DC Circuits II

- Circuits Review
- RC Circuits
- Exponential growth and decay

Quiz Tomorrow on Chapters 26, 27

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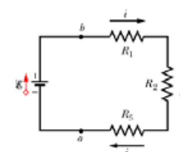
Circuits review so far

- **Resistance and resistivity** $R = \rho l / A$
- **Ohm's Law and voltage drops** $\Delta V = -iR$
- **Power and Joule heating** $P = iV$
- **Resistors in series and parallel**
- **Loop and junction rules**

Review: Series and Parallel Resistors

Series:

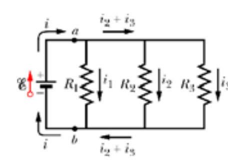
$$R = R_1 + R_2 + R_3$$



Why?

Parallel:

$$1/R = 1/R_1 + 1/R_2 + 1/R_3$$



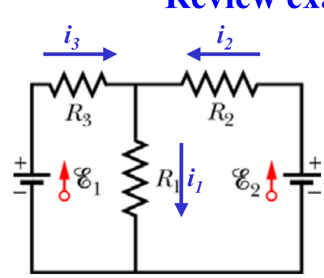
Why?

Review: Rules for Multiloop Circuits

- ***The net voltage change around any loop is zero.***
  "Energy conservation"
- ***The net current into any junction is zero.***
  "Charge conservation"

Using these two rules we can always get enough equations to solve for the currents if we are given the emfs and resistances.

Review example



Left-hand loop:

$$\mathcal{E}_1 - i_3 R_3 - i_1 R_1 = 0$$

Right-hand loop:

$$\mathcal{E}_2 - i_2 R_2 - i_1 R_1 = 0$$

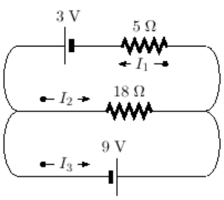
Junction:

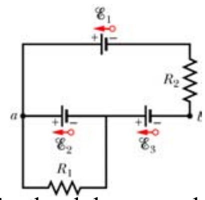
$$i_1 = i_2 + i_3$$

Algebra: solve 3 equations for 3 unknowns i_1, i_2, i_3

If any $i < 0$, current flows the opposite direction.

Simpler Examples





Textbook homework problem 27-19

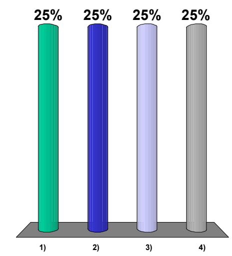
Both these problems can be solved for ***one unknown at a time,*** without messy algebra.

Q.27-3

Resistors R_1 and R_2 are connected in series. If $R_2 > R_1$, what can you say about the resistance R of the combination?

1. $R > R_2$
2. $R_2 > R > R_1$
3. $R_1 > R$
4. None of the above

20



Q.27-3

- **Resistors R_1 and R_2 are connected in series.**
- **$R_2 > R_1$.**
- **What can you say about the resistance R of this combination?**

Solution:

$$R = R_1 + R_2 \quad so \quad R > R_2$$

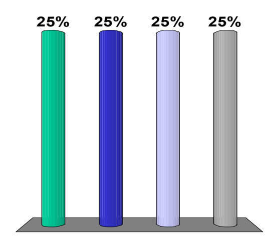
(1) $R > R_2$ (2) $R_2 > R > R_1$

(3) $R_1 > R$ (4) *None of the above*

Q.27-4

Resistors R_1 and R_2 are connected in parallel. If $R_2 > R_1$, what can you say about the resistance R of the combination?

1. $R > R_2$
2. $R_2 > R > R_1$
3. $R_1 > R$
4. None of the above



Q.27-4

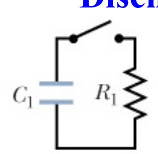
- **Resistors R_1 and R_2 are connected in parallel.**
- **$R_2 > R_1$.**
- **What can you say about the resistance R of this combination?**

Solution: $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad so \quad \frac{1}{R} > \frac{1}{R_1}$$

(1) $R > R_2$ (2) $R_2 > R > R_1$

(3) $R_1 > R$ (4) *None of the above*

Discharging a Capacitor



Capacitor has charge Q_0.
At time t=0, close switch.
What is charge q(t) for t>0?

Obviously q(t) is a function which decreases gradually, approaching zero as t approaches infinity.

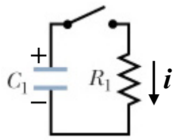
What function would do this?

$$q(t) = Q_0 e^{-t/\tau}$$

But what is the time constant τ?

Analyze circuit equation: find $\tau = RC$

Discharging a Capacitor



Sum voltage changes around loop:

$$Q/C - iR = 0\ ,\ i = \frac{Q}{RC}$$

But

$$i = -\frac{dQ}{dt}$$

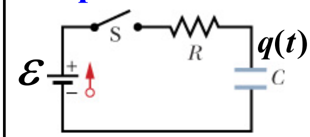
Get ***differential equation*** for *Q(t)*:

