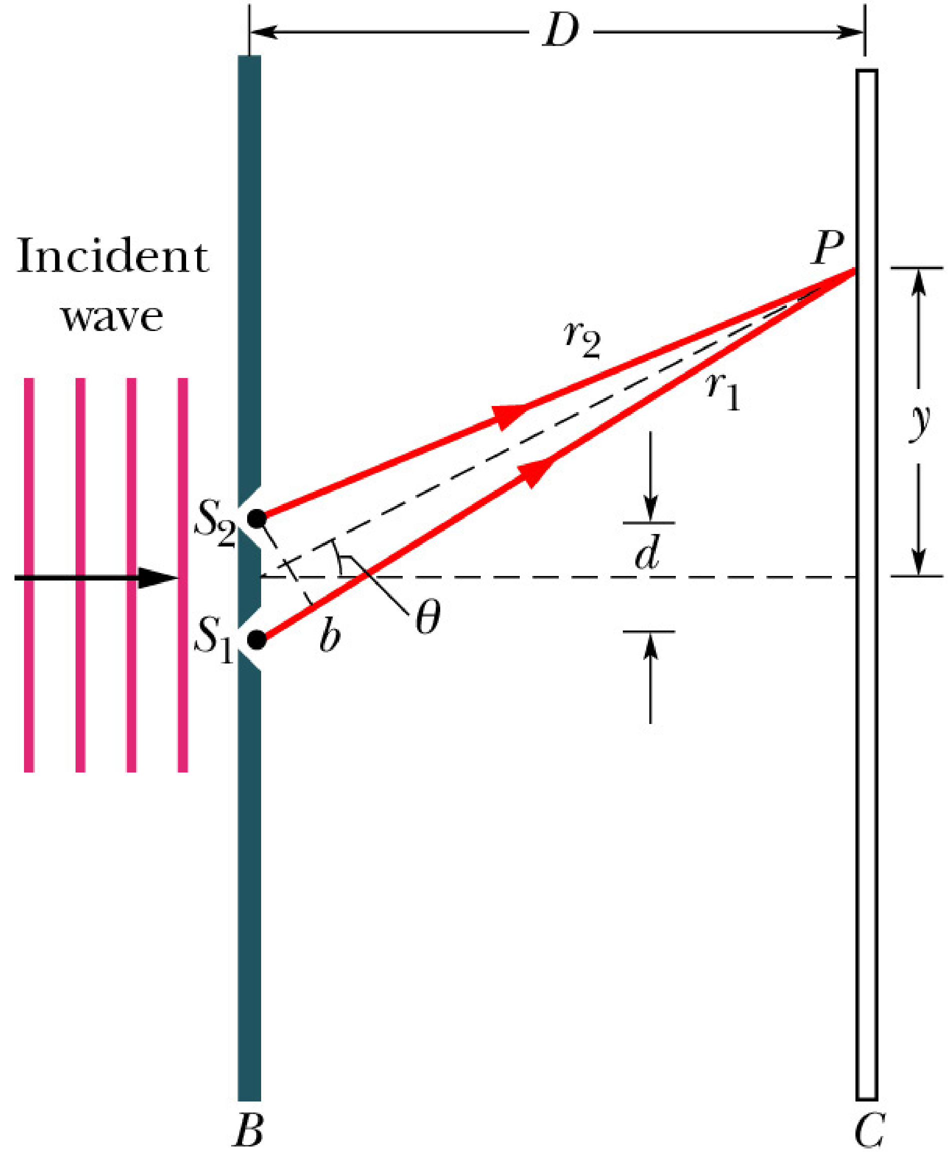
$$m\lambda = dy / D$$

$$y = m\lambda D / d$$

Q.36-1

$$\lambda = 600\ nm$$

$$r_1 - r_2 = 900\ nm$$



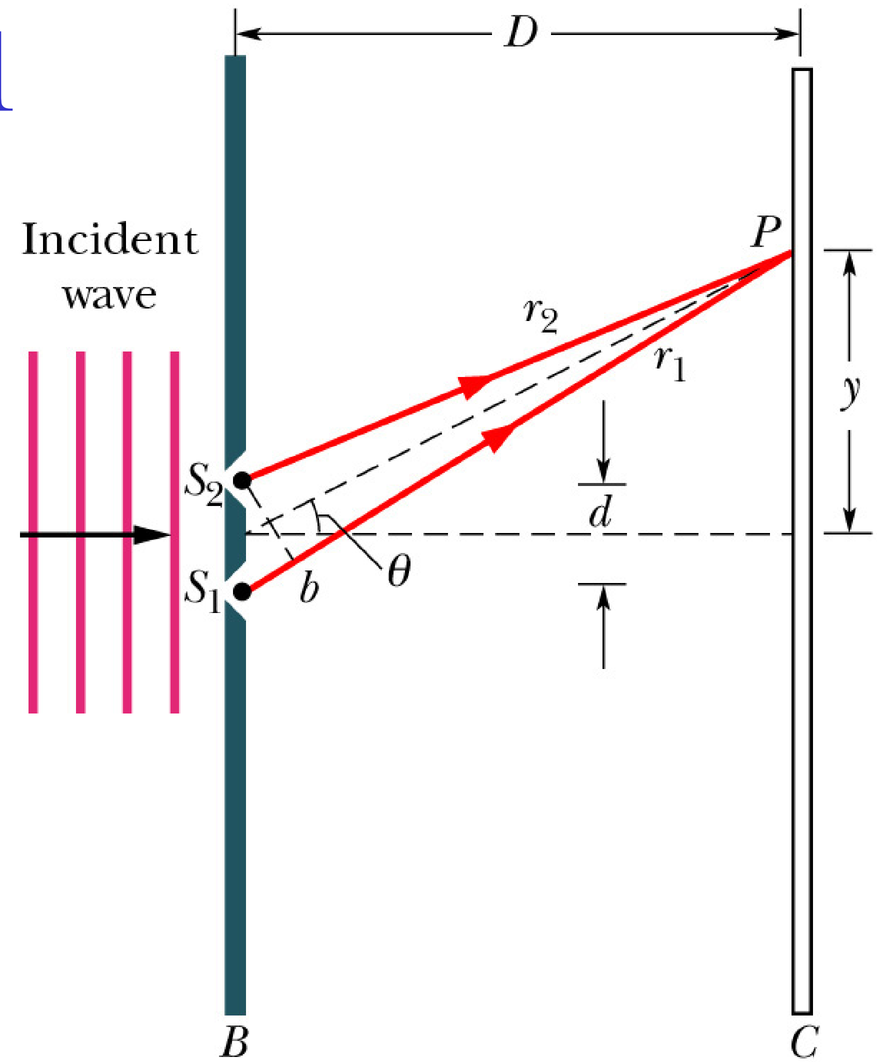
What is the phase difference between the two waves at P?

(1) $\pi/2$ (2) π (3) $3\pi/2$ (4) 2π (5) 3π

$\lambda = 600\ nm$

Q.36-1

$r_1 - r_2 = 900\ nm$



What is the phase difference between the two waves at P?

$$\Delta L = 1.5\lambda$$

$$\therefore \Delta\phi = 1.5(2\pi) = 3\pi$$

(1) $\pi / 2$ (2) π (3) $3\pi / 2$ (4) 2π $\boxed{(5)\ 3\pi}$

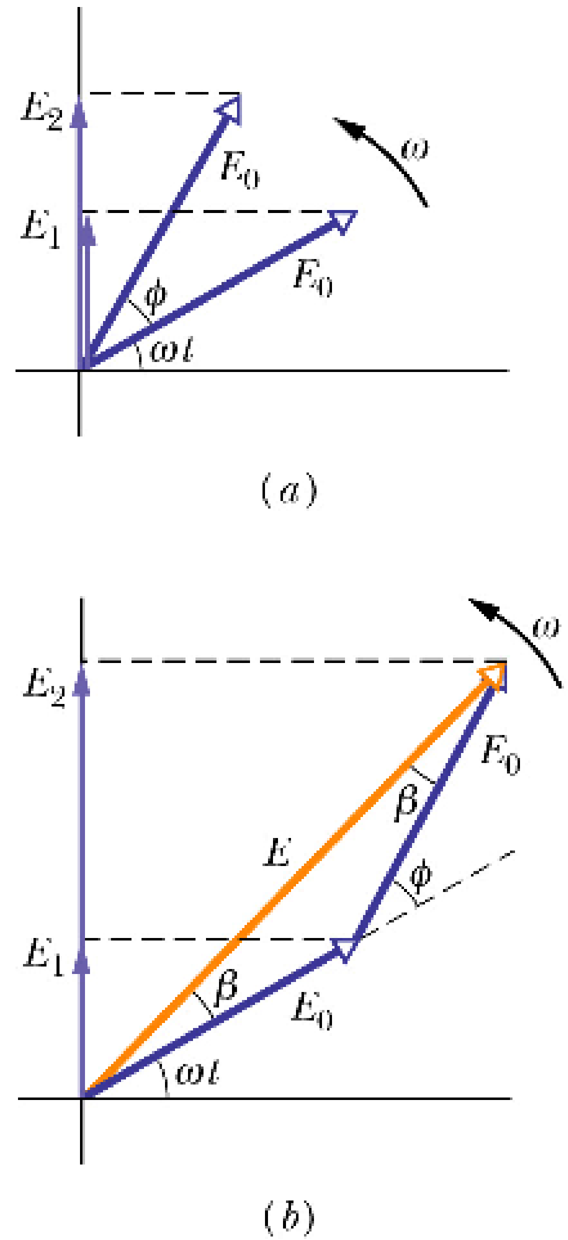
Interference: The General Case

What if the phase difference neither 0 nor 180° but something in between? How can we calculate the resultant amplitude?

Use phasors!

Phasor Diagram

Just as for AC circuits, we can add two oscillating functions using phasors. The ***lengths*** of the phasors are the ***amplitudes*** of the waves and the ***angle*** between the phasors is the ***phase difference*** between the waves. Then the ***length*** of the resultant phasor is the ***amplitude*** of the total wave.



Adding Vectors

A good way to get the length of the sum of two vectors is to use the ***dot product:***

If $$\vec{C} = \vec{A} + \vec{B}$$

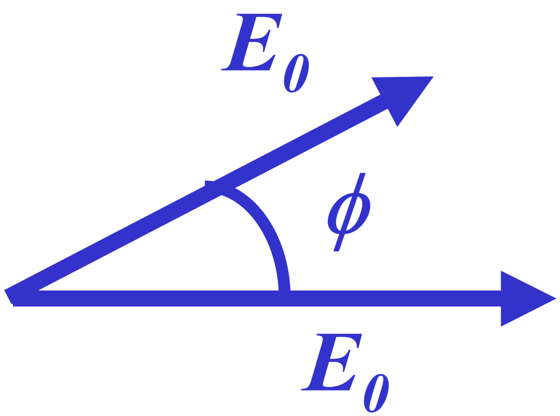
Then $$C^2 = \vec{C} \cdot \vec{C} = \left(\vec{A} + \vec{B}\right) \cdot \left(\vec{A} + \vec{B}\right)$$

$$= A^2 + B^2 + 2\vec{A} \cdot \vec{B}$$

$$= A^2 + B^2 + 2AB\cos\theta$$

Intensity Formula

Suppose two light waves have ***equal intensities*** I_0 and a ***phase difference*** of ϕ. When these waves interfere, what will be the total intensity I?



