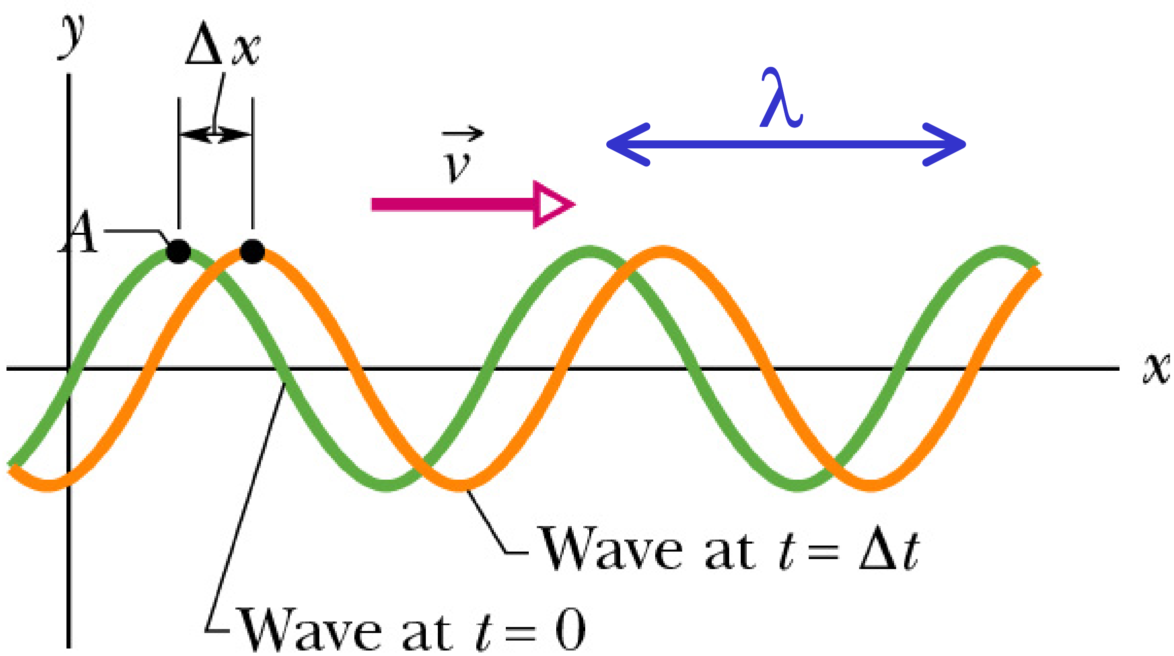
# Light Waves

- Today
  - Wavelengths, frequencies, polarization
  - Energy, momentum and photons
  - Reflection and refraction

# Review of Waves (Ch. 16)




Wavelength = $\lambda$

Frequency = **f**

Velocity = **v** = f $\lambda$

$$y = y_0 \sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$$

$$v = f\lambda = \omega / k$$

# Electromagnetic Waves

Try wave solutions for Maxwell Equations:

$$\vec{E}(x,t) = \vec{E}_0 \sin(kx - \omega t)$$
$$\vec{B}(x,t) = \vec{B}_0 \sin(kx - \omega t)$$

