OPTICS

- **Today:**
    - **Review refraction and lenses**
    - **Fermat's principle**
    - **Optical Instruments**

Review: Images

We can use reflection and refraction to do lots of things with light, such as forming images. This is called geometric optics, and is of course the basis for a big industry. Again we have to master some terminology. The key distinction is between *real* and *virtual* images.

REAL IMAGE: The light is *really* brought to a focus, such as when you start a fire using sunlight and a lens.

VIRTUAL IMAGE: The light only *appears* to come from it, as when you seem to see Halley Berry inside your TV set, or your face behind the bathroom mirror.

Optics Review

Formulas for spherical mirrors and thin lenses in the small angle approximation:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p}$$

- f = focal length: + = converging, – = diverging
- p = object distance: + = real, – = virtual
- i = image distance: + = real, – = virtual
- m = magnification: + = erect, – = inverted

Three more points

1. Dispersion

   Index of refraction depends on wavelength!

   Prism spectrometer

2. Circular polarization

   Light can carry angular momentum

3. Fermat's principle of least time

   Gives Snell's Law

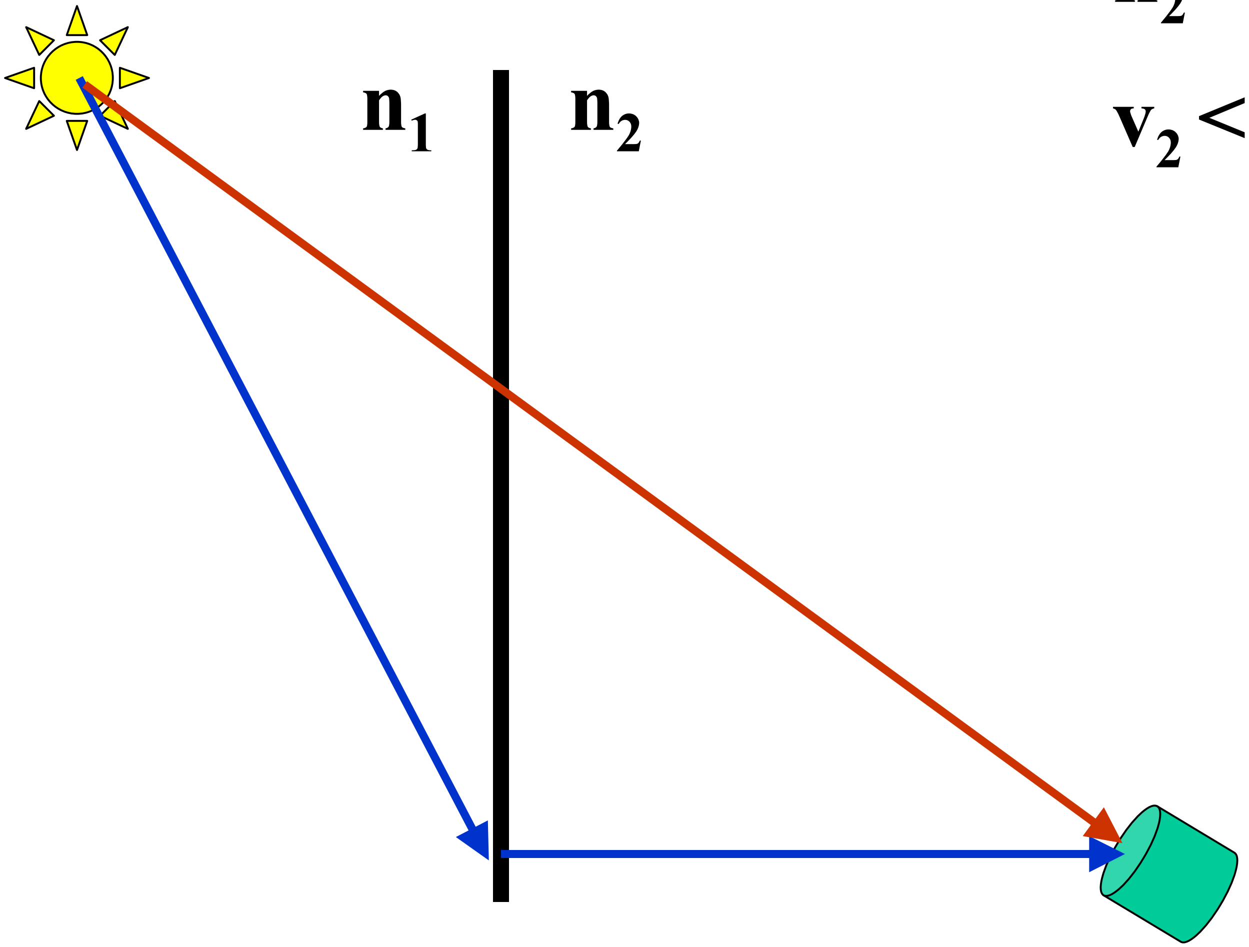
Fermat’s Principle

The path chosen by a light ray will be the one which *minimizes the time*.

$v = c/n$

$n_2 > n_1$

$v_2 < v_1$



Fermat’s Principle

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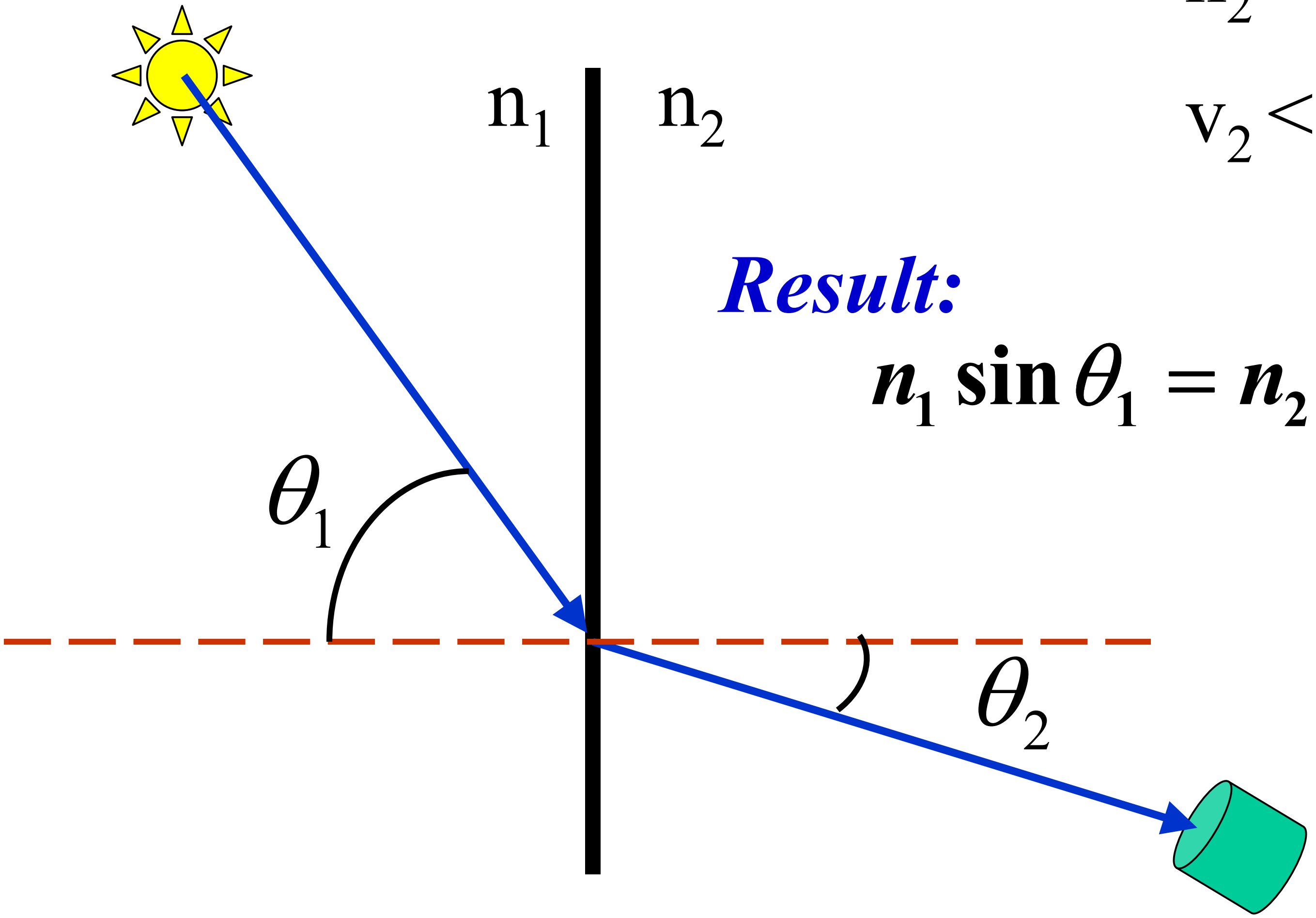
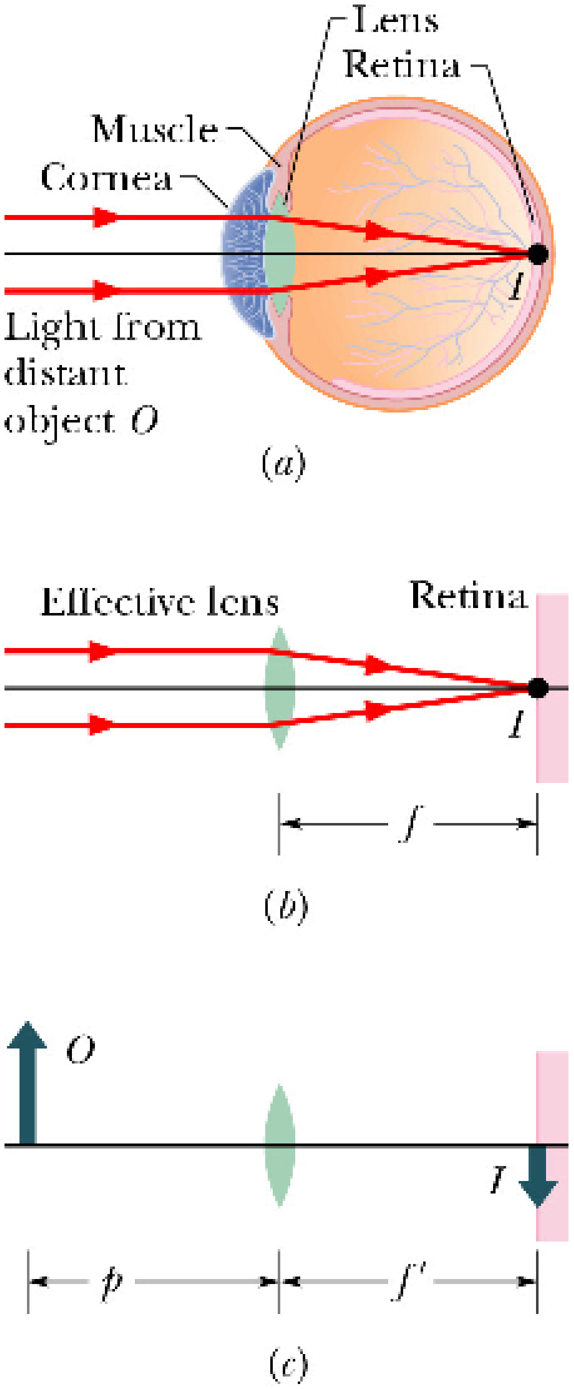
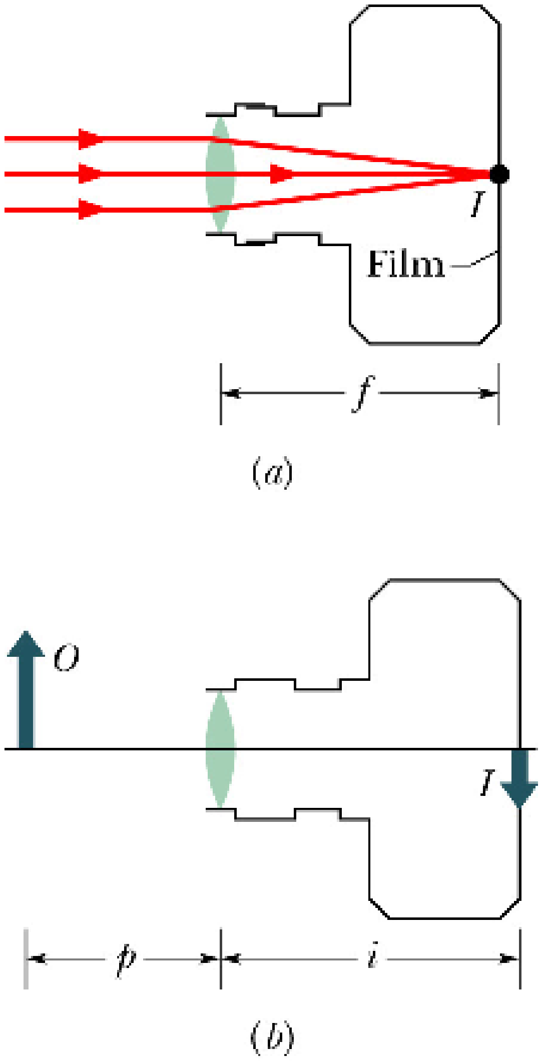