$$E^2 = E_1^{\,2} + E_2^{\,2} + 2\vec{E}_1 \cdot \vec{E}_2$$

$$E^2 = 2E_0^{\,2} + 2E_0^{\,2}\cos\phi$$

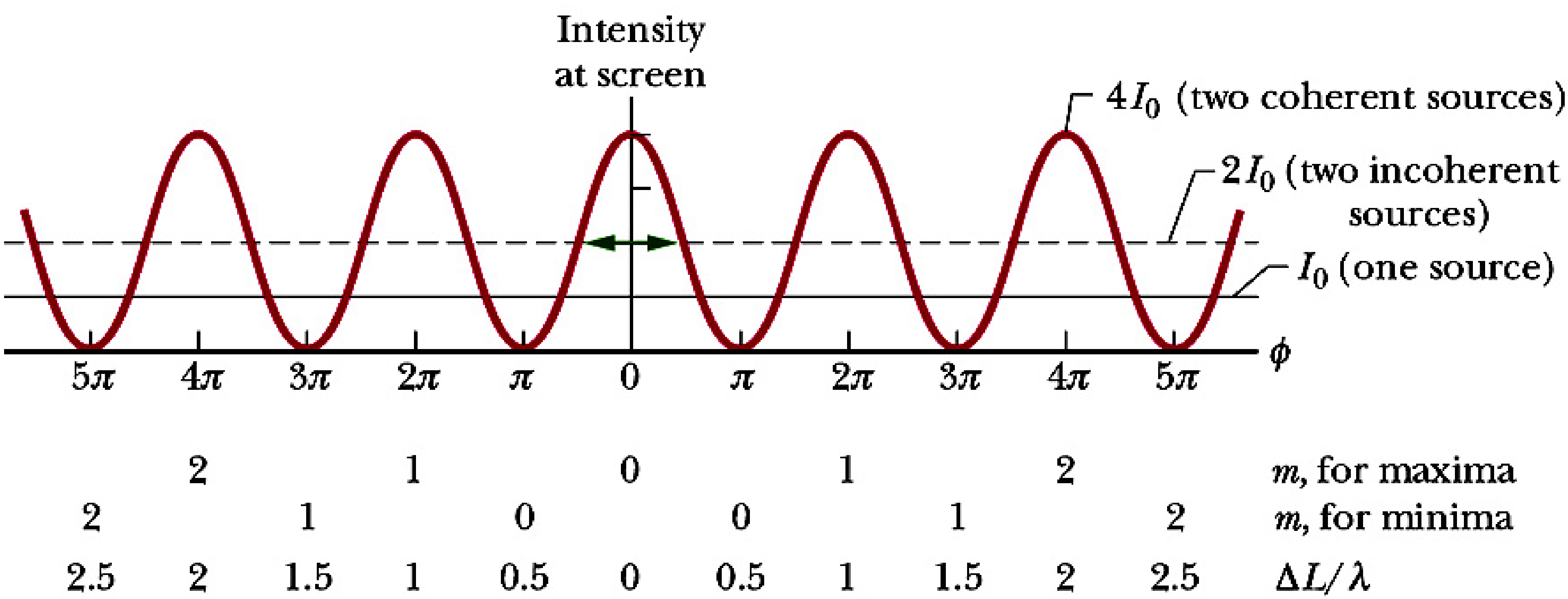
$$= 4E_0^{\,2}\cos^2\left(\frac{1}{2}\phi\right)$$

But $I \propto E^2$ ***so:***

$$I = 4I_0\cos^2\left(\frac{1}{2}\phi\right)$$

Text Eq. 36-21

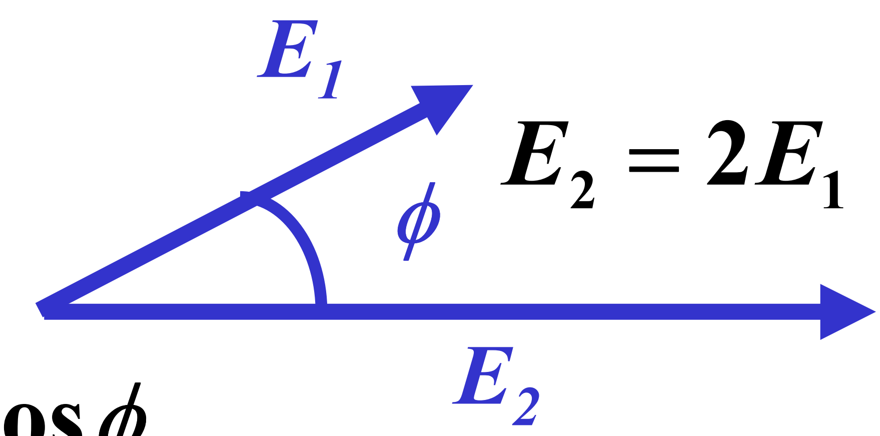
Double-slit intensity



Intensity Example

Problem 35-29.

Two waves interfere with phase difference $\phi = 60^\circ$.
One wave has intensity I_0, the other $4I_0$. What is the resulting intensity?



$$E_2 = 2E_1$$

$$E^2 = E_1^{\,2} + E_2^{\,2} + 2E_1E_2\cos\phi$$

$$E^2 = E_0^{\,2} + 4E_0^{\,2} + 2E_0(2E_0)(1/2)$$

$$= 5E_0^{\,2} + 2E_0^{\,2}$$

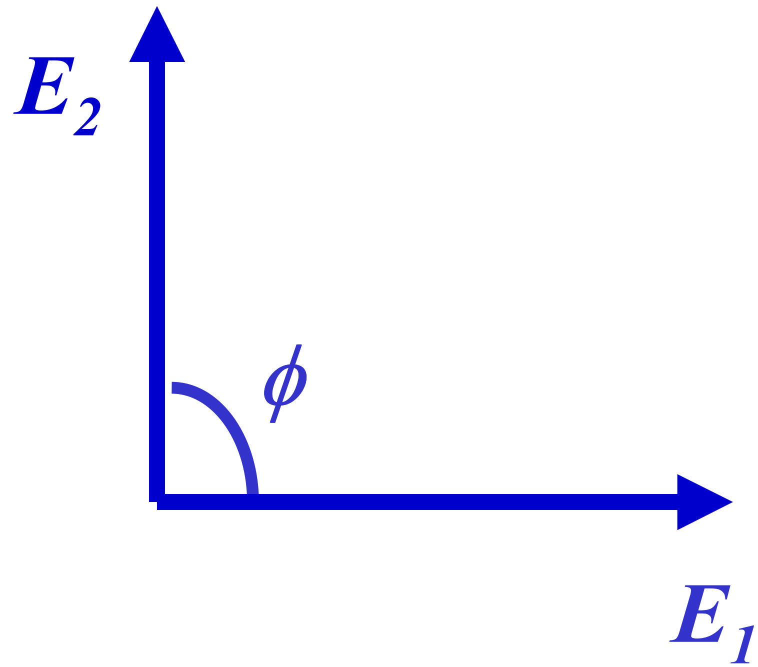
But $I = (Const)E^2$

so $I = 5I_0 + 2I_0 = \boxed{7I_0}$

Q.36-2

Two waves interfere with phase difference $\phi = 90°$

Each wave individually has intensity I_0**. What is the resulting intensity?**

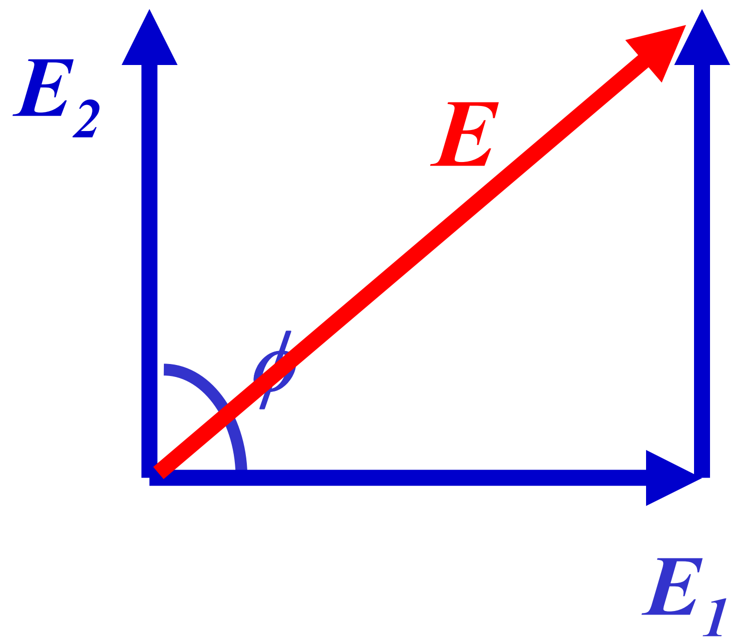


(1) $2I_0$ **(2)** $\sqrt{2}I_0$ **(3)** I_0 **(4)** $I_0/\sqrt{2}$ **(5)** $I_0/2$

Q.36-2

Two waves interfere with phase difference $\phi = 90°$

Each wave individually has intensity I_0. What is the resulting intensity?



$$E^2 = E_1^{\ 2} + E_2^{\ 2}$$

$$But\ I \propto E^2$$

$$\therefore \quad I = I_0 + I_0 = 2I_0$$

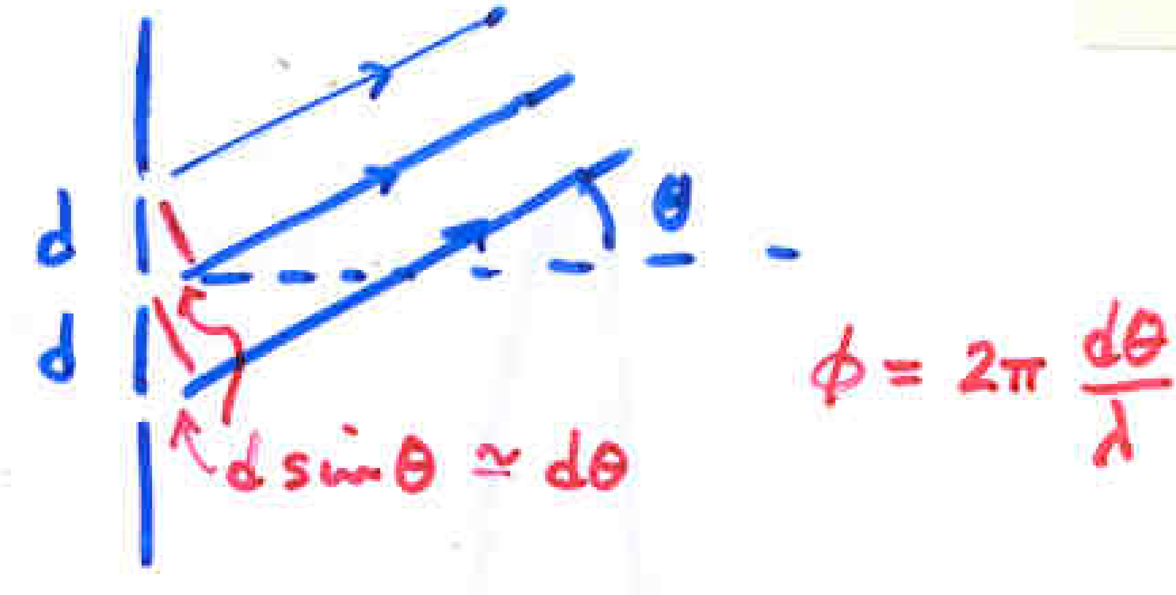
$\boxed{(1)\ 2I_0}$ (2) $\sqrt{2}I_0$ (3) I_0 (4) $I_0/\sqrt{2}$ (5) $I_0/2$

Interference with 3 Slits

Path difference between rays from adjacent slits:

$$\Delta L = d\sin\theta$$

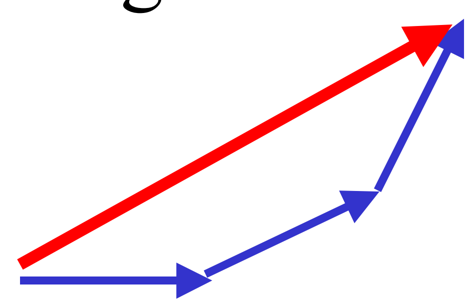
$$\theta \approx \Delta L / d$$



Phase difference between rays from adjacent slits:

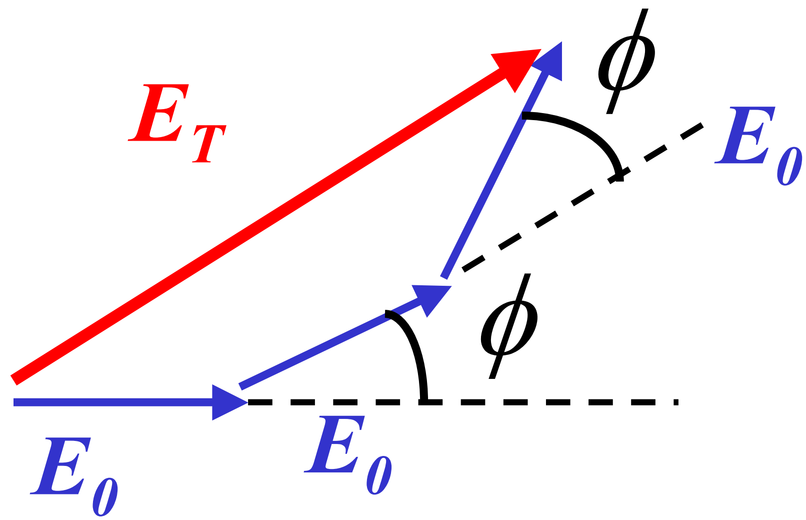
$$\phi = 2\pi\frac{\Delta L}{\lambda} = 2\pi\frac{d\theta}{\lambda}$$

Get intensity from phasor diagram:



Intensity for 3 Slits

What is intensity as a function of angle?



$$I = E_T^{\ 2}$$

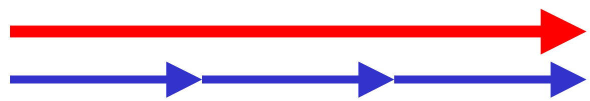
$$I_0 = E_0^{\ 2}$$

$$\phi = 2\pi \frac{\Delta L}{\lambda} = 2\pi \frac{d \sin\theta}{\lambda}$$

We could do a lot of trigonometry and figure out the general result for I as a function of I_0 and ϕ but let's not. Just get location of fringes using the phasor diagram, as θ and ϕ increase.