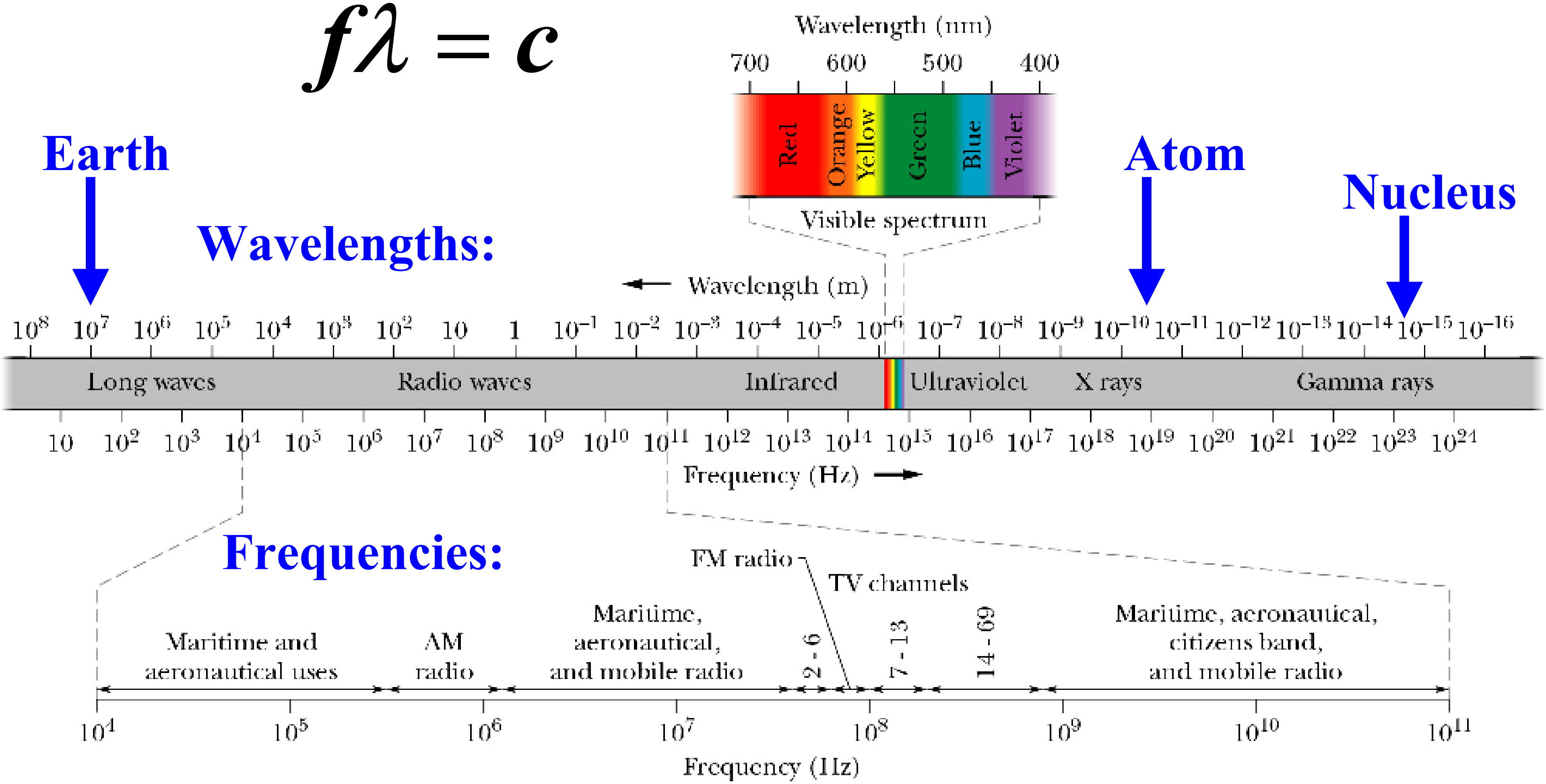
If we try these functions, for **any** given ω, we find they **do** satisfy Maxwell's Equations, and the wave speed is equal to the speed of light!

$$v = f\lambda = c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \underline{3.0 \times 10^8\ m/s}$$

Universal Constant

# The Electromagnetic Spectrum

$$f\lambda = c$$



# Radio Waves to X Rays

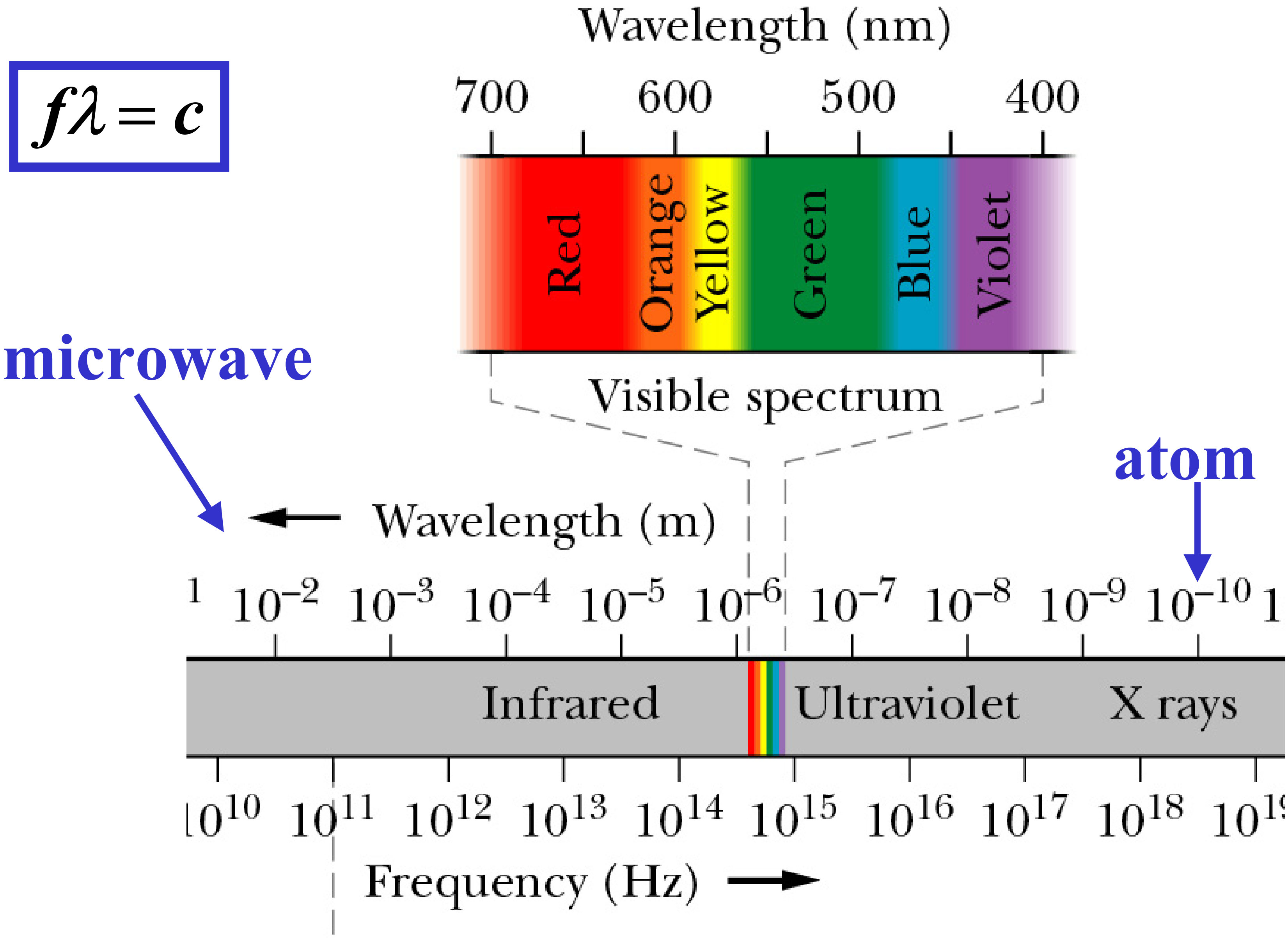
The range of wavelengths λ
(and frequencies f = c/λ) is amazing.
***All are electromagnetic waves governed by Maxwell's Equations!***