Result:

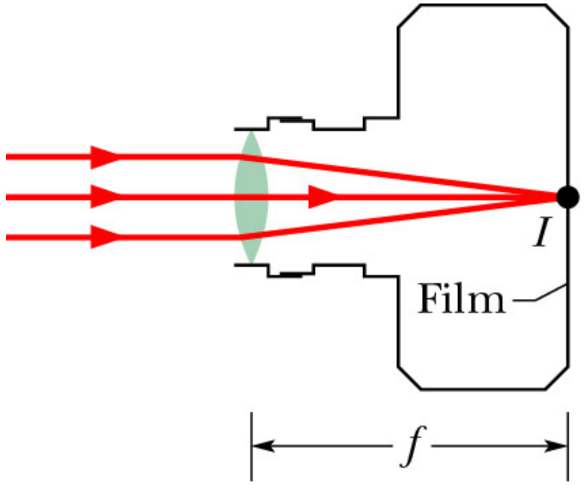
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

The Eye and the Camera

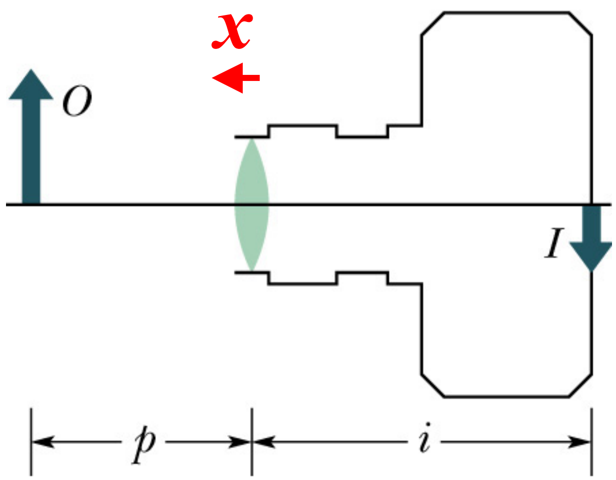




Focus a Camera



(a)



(b)

Problem 35-37

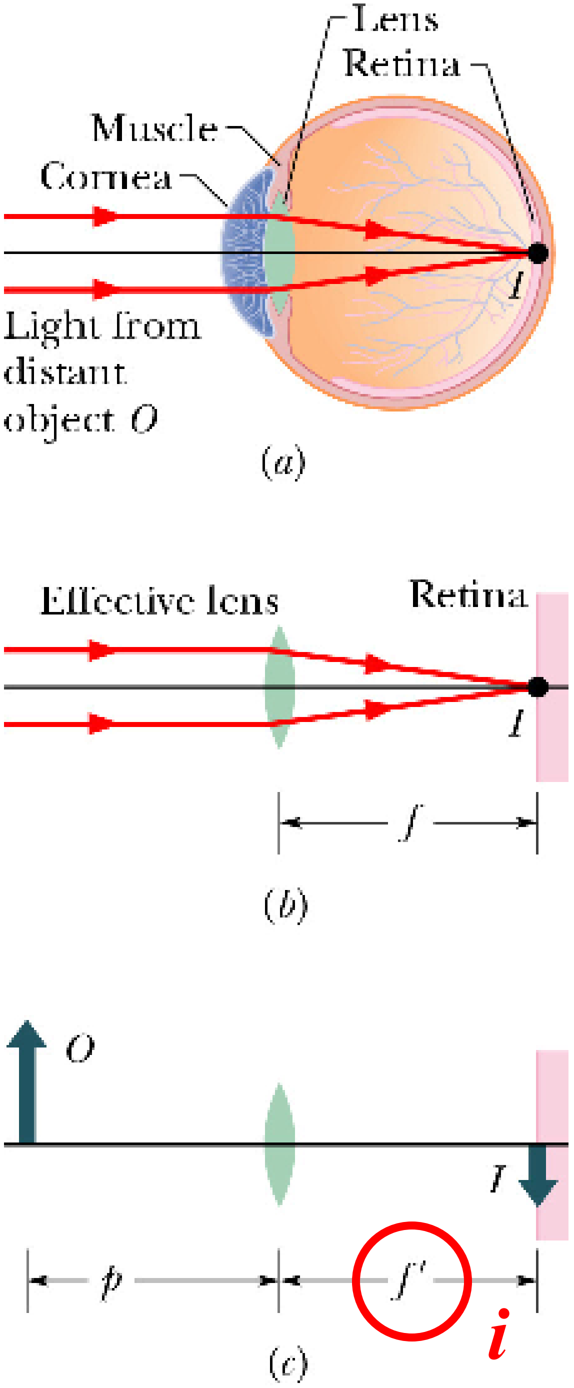
f=5cm, p=1m

- Set for infinity, focal point of lens is on the film
- Actually **focal plane.**
- For closer object, **move lens.**
- Which way and how much?

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{5} - \frac{1}{100} = .20 - .01 = .19$$

$$i = 1/0.19 = 5.26\ cm$$

$$x = i - f = \underline{2.6\ mm}$$



Focus the Eye

Problem 35-35

$$f = 2.50\ cm, \quad p = 40.0\ cm$$

- Set for infinity, focal point of lens is on the retina.
- For closer object, **reshape lens.**
- What should be new focal length?

$$\frac{1}{f'} = \frac{1}{p} + \frac{1}{i} = \frac{1}{40} + \frac{1}{2.5}$$

$$= .025 + .400 = .425$$

$$f' = 1/.425 = \underline{2.35\ cm}$$

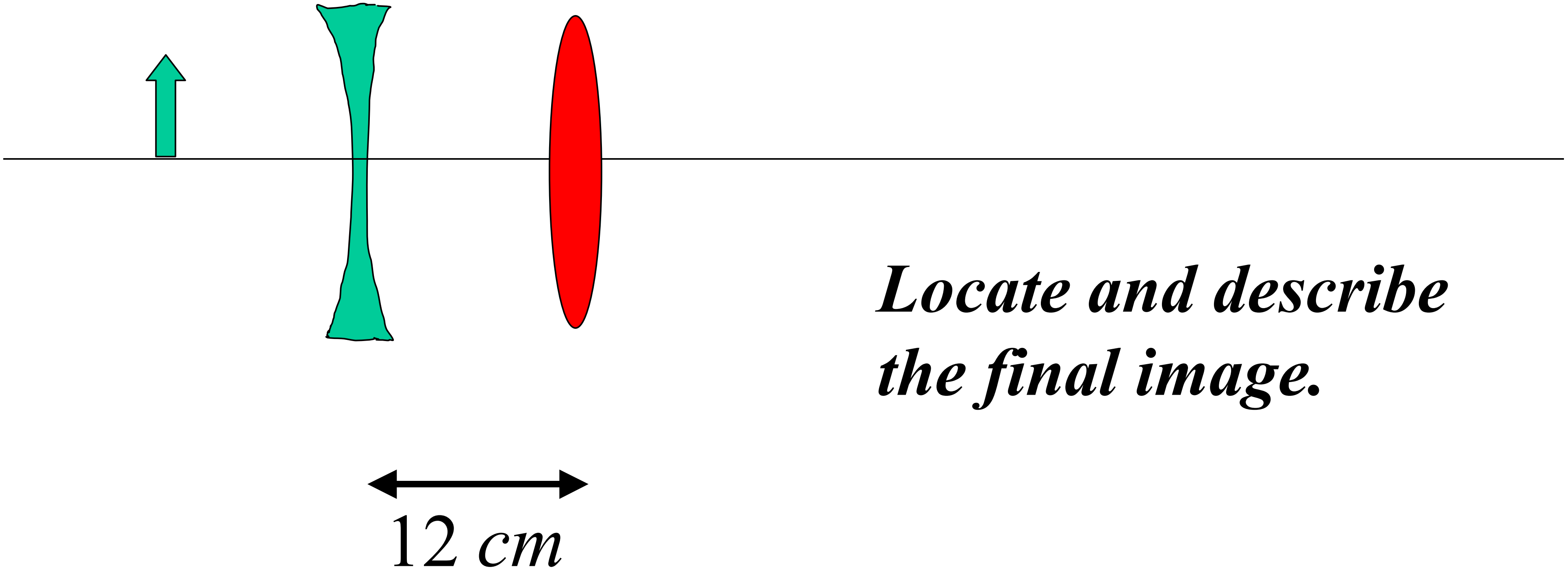
Systems of Lenses

For a system of two or more lenses, treat the lenses one at a time. The *image* formed by the first acts as the *object* for the second. Note that this can result in a *virtual object* ($p<0$).

Example: Problem 35-26

Diverging lens followed by converging lens.

$$f_1 = -15\ cm, \quad f_2 = +12\ cm, \quad p_1 = +10\ cm$$

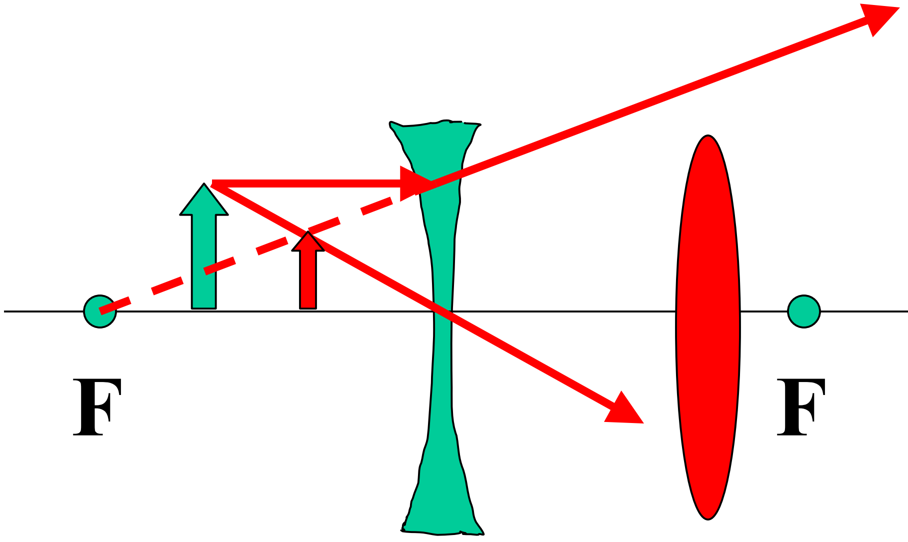


Locate and describe the final image.

$$f_1 = -15\ cm, \quad f_2 = +12\ cm, \quad p_1 = +10\ cm$$

First the principal rays for lens 1:

Image looks to be virtual, erect, and reduced.