3 Slits Continued

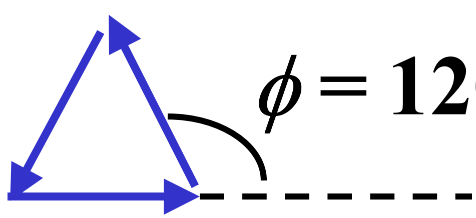
1. Central maximum:



$$\phi = 0 \qquad \theta = 0$$

$$E_T = 3E_0 \qquad I_T = 9I_0$$

2. First minimum:

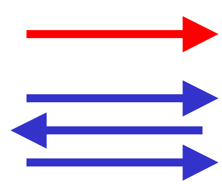


$\phi = 120°$

$$\phi = \frac{2\pi}{3} \qquad d\sin\theta = \frac{\lambda}{3}$$

$$E_T = 0 \qquad I_T = 0$$

3. Next maximum:



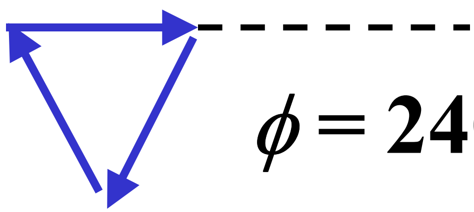
$\phi = 180°$

$$\phi = \pi \qquad d\sin\theta = \lambda/2$$

$$E_T = E_0 \qquad I_T = I_0$$

3 Slits Continued

4. Next minimum:

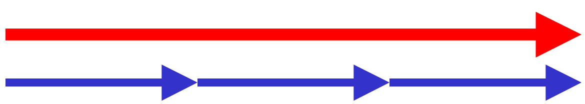


$$\phi = 240°$$

$$\phi = \frac{2}{3} 2\pi \qquad d \sin\theta = \frac{2}{3}\lambda$$

$$E_T = 0 \qquad I_T = 0$$

5. Next maximum:

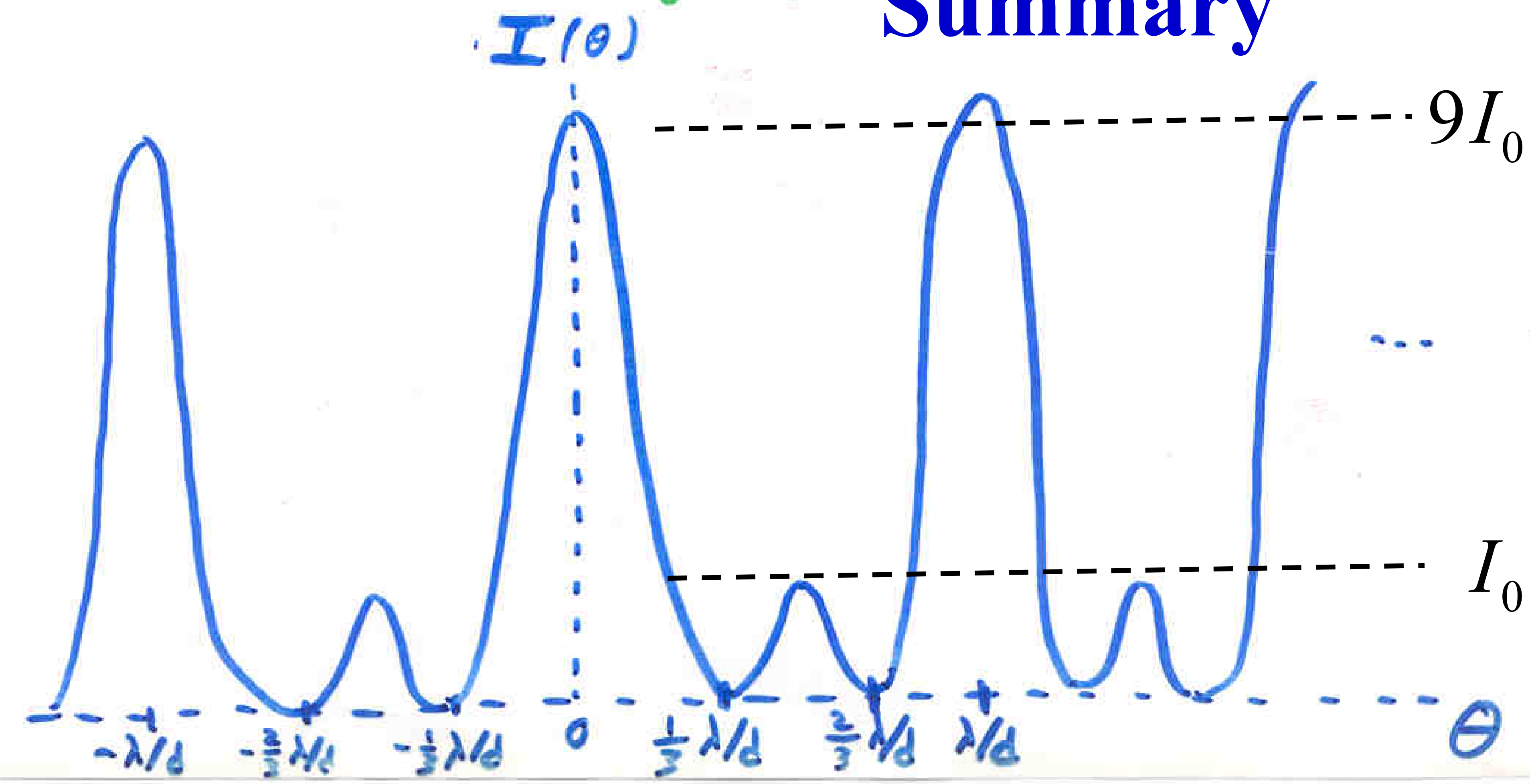


$$\phi = 360°$$

$$\phi = 2\pi \qquad d \sin\theta = \lambda$$

$$E_T = 3E_0 \qquad I_T = 9I_0$$

Summary



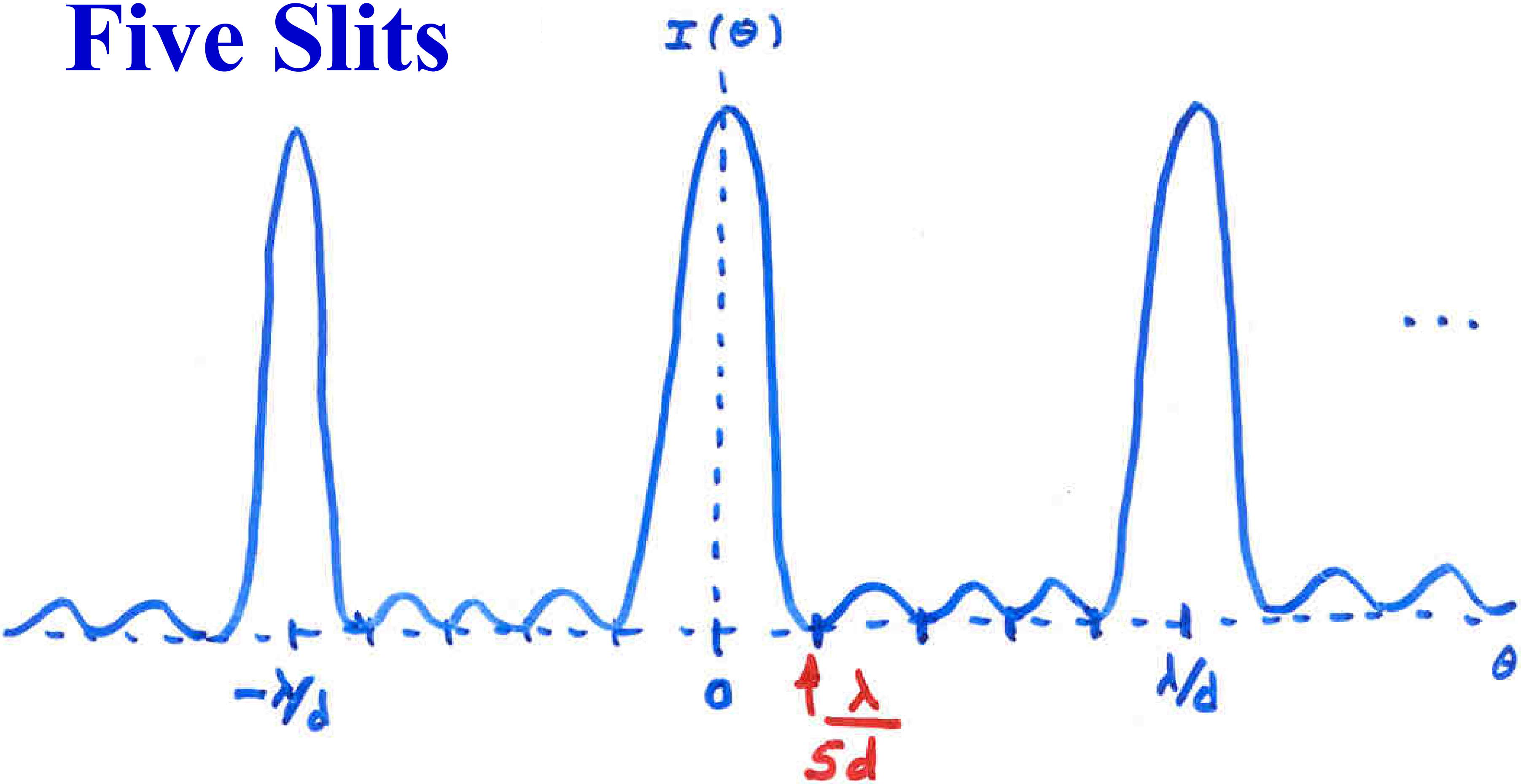
$$\phi = \quad 0 \quad \frac{2\pi}{3} \quad \pi \quad \frac{4\pi}{3} \quad 2\pi$$

$$\theta = \Delta L / d$$

$$\Delta L = \quad 0 \quad \frac{\lambda}{3} \quad \frac{\lambda}{2} \quad \frac{2\lambda}{3} \quad \lambda$$

$$\theta_{\max} = m\lambda / d$$

Five Slits



Note we still have $\theta_{\max} = m\lambda / d$
But more slits makes the peaks *sharper*.