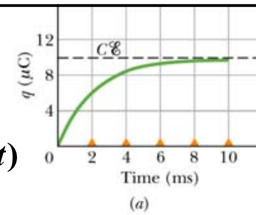
$$\frac{dQ}{dt} = -\frac{Q}{RC}$$

Solution: $Q(t) = Q_0 e^{-t/\tau}$

Where τ is the time constant $\tau = RC$

Charging a Capacitor

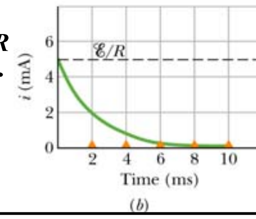




For small t, q=0 and $i = \mathcal{E}/R$

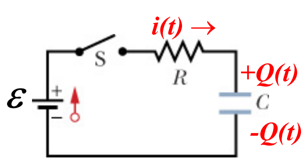
For large t, i=0 and $q = C\mathcal{E}$

But what are $q(t), i(t)$ **?**



Charging a Capacitor

Sum voltage changes:



$$\mathcal{E} - iR - Q/C = 0$$

$$i = \frac{dQ}{dt}$$

Get diff. eq.:

$$\frac{dQ}{dt} = \frac{\mathcal{E}}{R} - \frac{Q}{RC}$$

Solution

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$$\tau = RC$$

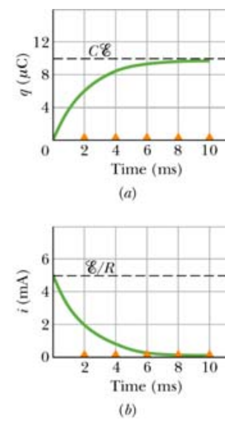
Charging a Capacitor

See solution gives desired behavior:

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$$\rightarrow 0 \quad as\ t \rightarrow 0$$

$$\rightarrow C\mathcal{E} \quad as\ t \rightarrow \infty$$



Exponential Growth and Decay

This ***simple differential equation*** occurs in many situations:

$$\frac{dQ}{dt} = (Const.)Q$$

If dQ/dt = +KQ, we have the “snowball” equation: growth rate proportional to size. ***Population growth.***

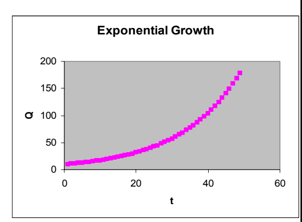
$$\frac{dQ}{dt} = +KQ \longrightarrow Q = Q_0 e^{+Kt}$$

If dQ/dt = -KQ, we have rate of ***decrease*** proportional to size. For example ***radioactive decay.***

$$\frac{dQ}{dt} = -KQ \longrightarrow Q = Q_0 e^{-Kt}$$

Try Exponential Solution

We know we want a result which increases faster and faster. One function which does this is the exponential function. So try that:



$$To\ solve\quad \frac{dQ}{dt} = KQ \quad try \quad Q(t) = Q_0 e^{t/\tau}$$

Questions:

1. Is this a solution?
2. If so, what is the ***“time constant”*** τ ?

$$\text{To solve}\quad \frac{dQ}{dt} = KQ \quad \text{try} \quad Q(t) = Q_0 e^{t/\tau}$$

$$Q(t) = Q_0 e^{t/\tau}$$

$$\frac{dQ}{dt} = Q_0 \frac{d}{dt} e^{t/\tau} = \frac{Q_0}{\tau} e^{t/\tau} = \frac{Q}{\tau}$$

But we want $\dfrac{dQ}{dt} = KQ$

So we **DO** have a solution **IF** $\tau = 1/K$

Doubling Time

If $Q(t) = Q_0 e^{t/\tau}$

how long does it take for Q to double?

$$\frac{Q(t+\Delta t)}{Q(t)} = \frac{e^{(t+\Delta t)/\tau}}{e^{t/\tau}} = e^{\Delta t/\tau}$$

$$\text{And} \quad e^{\Delta t/\tau} = 2 \quad \text{if} \quad \Delta t/\tau = \ln(2) = 0.693$$

$$\text{So} \quad \Delta t \cong 0.7\,\tau$$

Radioactive Decay

For an unstable isotope, a certain fraction of the atoms will disintegrate per unit time.

$$\text{For} \quad \frac{dQ}{dt} = -KQ \quad \text{use} \quad Q(t) = Q_0 e^{-t/\tau}$$

Now τ is called the mean life, and the half-life is $T_{1/2} = \tau \ln(2)$ = time for half the remaining atoms to disintegrate, and

$$T_{1/2} \cong 0.7\,\tau$$

Discharge of a Capacitor

Back to electricity. From the loop rule we got

$$\frac{dQ}{dt} = -\frac{Q}{RC} = -KQ$$

So the solution is $Q(t) = Q_0 e^{-t/\tau}$

But what are Q_0 and τ?

Initial condition: $Q_0 = Q(0)$

Time constant: $\tau = 1/K = RC$

Example

A 40 pF capacitor with a charge of 20 nC is discharged through a 50 MΩ resistor.

(a) What is the time constant?

$$\tau = RC = 40 \times 10^{-12} \times 50 \times 10^{6} = 2.0 \times 10^{-3}\ s$$

(b) At what time will ½ the charge remain?

$$T_{1/2} = 0.7\tau = 0.7 \times 2.0 \times 10^{-3}\ s = 1.4\ ms$$

(c) How much charge will remain after 5 ms?

$$Q(t) = Q_0 e^{-t/\tau} = 20 \times e^{-2.5} = 1.64\ nC$$

Circuits Summary

Things to remember about DC circuits:

- **Resistance and resistivity** $R = \rho l / A$
- **Ohm's Law and voltage drops** $\Delta V = -iR$
- **Power and Joule heating** $P = iV$
- **Resistors in series and parallel**
- **Loop and junction rules**
- **RC circuits: charging and discharging a capacitor**
- **RC time constant** $Q(t) = Q_0 e^{-t/\tau} \quad \tau = RC$

Quiz tomorrow on Chapters 26,27.