$$\frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{p_1}$$

$$= -\frac{1}{15} - \frac{1}{10} = -\frac{1}{6}$$

$$i_1 = -6\ cm$$

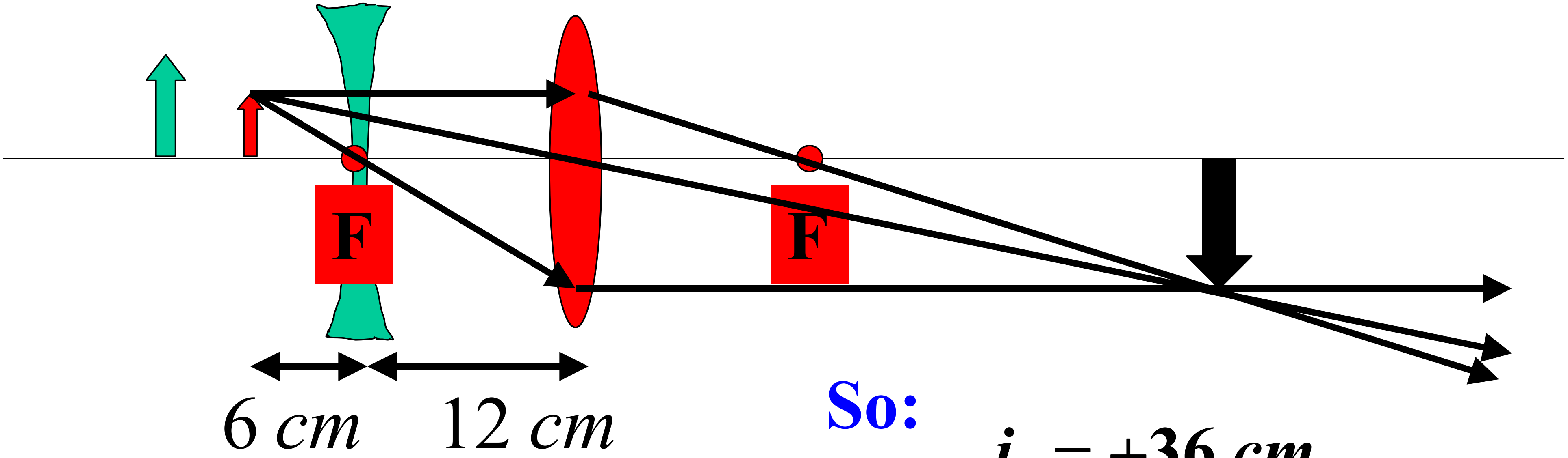
$$m_1 = -(-6)/10 = 0.6$$

$$f_1 = -15\ cm, \quad f_2 = +12\ cm, \quad p_1 = +10\ cm$$

$$i_1 = -6\ cm, \quad m_1 = 0.6, \quad \underline{p_2 = +18\ cm}$$

Now do the principal rays for lens 2:

Final image is real and inverted.



So:

$$i_2 = +36\ cm$$

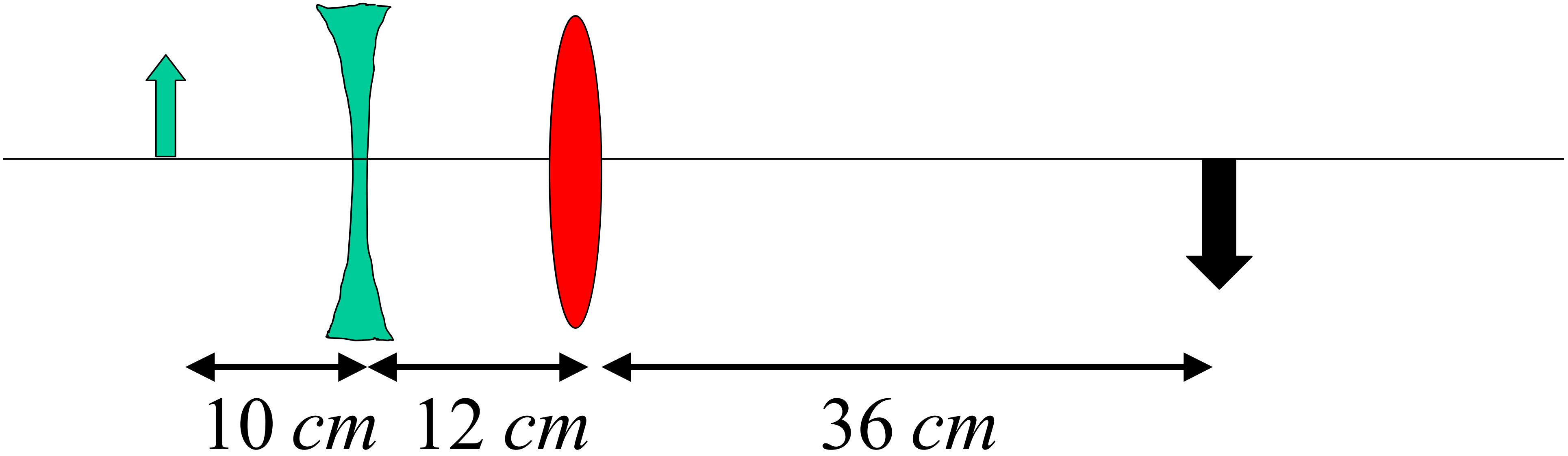
$$m_2 = -36/18 = -2$$

$$\frac{1}{i_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{12} - \frac{1}{18} = \frac{1}{36}$$

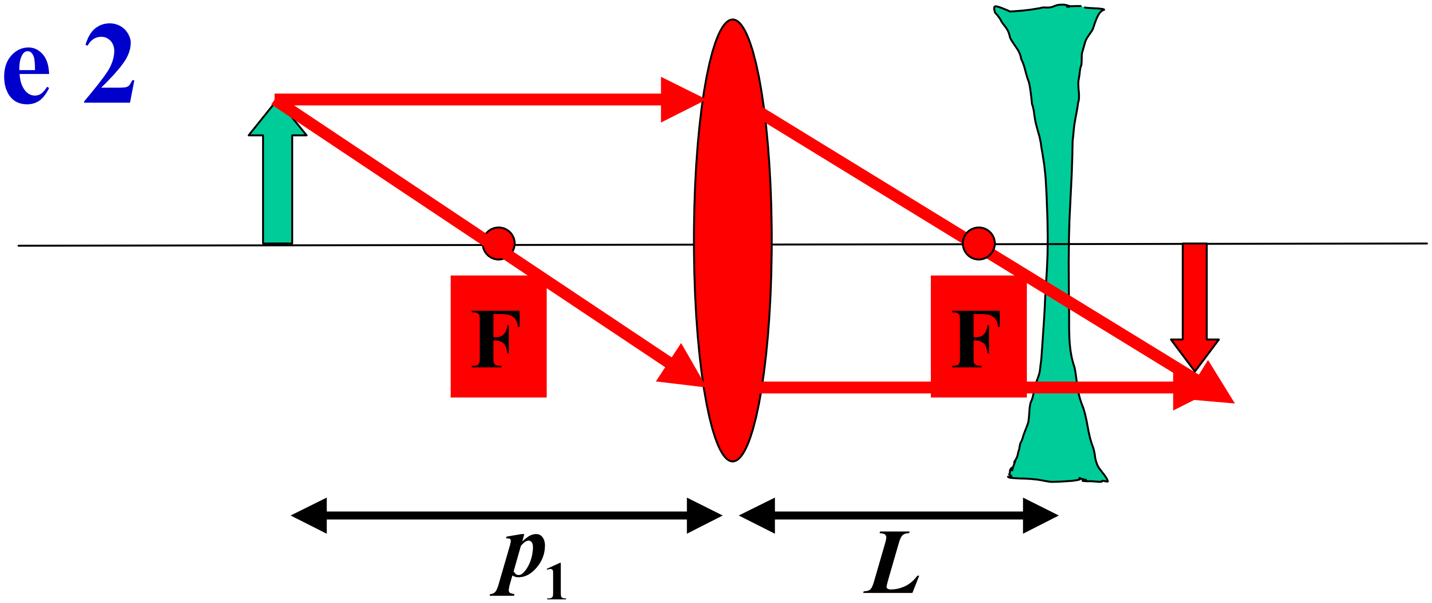
Overall magnification of two-lens system:

$$m_{tot} = m_1 m_2 = (0.6)(-2) = \underline{-1.2}$$

Final image is real, inverted, enlarged.



Example 2



$$f_1 = +3cm$$

$$f_2 = -4cm$$

$$L = 4cm$$

$$p_1 = 6cm$$

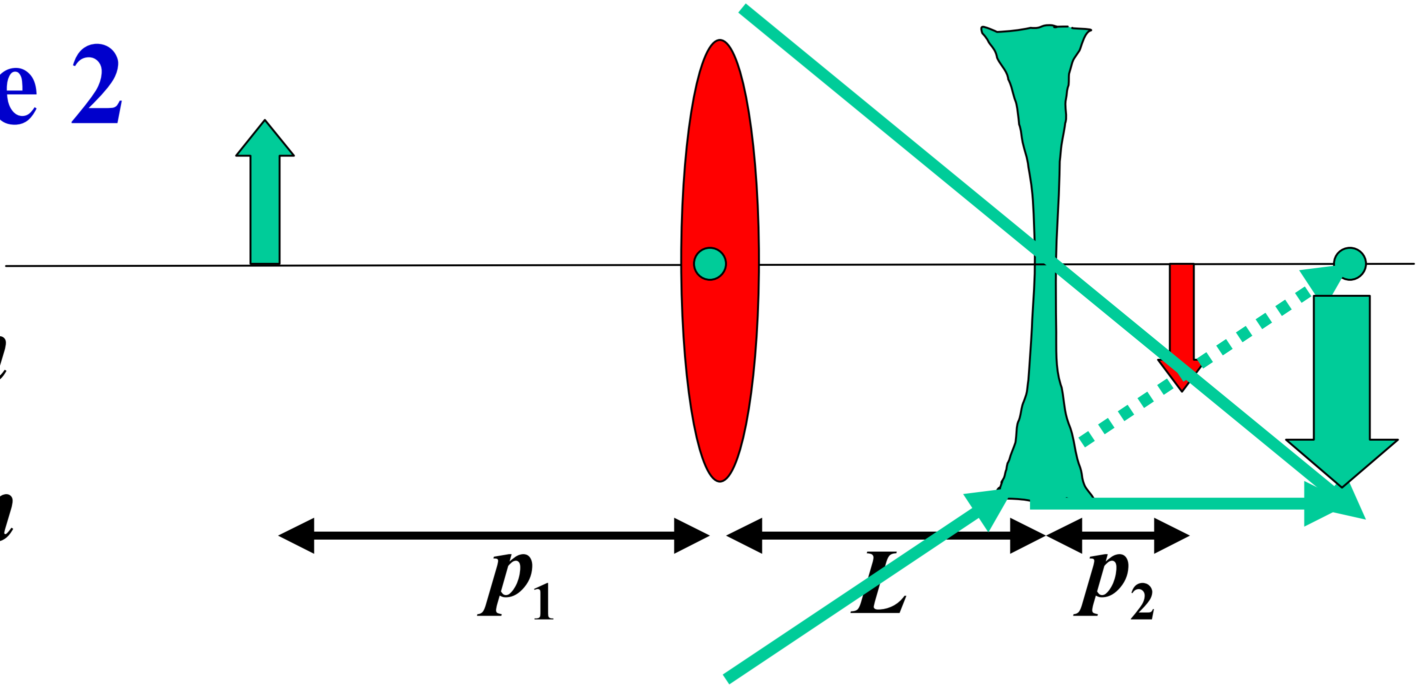
Find image due to lens 1:

$$i_1 = 6cm \qquad m_1 = -1$$

Real image due to lens 1.
Virtual object for lens 2.

$$\boxed{p_2 = -(i_1 - L) = -2cm}$$

Example 2



$$f_2 = -4cm$$

$$p_2 = -2cm$$

Find image due to lens 2:

$$i_2 = +4cm \qquad m_2 = +2$$

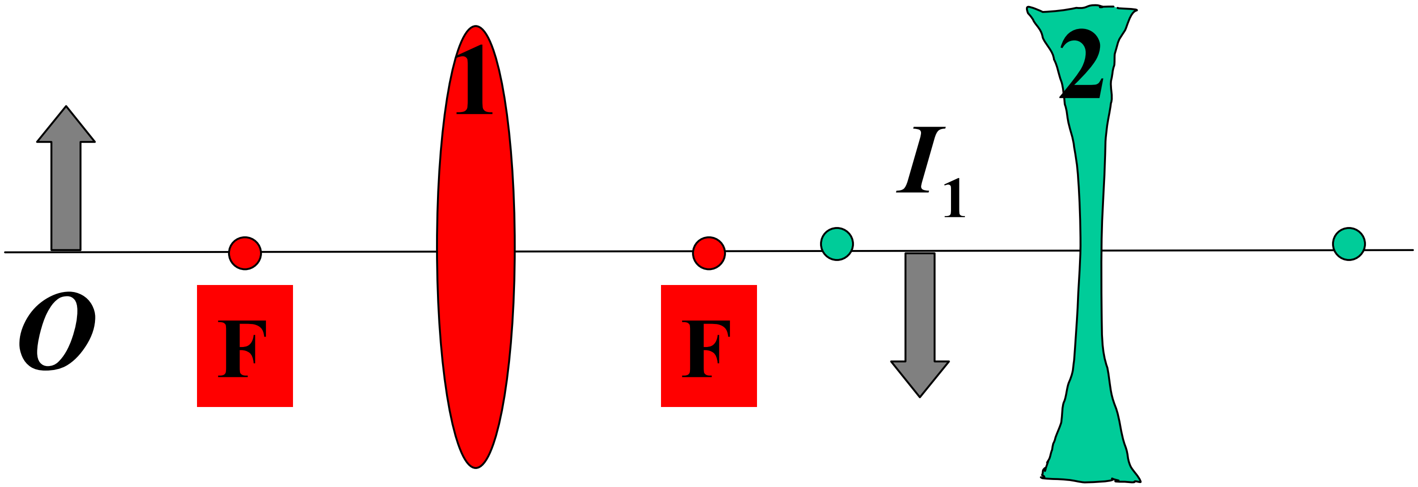
Real image due to lens 2.

$$m_{TOT} = m_1 \times m_2 = -2$$

Final result is real, inverted and enlarged.