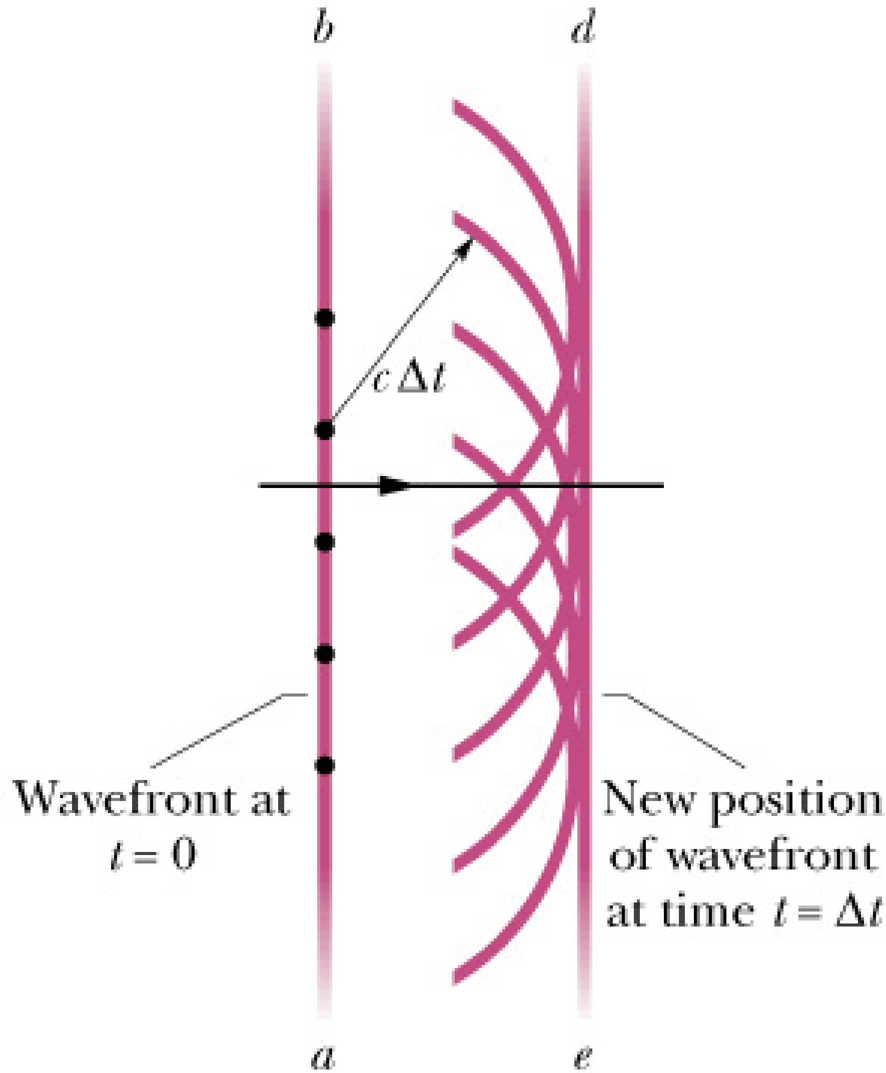
For ***<u>many</u>*** slits, we get a diffraction grating.

INTERFERENCE

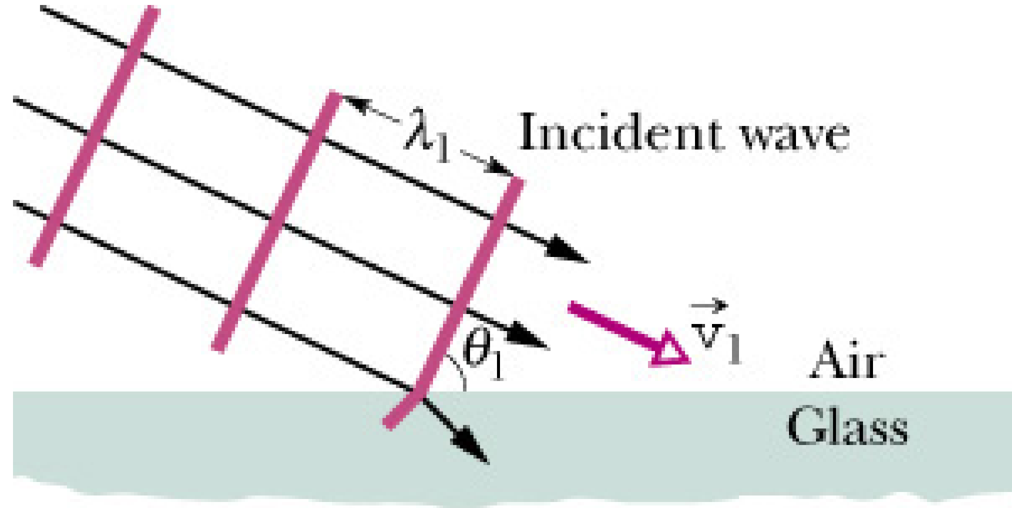
- **Today**
  - **Some loose ends**
    - **Huygens' Principle**
    - **Chromatic Dispersion**
    - **Sources of Waves**
  - **Interference review**
  - **Sample problems**

Huygens' Principle

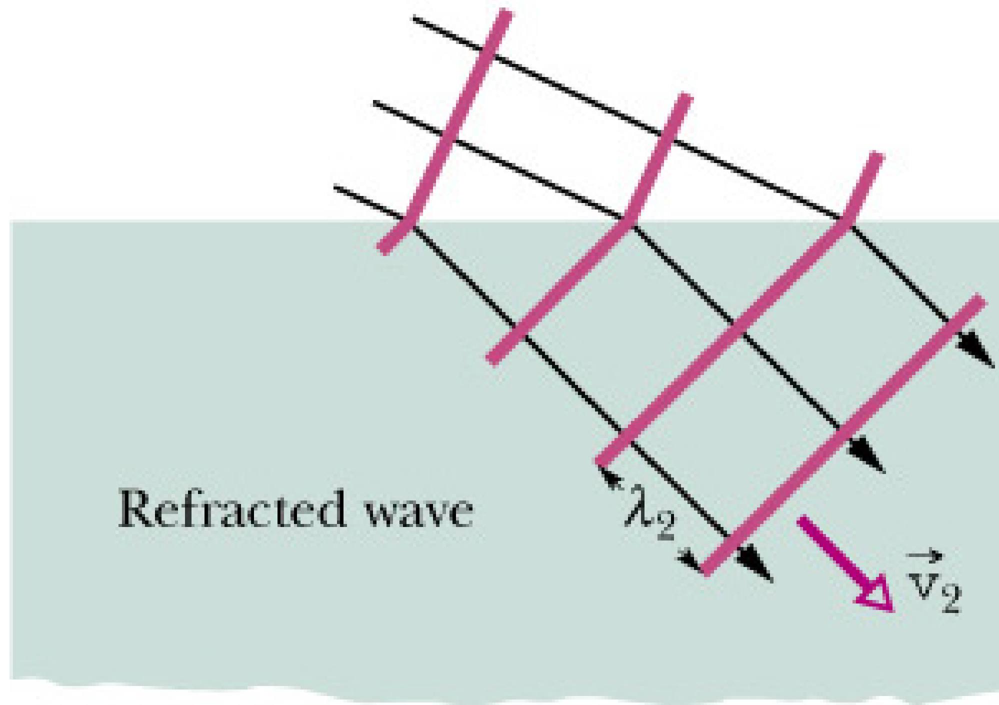
- **Follow the motion of a wave in three dimensions by drawing successive wave fronts.**
- **Velocity is perpendicular to wave front.**



Huygens Gives Refraction

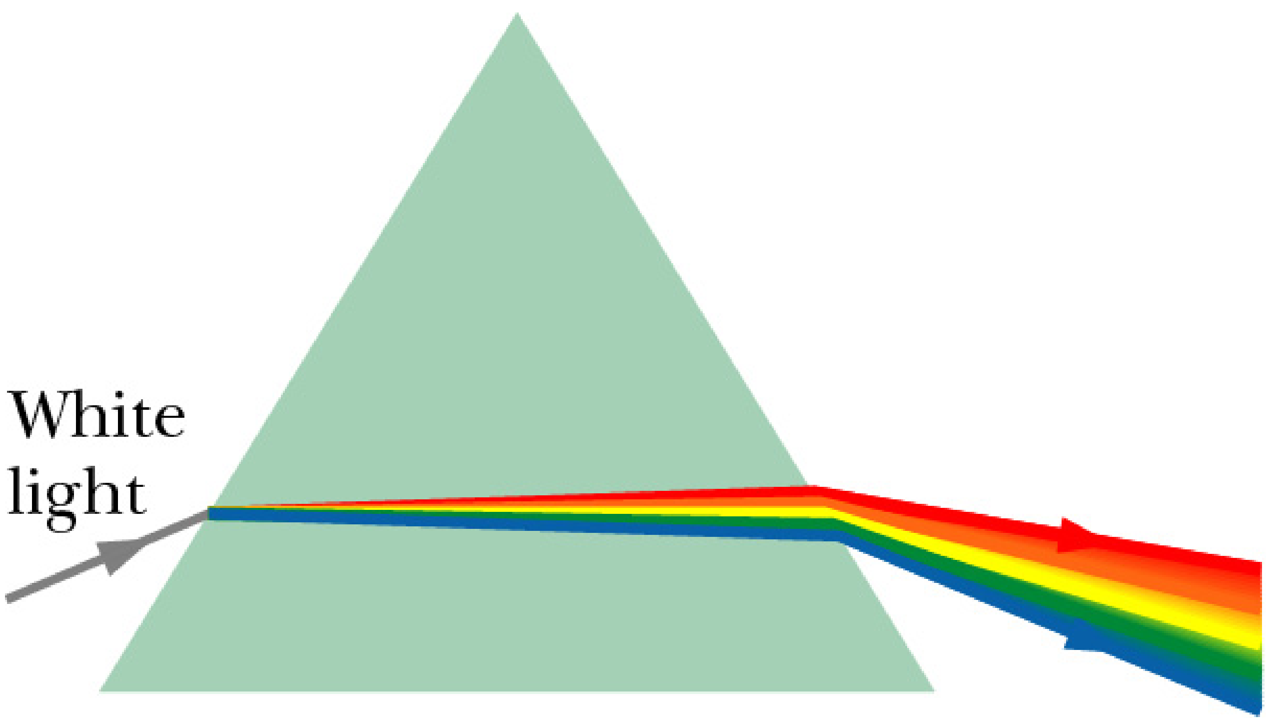


- Follow change in wave fronts when velocity decreases.
- Velocity is perpendicular to wave front.