Q.35-3

We have a real image formed by a converging lens 1 as shown. Now a diverging lens 2 is added as shown.

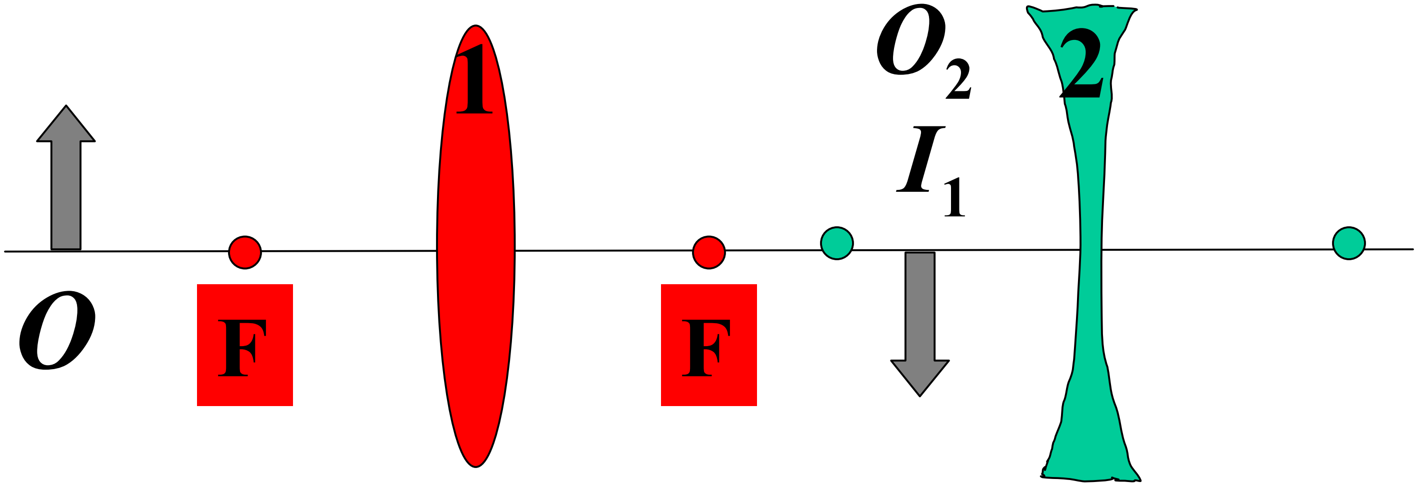


Will lens 2 have a real or virtual object?

(1) Real **(2) Virtual**

Q.35-3

We have a real image formed by a converging lens 1 as shown. Now a diverging lens 2 is added as shown.



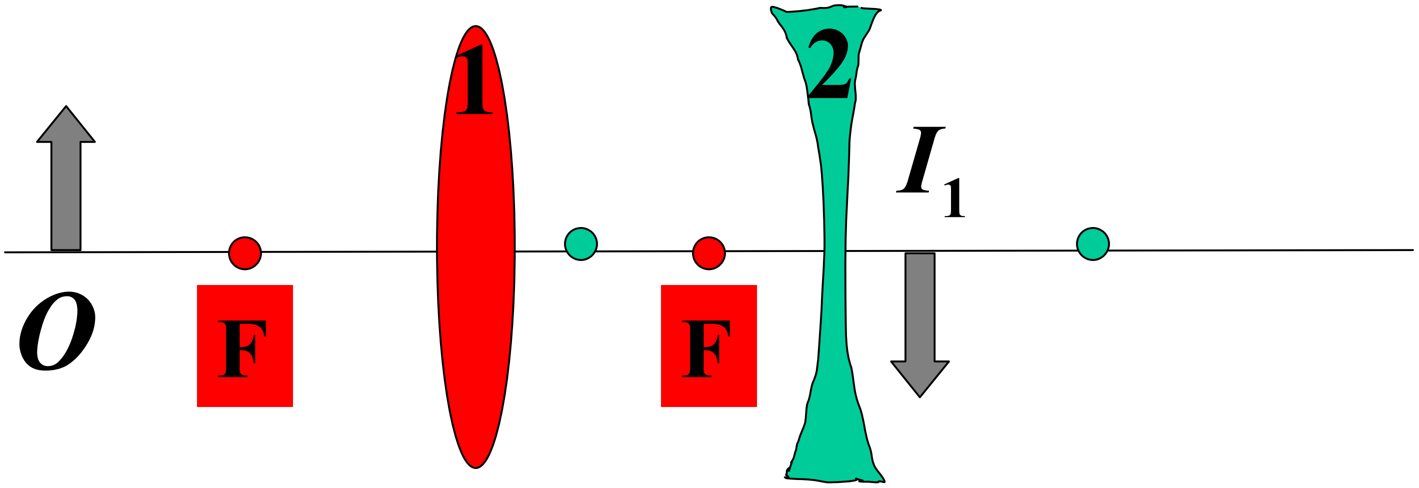
Will lens 2 have a real or virtual object?

[✓ (1) Real]

(2) Virtual

Q.35-4

We have a real image formed by a converging lens 1 as shown. Now a diverging lens 2 is added as shown.

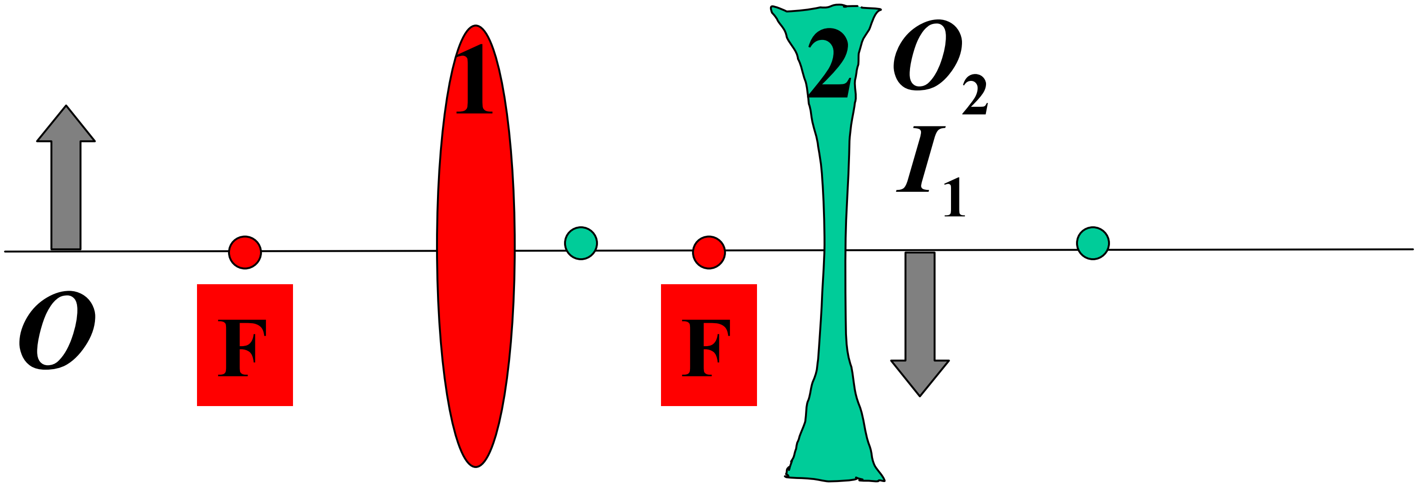


Will lens 2 have a real or virtual object?

(1) Real (2) Virtual

Q.35-4

We have a real image formed by a converging lens 1 as shown. Now a diverging lens 2 is added as shown.



Will lens 2 have a real or virtual object?

(1) Real [✓ (2) Virtual]