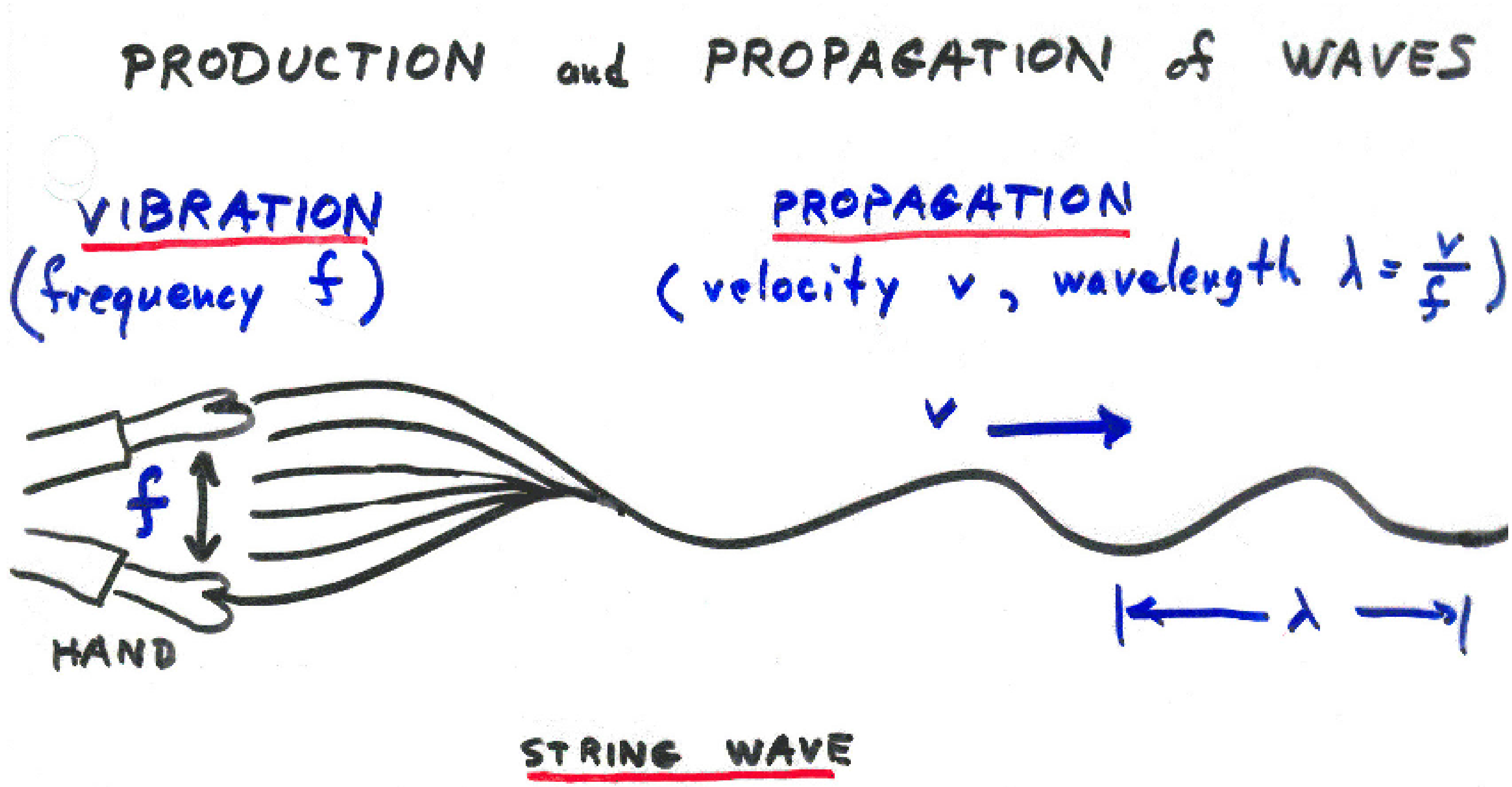


Chromatic Dispersion

This just means that the index of refraction is different for different wavelengths (colors) and so you can make a rainbow from white light using a glass prism. (Chapter 33)



Usually ***n*** is greater for blue light, so it gets bent more.



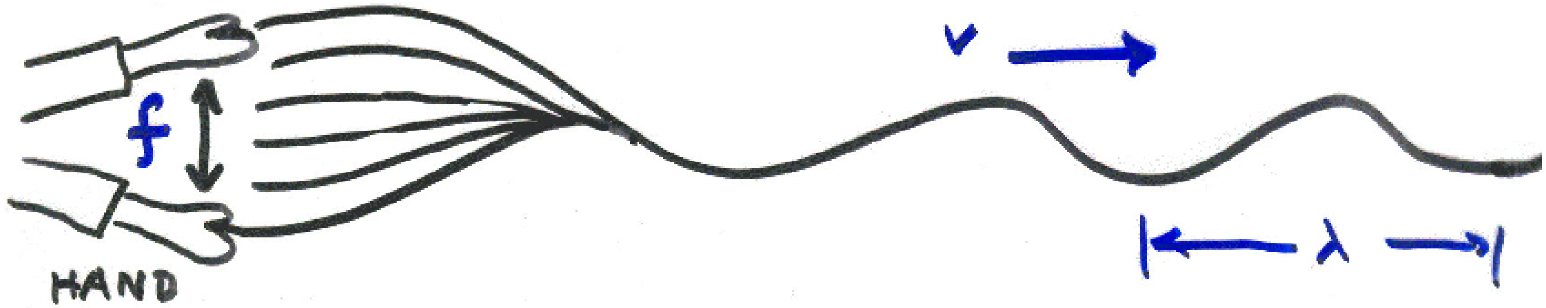
Frequency determined by source motion (hand).

Velocity determined by physics of medium (string). $v = \sqrt{\tau / \mu}$

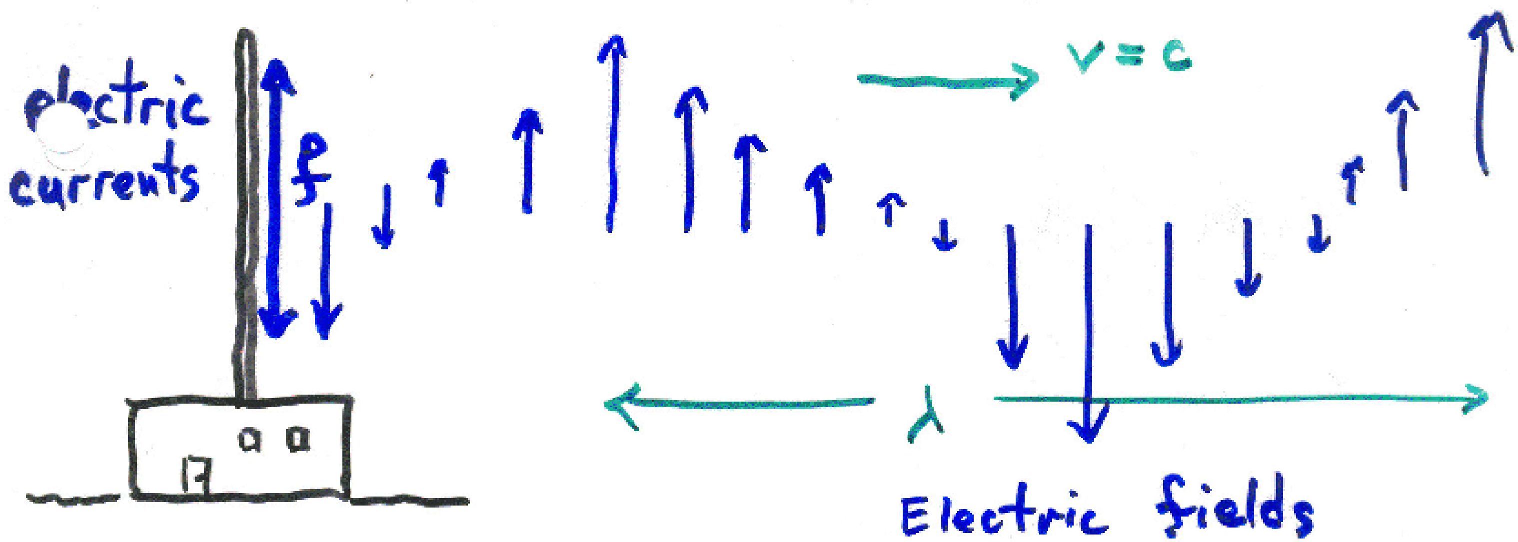
Wavelength determined by frequency and velocity: $\lambda = v / f$

VIBRATION
(frequency f)

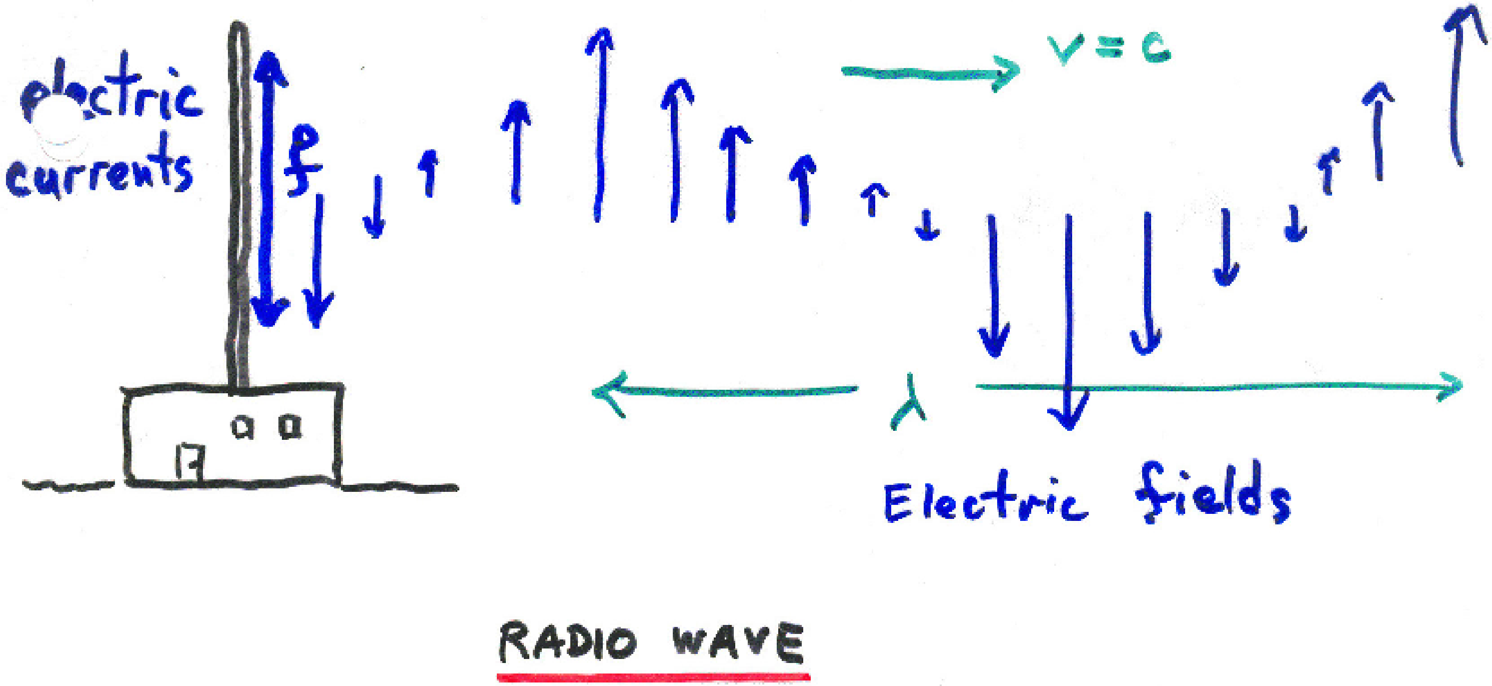
PROPAGATION
(velocity v, wavelength $\lambda = \frac{v}{f}$)



STRING WAVE



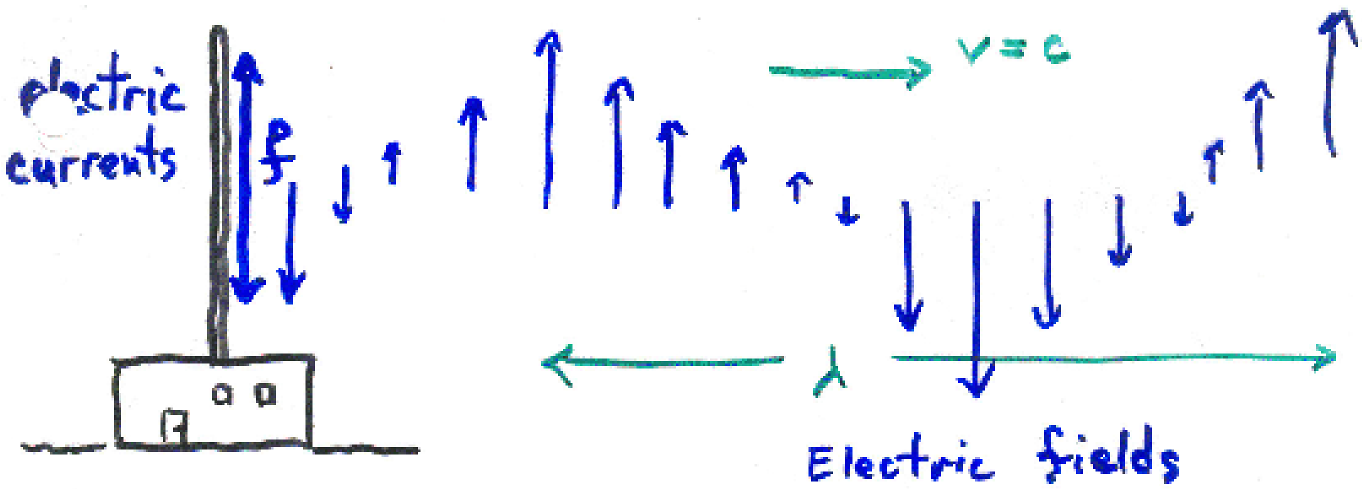
RADIO WAVE



Frequency determined by source motion (AC circuit).