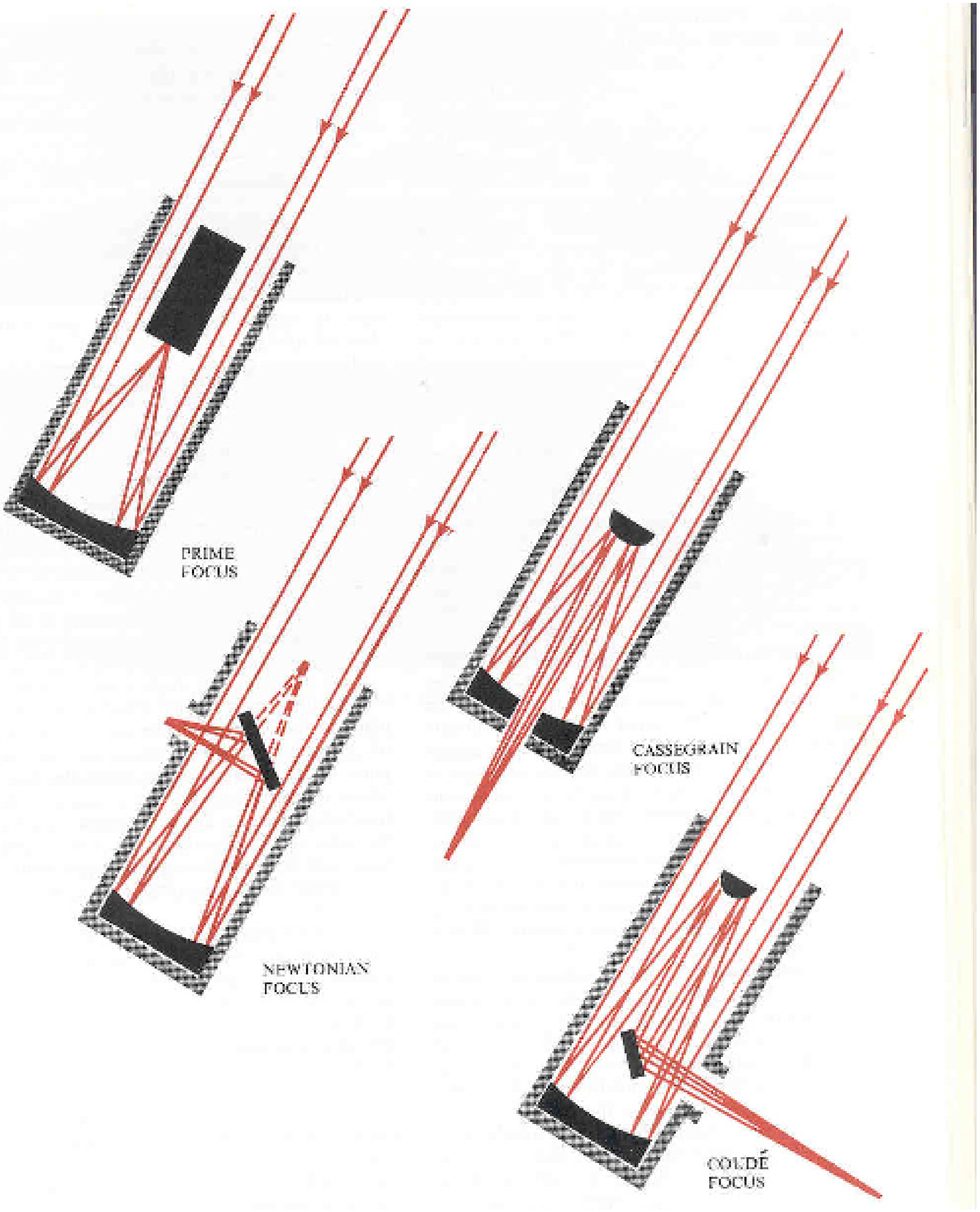
Astronomical Telescopes

Ritter Observatory

Ritter 1-meter reflector

Styles of Reflectors

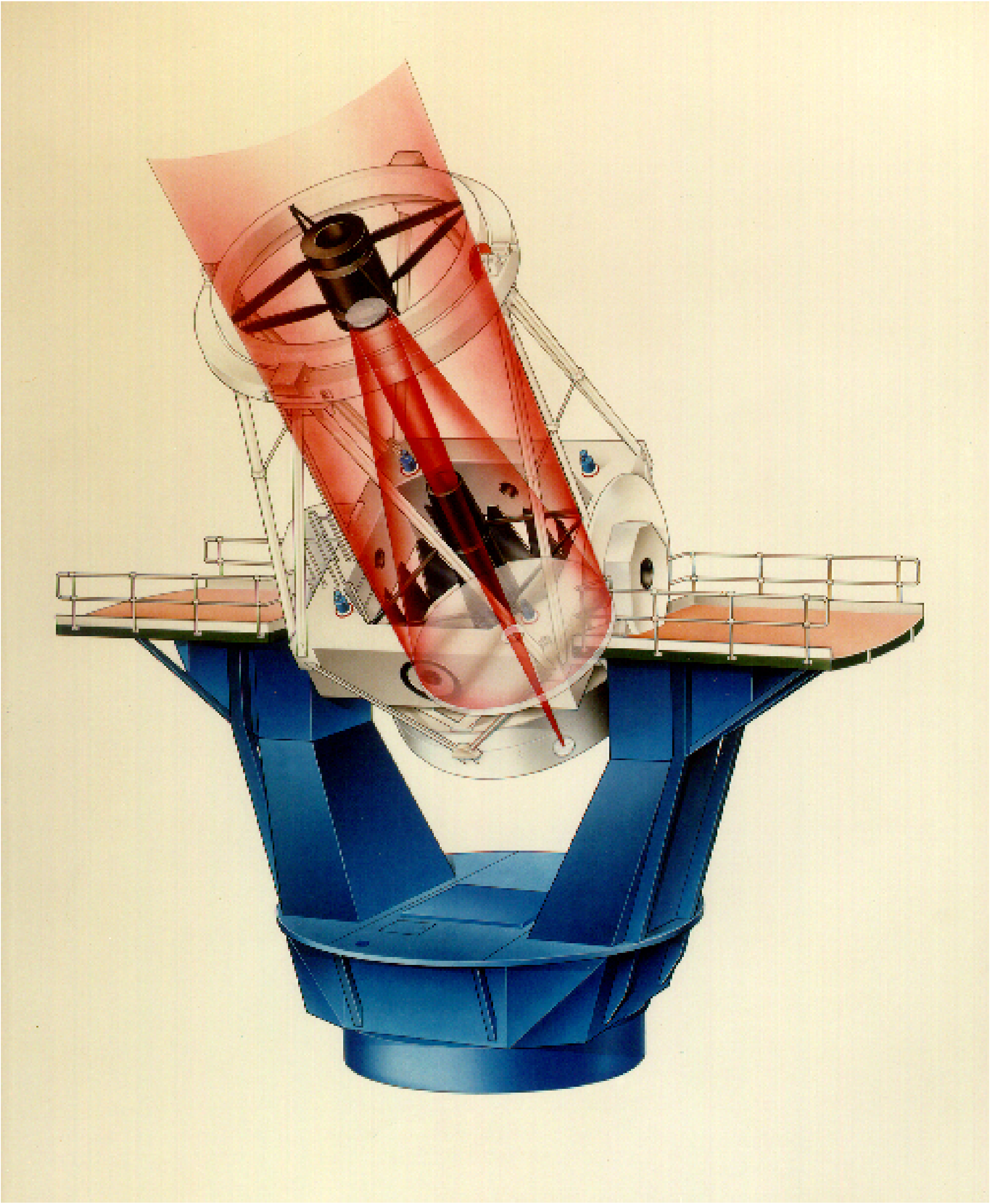
Goal: Collect as much light as possible, with the fewest possible reflections.



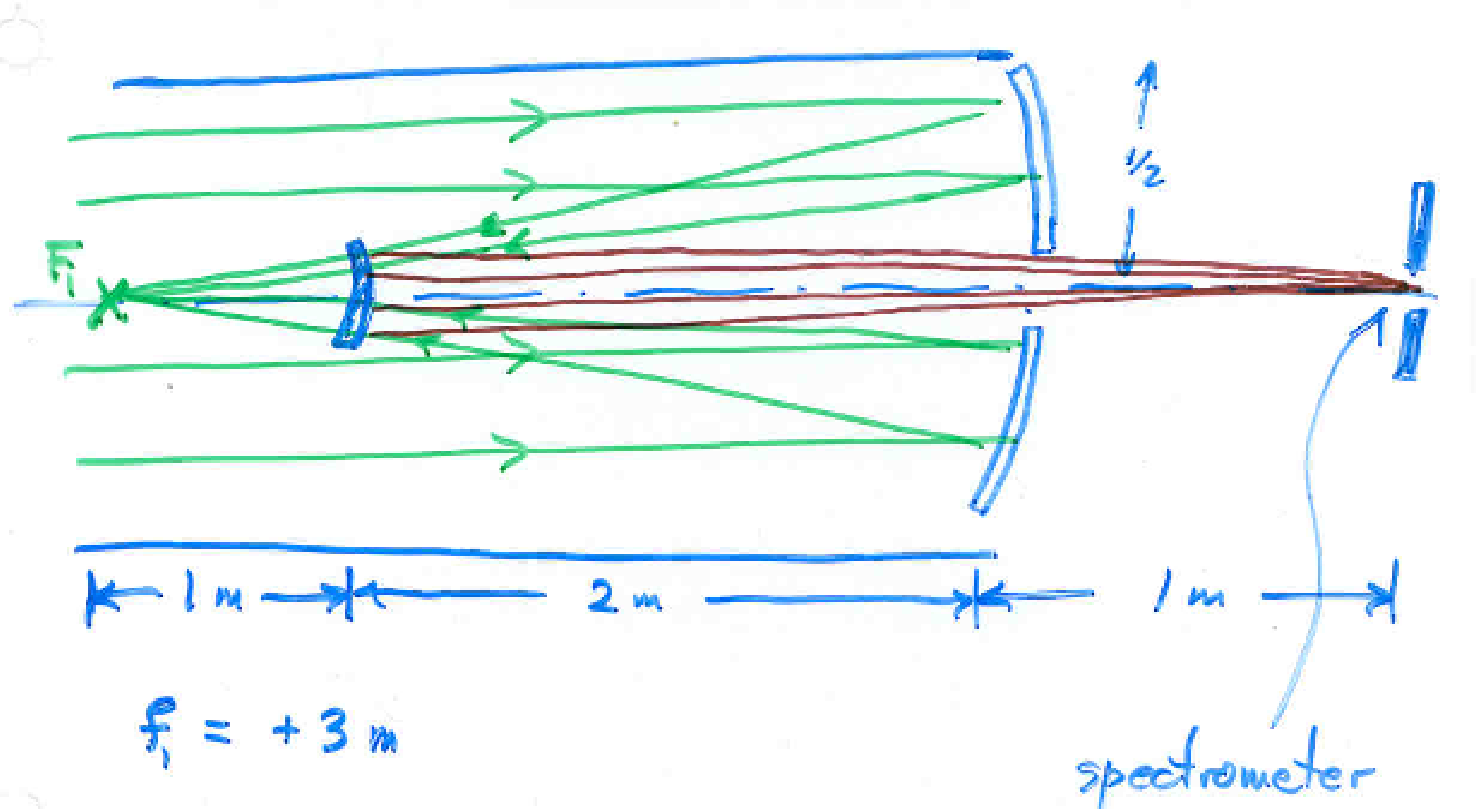
Herschel Telescope

European 4-meter reflector on the Canary Islands

Collects 16 times as much light as we do at Ritter.

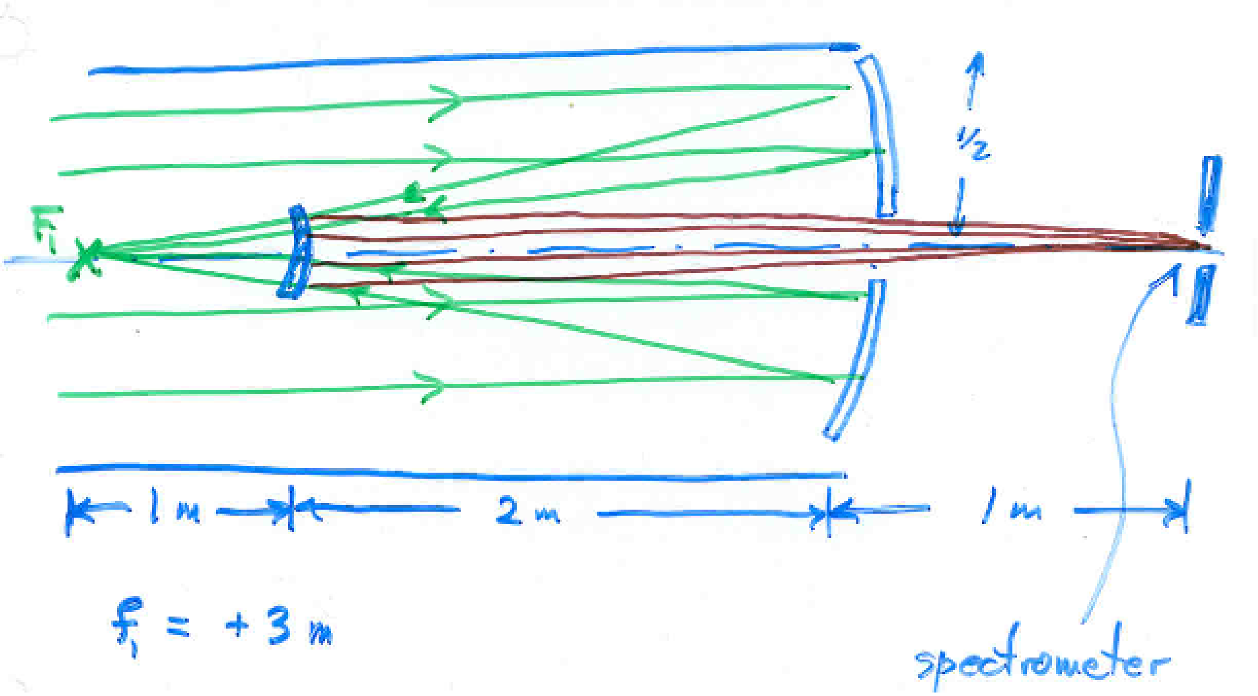


Ritter 1-meter Reflector



- **Problem:** **What do we need for f_2, the focal length of the secondary mirror?**

Solution



Mirror 1: Image formed at focus F_1

Mirror 2: Virtual object p= -1m, real image i=3m.

$$\frac{1}{f_2}=\frac{1}{p_2}+\frac{1}{i_2}=\frac{1}{-1}+\frac{1}{3}=-\frac{2}{3} \qquad f_2=-\frac{3}{2}=\underline{-1.5\,m}$$

So we want a diverging mirror with f=1.5m.