Velocity determined by physics of medium: $v = c = 1/\sqrt{\varepsilon_0 \mu_0}$

Wavelength determined by frequency and velocity: $\lambda = v / f$



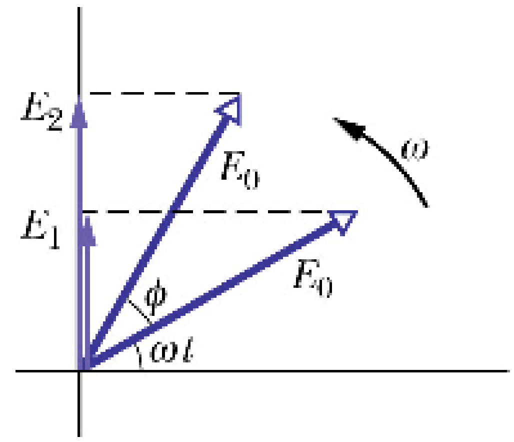
RADIO WAVE

LIGHT WAVE:

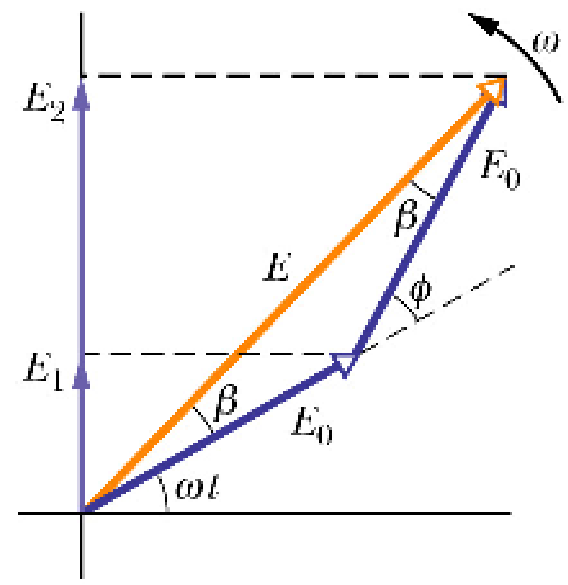
Same as radio wave except electric currents are inside atoms; also f is much larger and λ much smaller; but v is same.

Review: Phasor Diagram

Just as for AC circuits, we can add two oscillating functions using phasors. The ***lengths*** of the phasors are the ***amplitudes*** of the waves and the ***angle*** between the phasors is the ***phase difference*** between the waves. Then the ***length*** of the resultant phasor is the ***amplitude*** of the total wave.



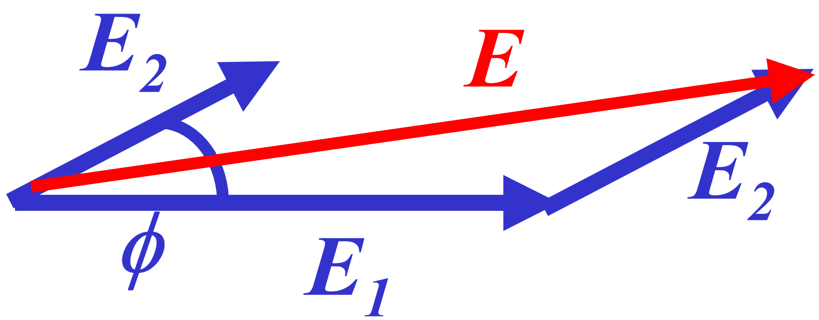
(a)



(b)

Interference of two waves

Suppose two interfering waves have *intensities* I_1, I_2 and a *phase difference* of ϕ. What is the total intensity I?



$$I = E^2 = \vec{E} \cdot \vec{E} = (\vec{E}_1 + \vec{E}_2) \cdot (\vec{E}_1 + \vec{E}_2)$$

$$= E_1^{\,2} + E_2^{\,2} + 2\vec{E}_1 \cdot \vec{E}_2$$

$$= E_1^{\,2} + E_2^{\,2} + 2E_1E_2 \cos\phi$$

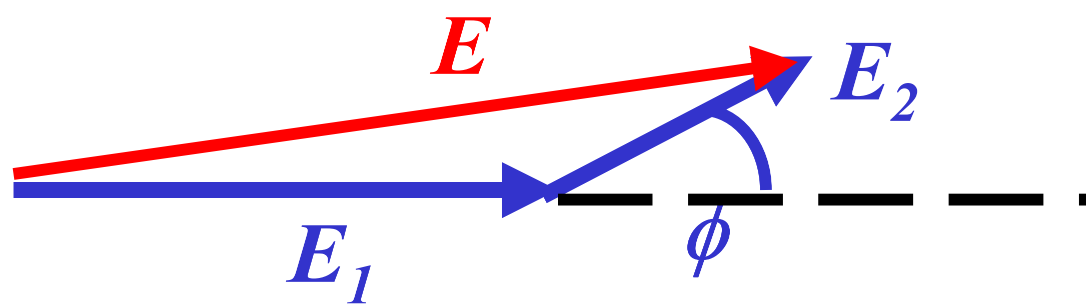
$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

Example #1

Two waves interfere with phase difference $\phi = 37°$.

One wave has intensity 25 W/m², the other 9 W/m².

What is the resulting intensity?



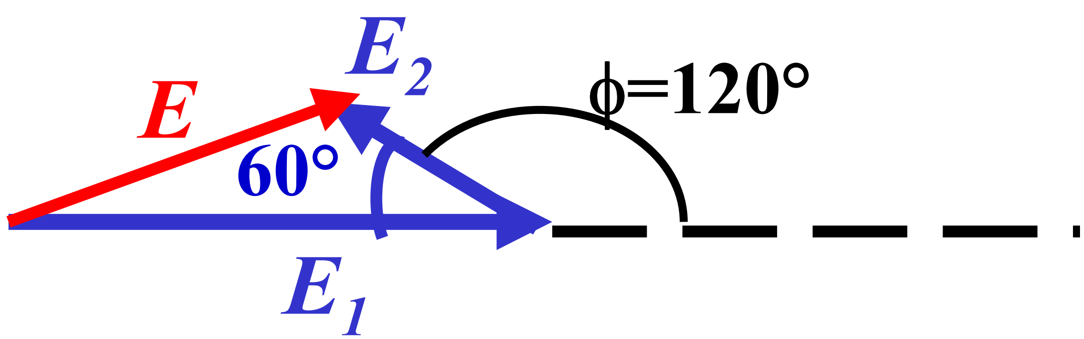
$$
\begin{aligned}
I &= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \\
&= 25 + 9 + 2\sqrt{25\cdot 9}\cos 37° \\
&= 34 + 2\cdot 5\cdot 3\cdot\left(4/5\right) = \underline{58\ W/m^2}
\end{aligned}
$$

Example #2

Two waves have intensities 25 W/m^2 and 9 W/m^2. What is the resulting intensity if they come together with a path difference of λ/3?

$$\Delta L = \lambda / 3$$

$$\Delta\phi = 360° / 3 = 120°$$



$$\cos 120° = -\cos 60°$$
$$= -1/2$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$
$$= 25 + 9 + 2\sqrt{25 \cdot 9}\cos 120°$$
$$= 34 - 2 \cdot 5 \cdot 3 \cdot (1/2) = 34 - 15 = \underline{19\ W/m^2}$$

Q.35-3

Two waves interfere with phase difference $\phi = 90°$

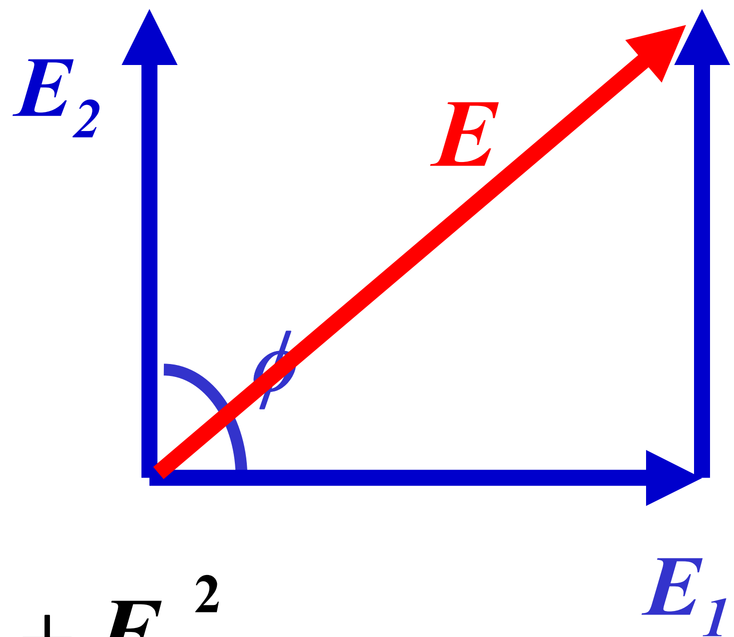
Individually the wave intensities are $I_1 = 9\ W/m^2$ and $I_2 = 16\ W/m^2$. What is the resulting intensity?

1. **$1\ W/m^2$**
2. **$7\ W/m^2$**
3. **$18\ W/m^2$**
4. **$25\ W/m^2$**
5. **$49\ W/m^2$**

Q.35-3

Two waves interfere with phase difference $\phi = 90°$

$I_1 = 9\ W/m^2$
$I_2 = 16\ W/m^2$
Find resulting intensity



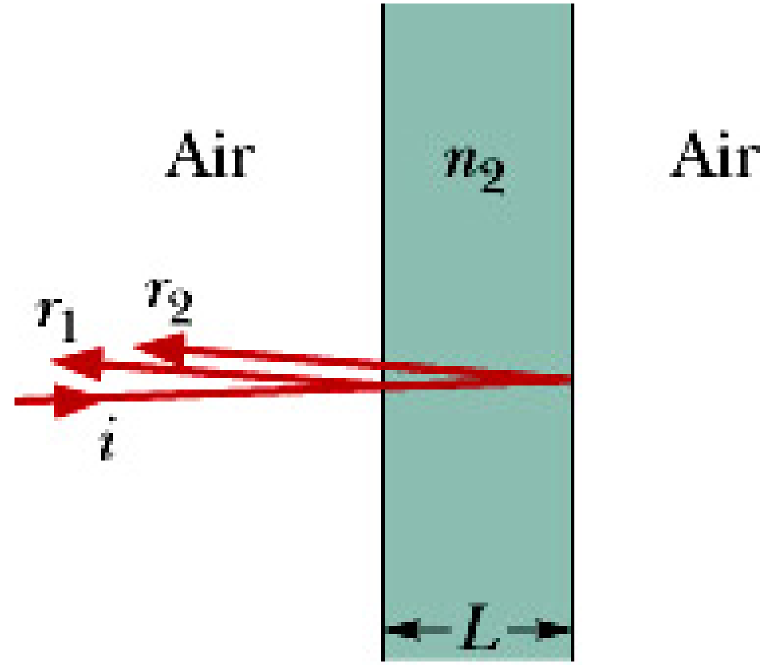
1. **1 W/m²**
2. **7 W/m²**
3. **18 W/m²**
4. $\boxed{\textbf{25 W/m}^2}$
5. **49 W/m²**

$$E^2 = E_1^{\ 2} + E_2^{\ 2}$$

$$But\ I \propto E^2$$

$$\therefore \quad I = I_1 + I_2 = 25\ W/m^2$$

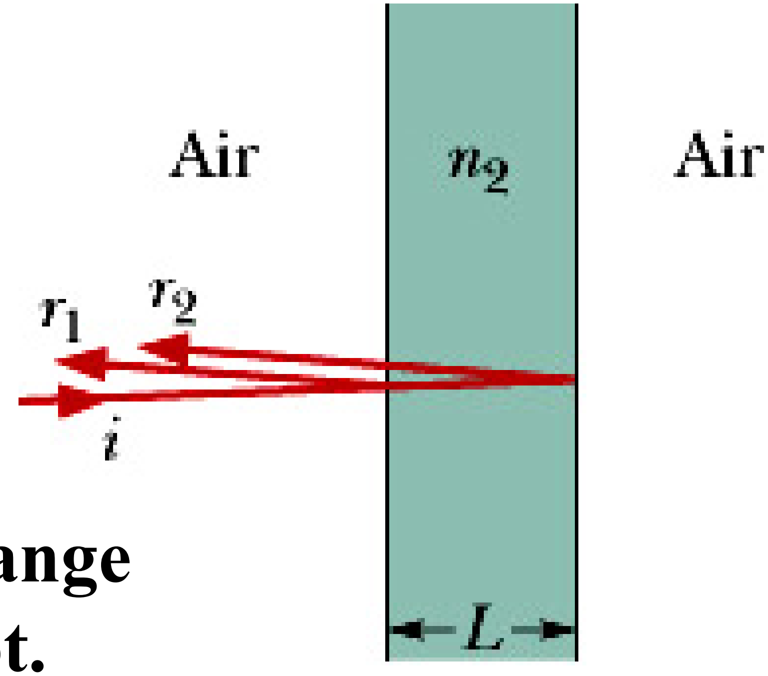
Q.35-4



Light of wavelength λ in air is reflected from a thin film of index n, with air on both sides. What is the condition for the minimum thickness L to give *constructive* interference?

1. $2L = 2\lambda/n$
2. $2L = n\lambda$
3. $2L = \lambda/n$
4. $2L = n\lambda/2$
5. $2L = \lambda/2n$

Q.35-4



Minimum thickness for ***constructive*** interference?

One ray (r_1) has 180° phase change on reflection; the other does not.

1. $2L = 2\lambda/n$
2. $2L = n\lambda$
3. $2L = \lambda/n$
4. $2L = n\lambda/2$
5. $\boxed{2L = \lambda/2n}$

So we need path difference of ***half a wavelength*** to cancel this out.

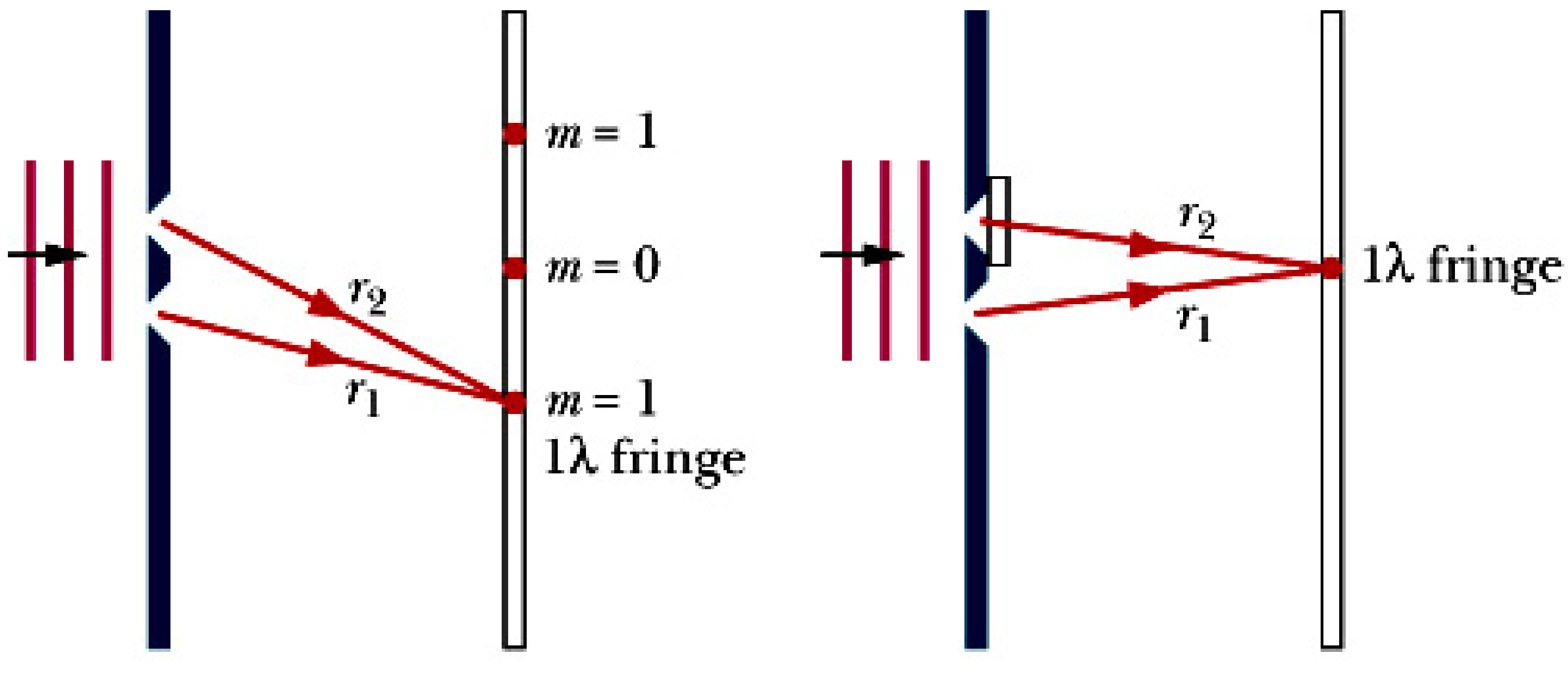
But wavelength in film is λ/n.

So we need $2L = (\lambda/n)/2$

Sample Problem 35-3

Change a double-slit pattern by placing a thin film over one slit. Cause the first bright fringe below the axis to move up to the axis.

How thick is the film?



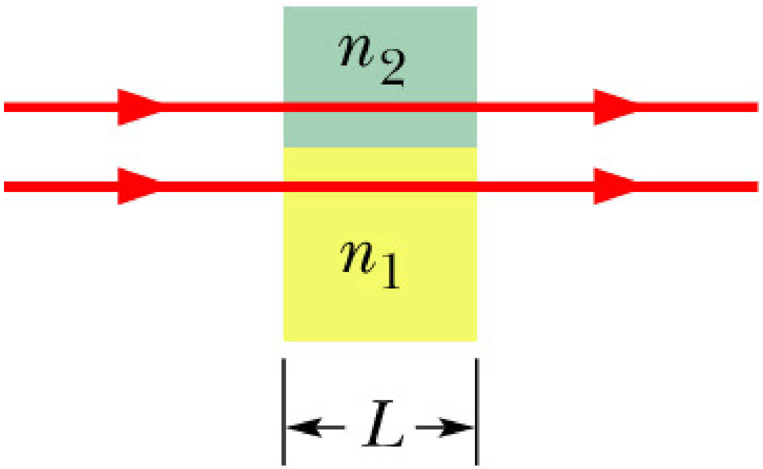
(a) (b)

Phase change and refractive index

Remember our result for phase accumulation in transparent material:

$$\phi = \left(\frac{2\pi}{\lambda}\right)x \quad and \quad \lambda = \frac{\lambda_0}{n}$$

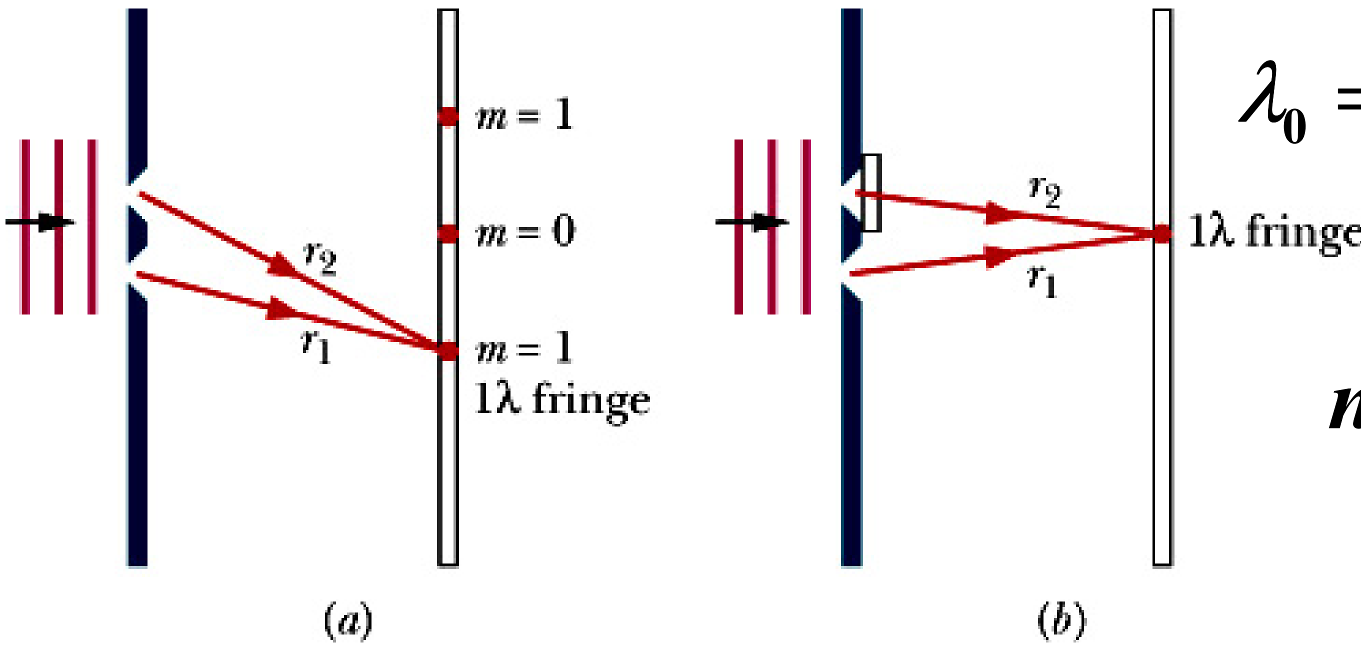
$$so \quad \phi = \frac{2\pi n}{\lambda_0}x$$



So as light passes through a thin film of index n and thickness L it picks up an extra phase, compared to the same distance in air

$$\Delta\phi = \frac{2\pi}{\lambda_0}nL - \frac{2\pi}{\lambda_0}L = \frac{2\pi L}{\lambda_0}(n-1)$$