Optics Review

Formulas for spherical mirrors and thin lenses in the small angle approximation:

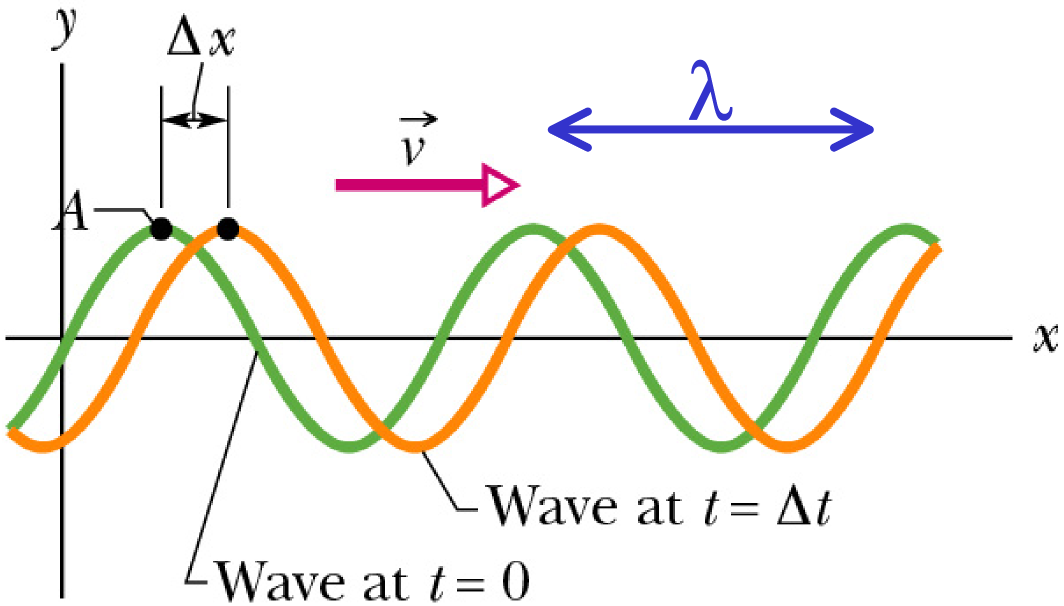
$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \qquad m = -\frac{i}{p}$$

- f = focal length: + = converging, – = diverging
- p = object distance: + = real, – = virtual
- i = image distance: + = real, – = virtual
- m = magnification: + = erect, – = inverted

INTERFERENCE

- Today Ch. 35 Interference
  - The general idea
  - Examples
    - Two slits
    - Phase change on reflection
    - Thin films
    - Interferometers
  - Intensities

Review of Waves (Ch. 16)



Wavelength = λ
Frequency = **f**
Velocity = **v** = f λ

$$y = y_0 \sin(kx - \omega t)$$

$$\boxed{k = \frac{2\pi}{\lambda}} \qquad \omega = 2\pi f$$

$$v = f\lambda = \omega / k$$

$$Amplitude = y_0$$

$$Intensity \propto y_0^{\,2}$$

Interference of Two Waves

Adding two waves of the same frequency:

$$E_1 = E_1^0 \sin(kx - \omega t)$$

$$E_2 = E_2^0 \sin(kx - \omega t + \phi)$$

$$E_T = E_1 + E_2 = ?$$

Answer: $\boxed{E_T = E_T^0 \sin(kx - \omega t + \phi_T)}$

Result is a wave of the same frequency. Usually we want the ***amplitude*** E_T^0 or the ***intensity*** I_T.

Phase and Path Differences

One way to get a *phase difference* Δϕ between two waves is to arrange for a *path difference* ΔL.

The *general relation* between phase difference and path difference is

$$\boxed{\Delta\phi = k\Delta L = 2\pi\frac{\Delta L}{\lambda}}$$

Remember k is phase per unit length: $E = E^0 \sin(kx - \omega t)$

Simple Interference

Constructive case:

$$\boxed{E_T{}^0 = E_1{}^0 + E_2{}^0}$$

$$\phi = m(2\pi) \qquad \Delta L = m\lambda$$

Note if $E_1{}^0 = E_2{}^0$ *then* $E_T{}^0 = 2E_1{}^0$ *and* $\underline{I_T = 4I_1}$

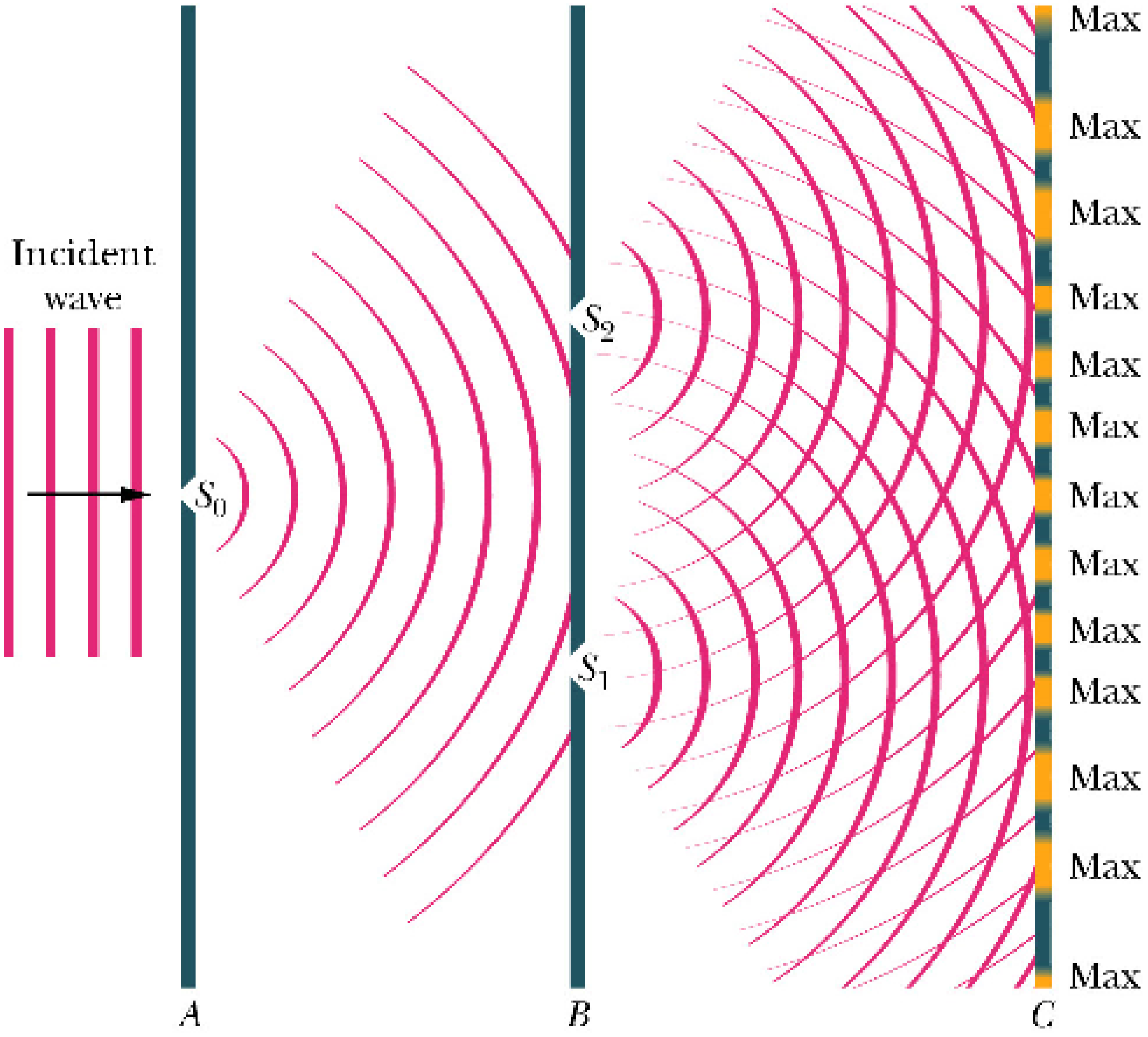
Destructive case:

$$\boxed{E_T{}^0 = E_1{}^0 - E_2{}^0}$$

$$\phi = (m + \frac{1}{2})2\pi \qquad \Delta L = (m + \frac{1}{2})\lambda$$

Note if $E_1{}^0 = E_2{}^0$ *then* $E_T{}^0 = 0$ *and* $\underline{I_T = 0}$

The Double Slit Experiment



Interference "fringes" due to alternating constructive and destructive interference between rays from S_1 and S_2.

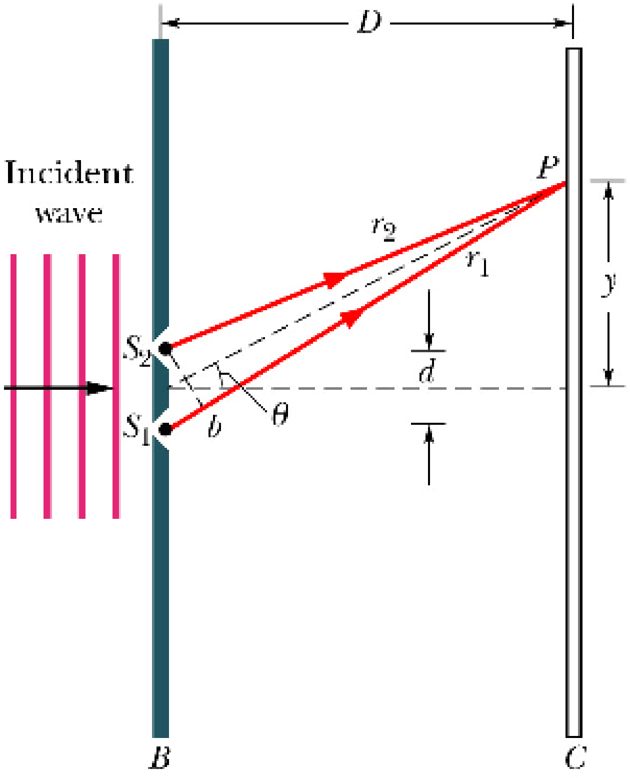
Double Slit Path Differences

For point P at angle θ triangle shows $\Delta L = d \sin\theta$

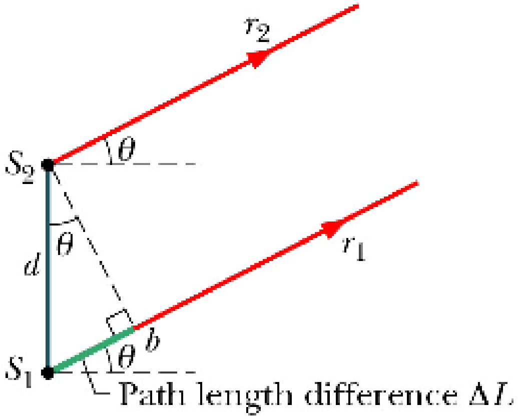
For constructive interference we need $\Delta L = m\lambda$

where m=0,1,2,… is any integer.