Sample Problem 35-3 Solution



$$\lambda_0 = 600\ nm$$

$$n = 1.50$$

Here we want the two rays to have $\Delta\phi = 2\pi$

But the film gives $\Delta\phi = \dfrac{2\pi}{\lambda_0} L(n-1)$

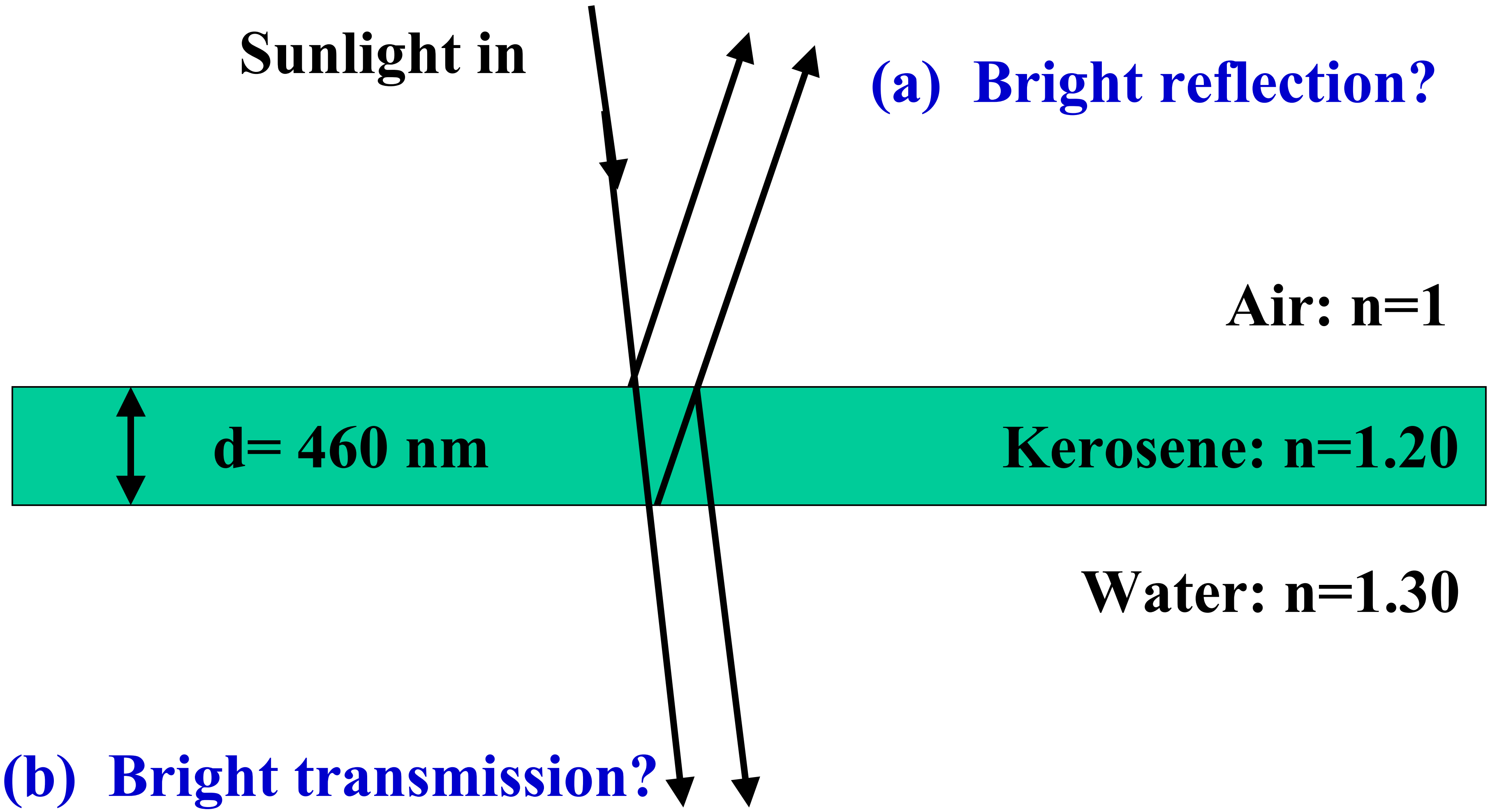
So we need

$$(n-1)L = \lambda_0 \quad so \quad L = \frac{\lambda_0}{n-1} = \frac{600\ nm}{0.5} = \underline{1200\ nm}$$

Problem 35-55

••55 A disabled tanker leaks kerosene (n = 1.20) into the Persian Gulf, creating a large slick on top of the water (n = 1.30). (a) If you are looking straight down from an airplane, while the Sun is overhead, at a region of the slick where its thickness is 460 nm, for which wavelength(s) of visible light is the reflection brightest because of constructive interference? (b) If you are scuba diving directly under this same region of the slick, for which wavelength(s) of visible light is the transmitted intensity strongest?

Text Problem 55

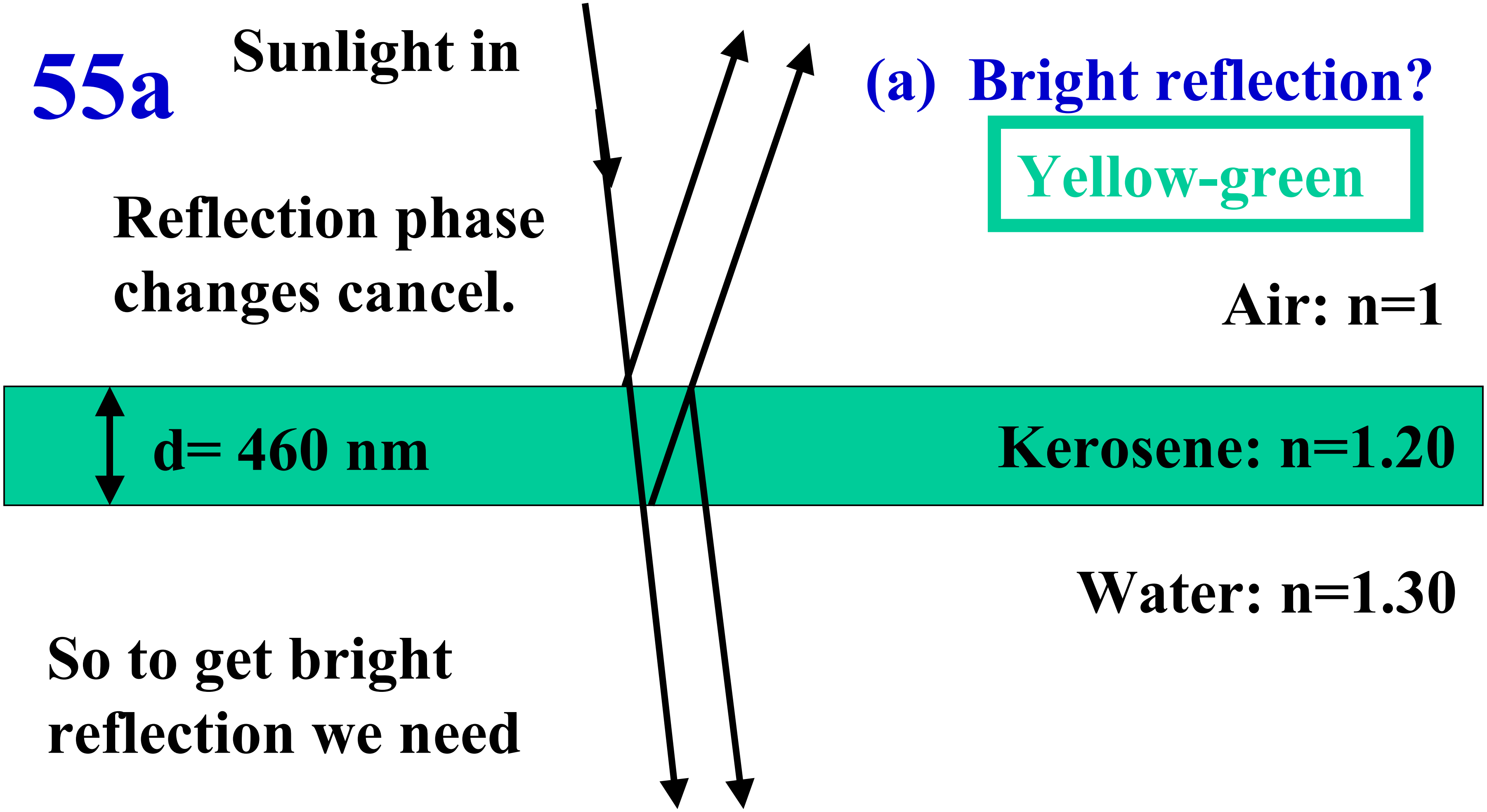


55a

(a) Bright reflection?

$\boxed{\text{Yellow-green}}$

Reflection phase changes cancel.



So to get bright reflection we need

$$\Delta L = 2d = m\lambda = m\lambda_0 / n$$

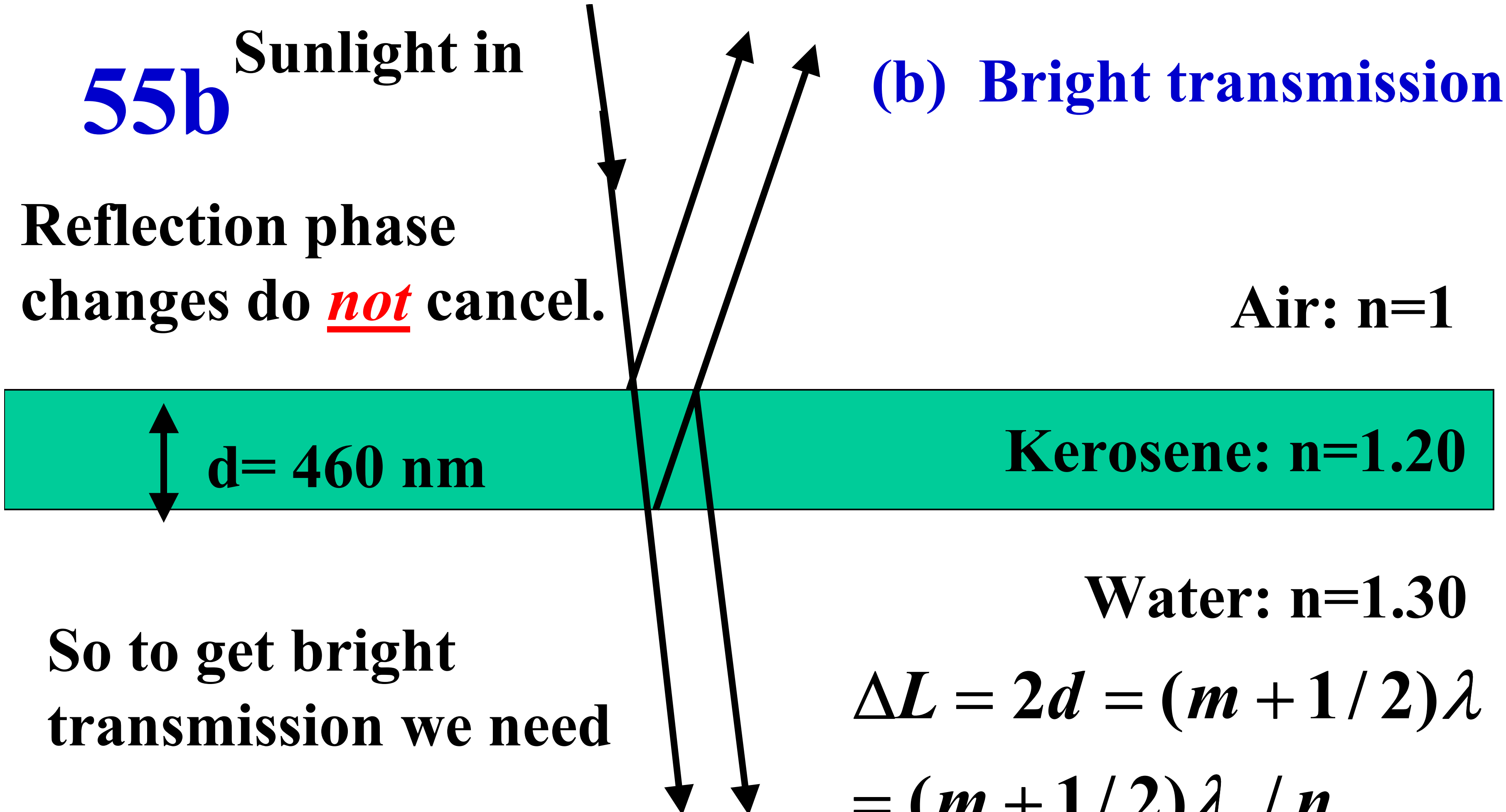
$$\lambda_0 = 2dn / m = 2(460)(1.2) / m = (1104\ nm) / m$$

In visible range: $\lambda_0 = (1104\ nm) / 2 = \underline{552\ nm}$

55b

(b) Bright transmission?

Reflection phase changes do ***not*** cancel.



So to get bright transmission we need

$$\Delta L = 2d = (m + 1/2)\lambda$$

$$= (m + 1/2)\lambda_0 / n$$

$$\lambda_0 = 2dn/(m+1/2) = (1104\ nm)/(m+1/2)$$

In visible range? $\lambda_0 = (1104\ nm)/1.5 = 736\ nm \quad (IR)$

$$\lambda_0 = (1104\ nm)/2.5 = \underline{442\ nm}$$

Violet