So the bright fringes are at angles given by $\boxed{d \sin\theta = m\lambda}$



(a)



(b)

Bright and Dark Fringes

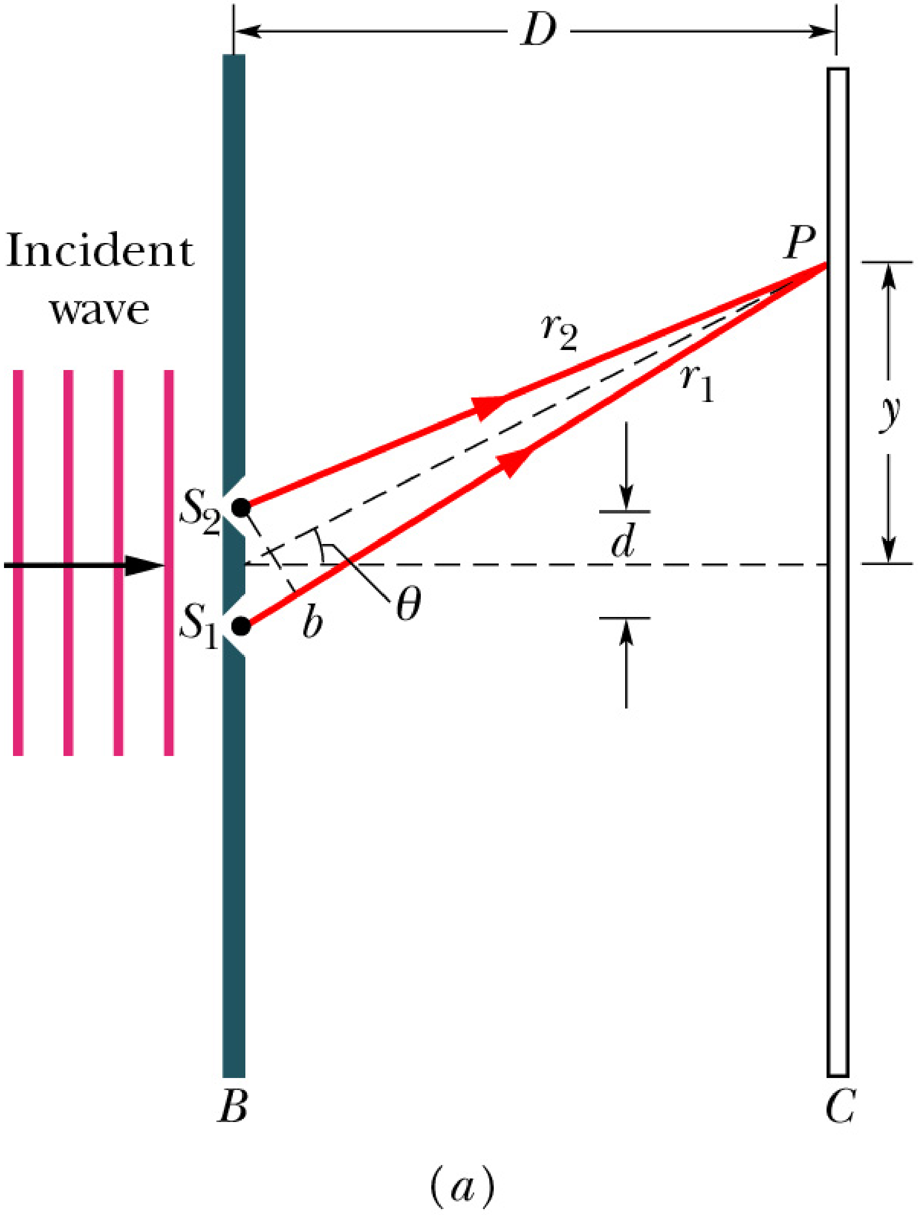
So the *bright fringes* are at angles given by

$$d \sin\theta = m\lambda$$

And the *dark fringes* are at angles given by

$$d \sin\theta = (m + \frac{1}{2})\lambda$$

Locating the Fringes



For a bright spot we need $d\sin\theta = m\lambda$.

From the figure we see $\tan\theta = y/D$.

But for small angles we have $\sin\theta \approx \tan\theta$.

So the bright lines are at

$$m\lambda = dy/D$$

$$y = m\lambda D/d$$

Double Slit Example

Given a double slit experiment with wavelength 450 nm, slit separation 0.3 mm, distance to screen 2 m, where will be the bright fringes?

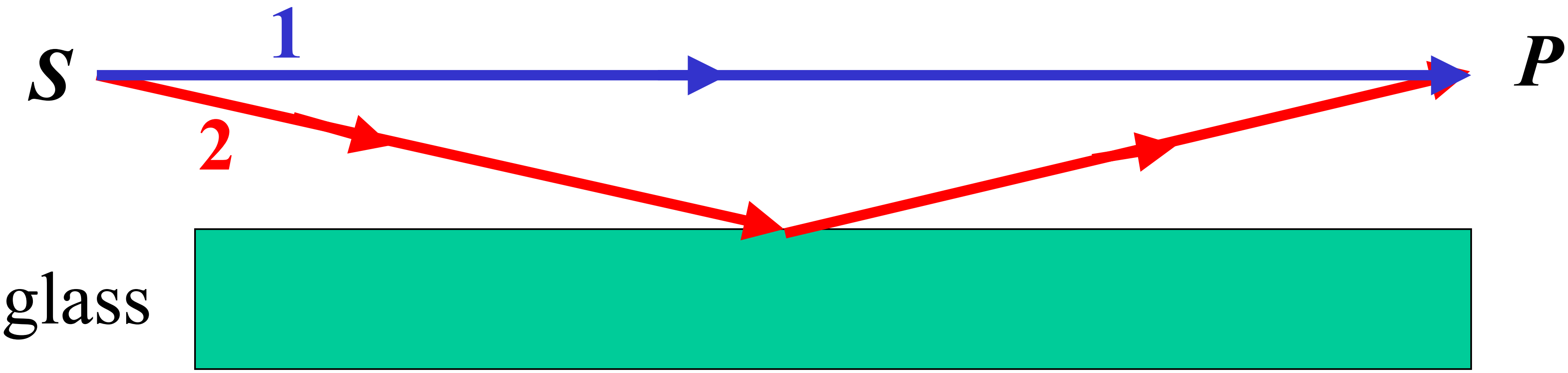
$$y = m\lambda D / d \quad with \quad m = 0,1,2,\cdots$$

$$\lambda D / d = \frac{.45 \times 10^{-6} \times 2}{.3 \times 10^{-3}} = 3\,mm$$

So the bright lines are at $\underline{y = 0, 3\,mm, 6\,mm, 9\,mm, etc.}$

Note the angles really are small: $\theta \approx \frac{6\,mm}{2\,m} = .003\,rad = 0.17°$

Phase Change on Reflection



- Two rays from ***S*** arrive at ***P***. Path difference gives interference. Expect as angle → 0, get ***constructive***.
- **Wrong! It's *destructive*. Why?**
- **Because reflection from medium of higher *n* always gives a phase change of 180°.**
- **(Maxwell says so!)**

Michelson Interferometer

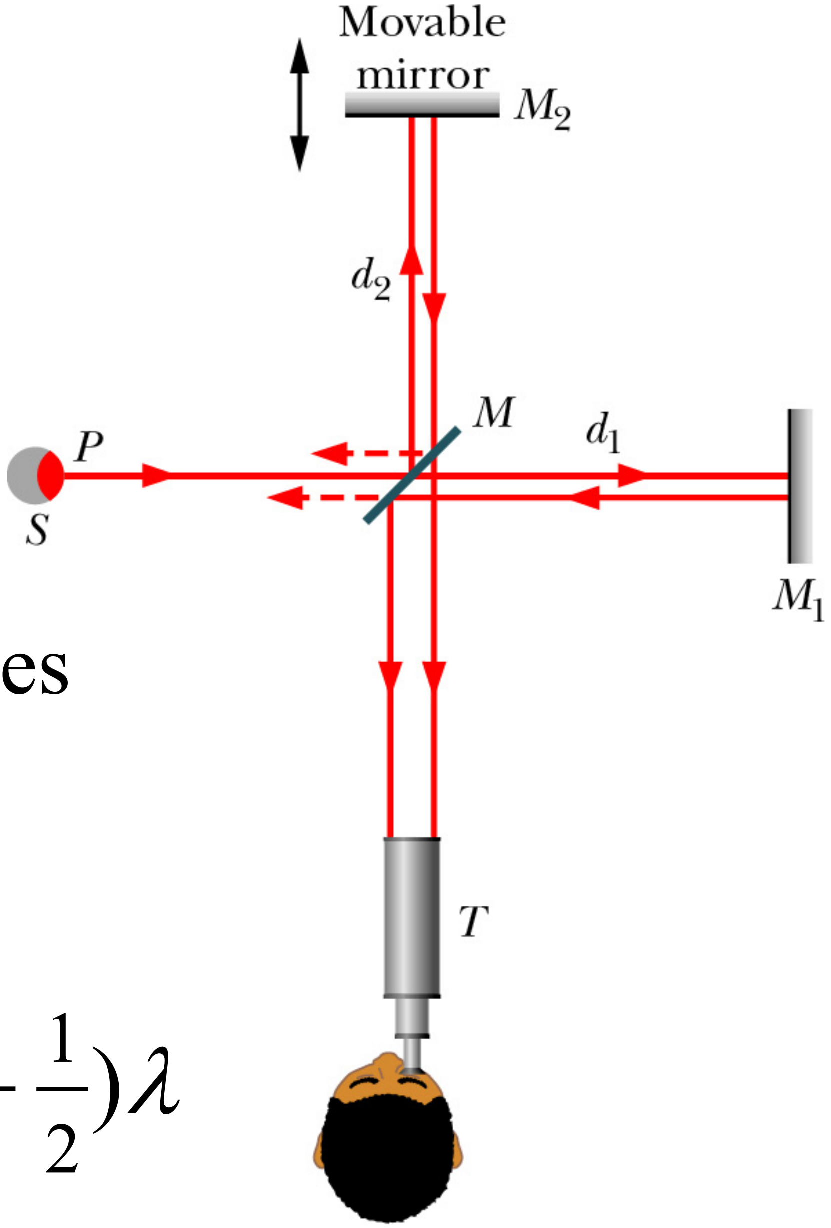
Another device for getting optical interference.

$$\Delta L = 2d_1 - 2d_2$$

As d2 is changed, we see series of bright and dark fringes.

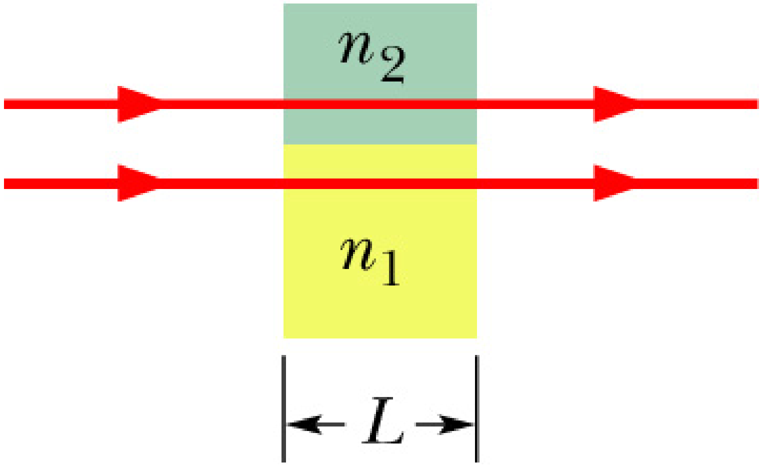
Bright when $\Delta L = m\lambda$

And dark when $\Delta L = (m + \frac{1}{2})\lambda$



Phase Difference Due To Different Index of Refraction

Yet another way to get 2 light waves out of phase.



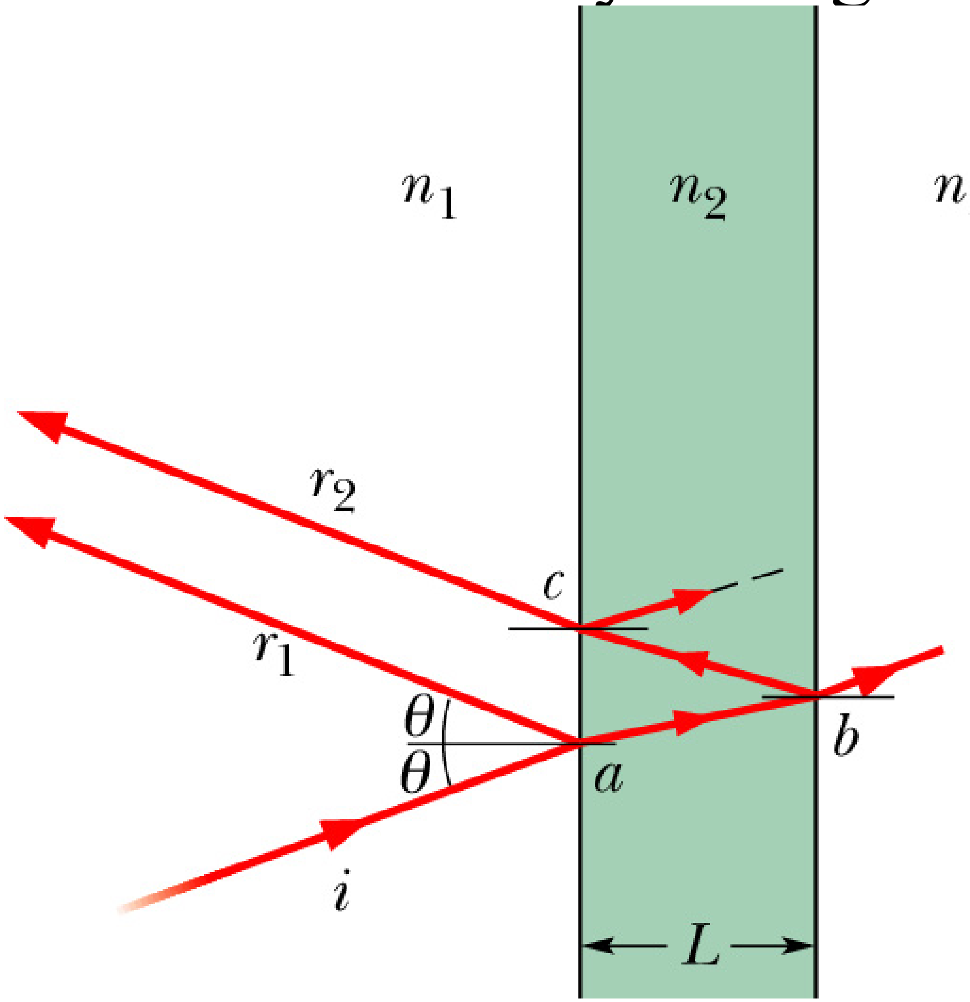
$$\phi_2 = k_2 L = (\frac{2\pi}{\lambda_2})L = (\frac{2\pi}{\lambda_0})n_2 L$$

$$\phi_1 = k_1 L = (\frac{2\pi}{\lambda_1})L = (\frac{2\pi}{\lambda_0})n_1 L$$

$$\Delta\phi = \phi_2 - \phi_1 = \underline{(\frac{2\pi}{\lambda_0})L(n_2 - n_1)}$$

Thin Film Interference

There are many ways to get a phase difference between two rays of light and so get interference.



When do rays r_1 and r_2 interfere destructively so there is no reflection?

For $\theta=0$ and $n_1<n_2<n_3$ the answer is easy: when the path difference 2L equals $\lambda/2$. (Or $3\lambda/2$, …)

Thin Film Example

Problem 36-33. Reflection of red light from a soap film with air on both sides. What thickness will give strong reflection? $\lambda_0 = 624\,nm$ $n = 1.33$

Wavelength in film: $\lambda = \lambda_0 / n = 624/1.33 = 469\,nm$

Phase change on reflection at front surface but not at back. So condition for strong reflection is

$2L = (m + \frac{1}{2})\lambda$ Solution: $L = \lambda/4 = 117\,nm$

$L = 3(117) = 352\,nm$

Recap

*

- **We have discussed conditions for constructive and destructive *interference* in terms of the *phase difference* Δφ:**
  - **Constructive: Δφ = 0, 360°, 720 °, …**
  - **Destructive: Δφ = 180°, 540°, …**

- **We have looked at 5 different ways to arrange for interference between two light waves:**
  - **Double slit, Reflection from glass surface, Thin films, Michelson interferometer, Different index of refraction.**

- **In most cases, we achieve a phase difference by arranging to have a *path difference* ΔL:**
  - **Constructive: ΔL = λ, 2λ, 3λ, …**
  - **Destructive: ΔL = λ/2, 3λ/2, 5λ/2, …**

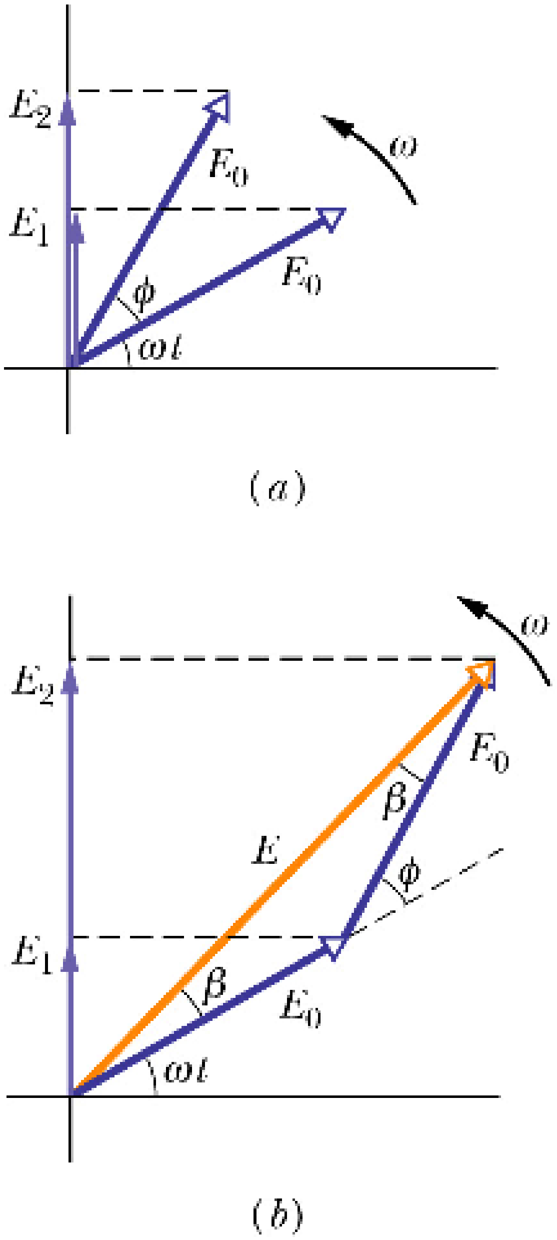
Interference: The General Case

What if the phase difference is neither 0 nor 180° but something in between? How can we calculate the resultant amplitude?

Use phasors!

Phasor Diagram

Just as for AC circuits, we can add two oscillating functions using phasors. The ***lengths*** of the phasors are the ***amplitudes*** of the waves and the ***angle*** between the phasors is the ***phase difference*** between the waves. Then the ***length*** of the resultant phasor is the ***amplitude*** of the total wave.



Adding Vectors

A good way to get the length of the sum of two vectors is to use the ***dot product:***

If $\vec{C} = \vec{A} + \vec{B}$

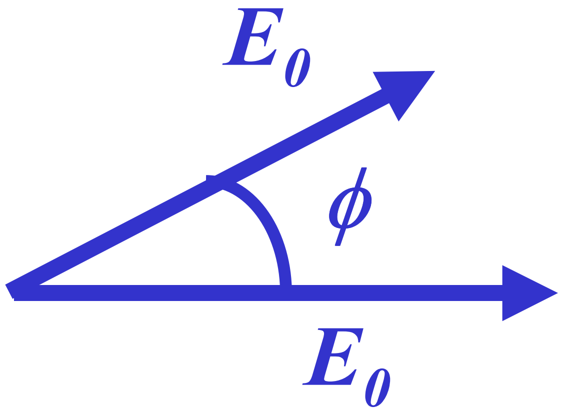
Then $C^2 = \vec{C} \cdot \vec{C} = \left(\vec{A} + \vec{B}\right) \cdot \left(\vec{A} + \vec{B}\right)$

$$= A^2 + B^2 + 2\vec{A} \cdot \vec{B}$$

$$= A^2 + B^2 + 2AB\cos\theta$$

Intensity Formula

Suppose two light waves have ***equal intensities*** I_0 and a ***phase difference*** of ϕ. When these waves interfere, what will be the total intensity I?



$$E^2 = E_1{}^2 + E_2{}^2 + 2\vec{E}_1 \cdot \vec{E}_2$$

$$E^2 = 2E_0{}^2 + 2E_0{}^2 \cos\phi$$

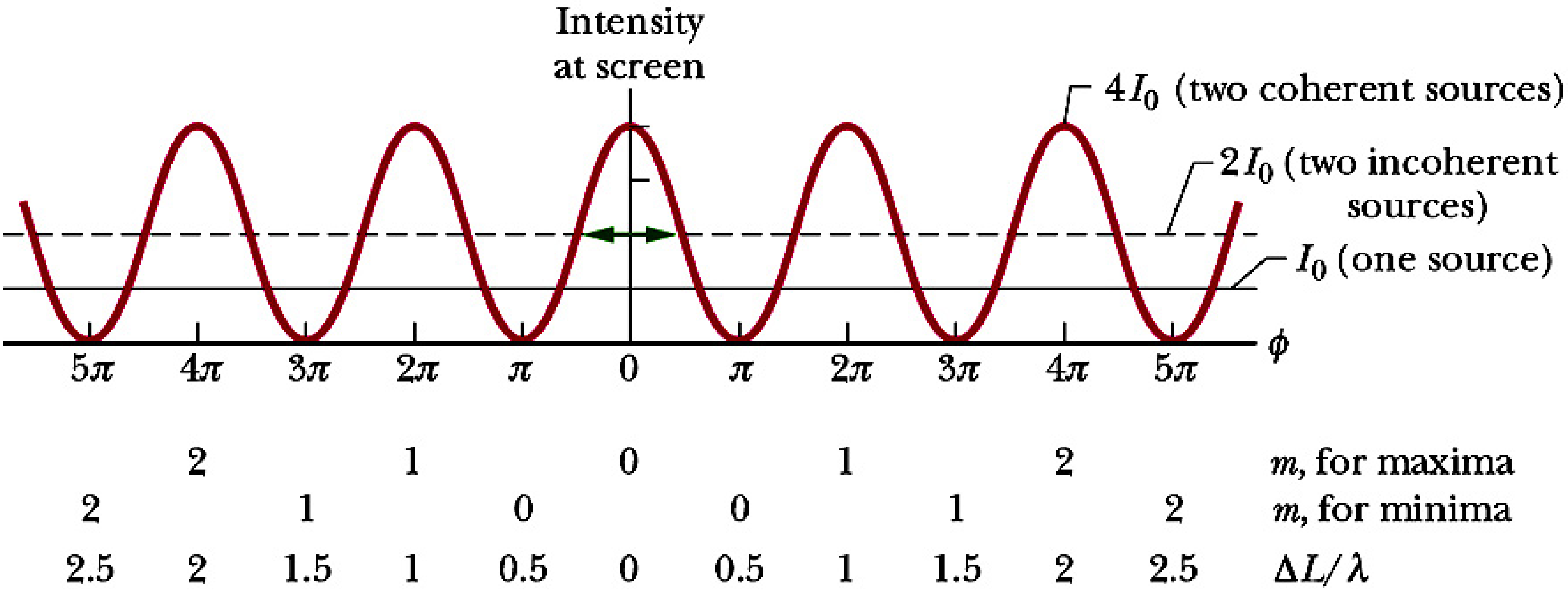
$$= 4E_0{}^2 \cos^2\left(\frac{1}{2}\phi\right)$$

But $I \propto E^2$ so:

$$I = 4I_0 \cos^2\left(\frac{1}{2}\phi\right)$$

Text equations 35-22,23; proved on p. 970.

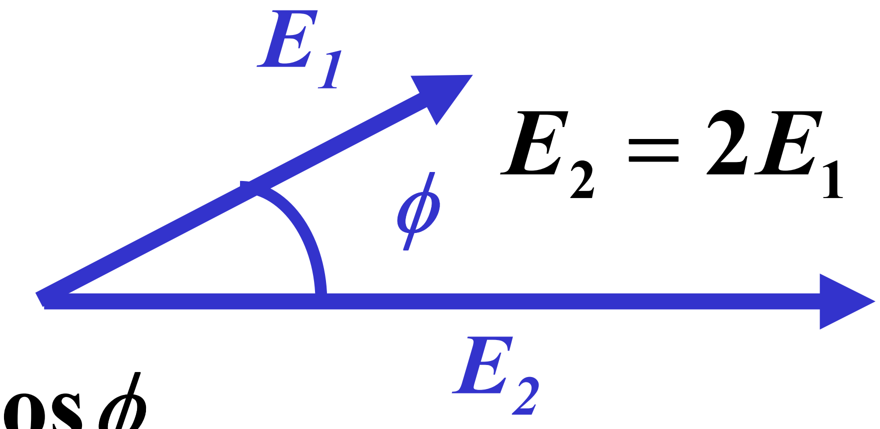
Double-slit intensity



Intensity Example

Problem 35-29 revised.

Two waves interfere with phase difference $\phi = 60°$.
One wave has intensity I_0, the other $4I_0$. What is the resulting intensity?



$$E_2 = 2E_1$$

$$E^2 = E_1^{\,2} + E_2^{\,2} + 2E_1E_2\cos\phi$$

$$E^2 = E_0^{\,2} + 4E_0^{\,2} + 2E_0(2E_0)(1/2)$$

$$= 5E_0^{\,2} + 2E_0^{\,2}$$

$$But \quad I = (Const)E^2$$

$$so \quad I = 5I_0 + 2I_0 = \boxed{7I_0}$$