**AM radio, f =1500 kHz:** $$\lambda = \frac{c}{f} = \frac{3\times10^{8}}{1.5\times10^{6}} = 200\,m$$

**X ray, the size of an atom, λ = 0.1 nm:** $$f = \frac{c}{\lambda} = \frac{3\times10^{8}}{1\times10^{-10}} = 3\times10^{18}\,Hz$$

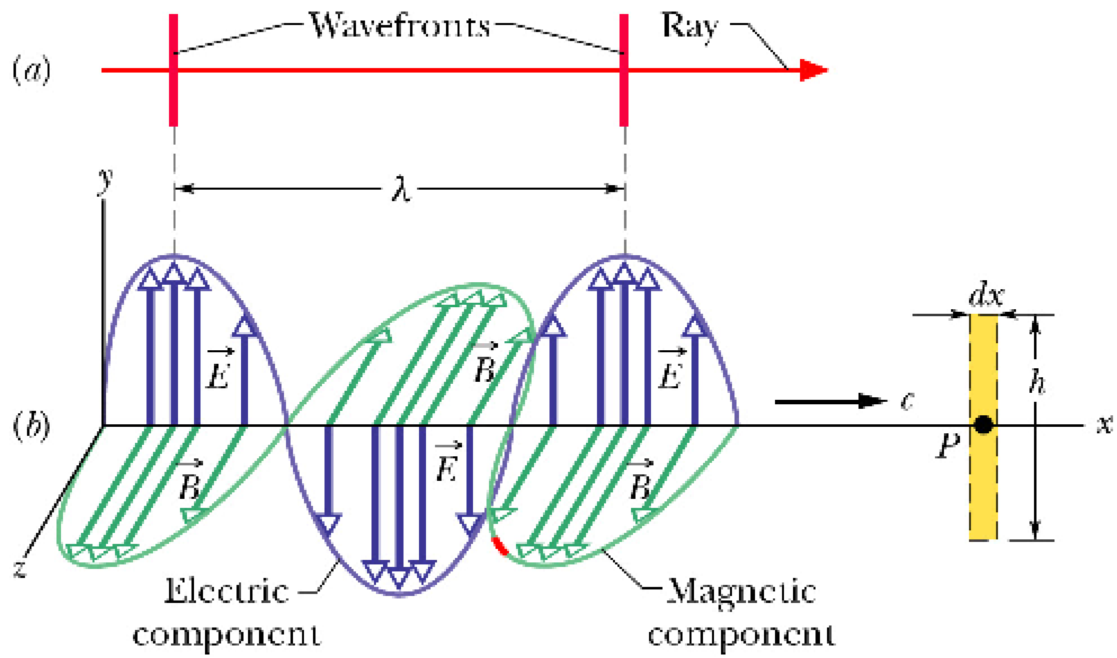
# A Closer Look

$$\boxed{f\lambda = c}$$

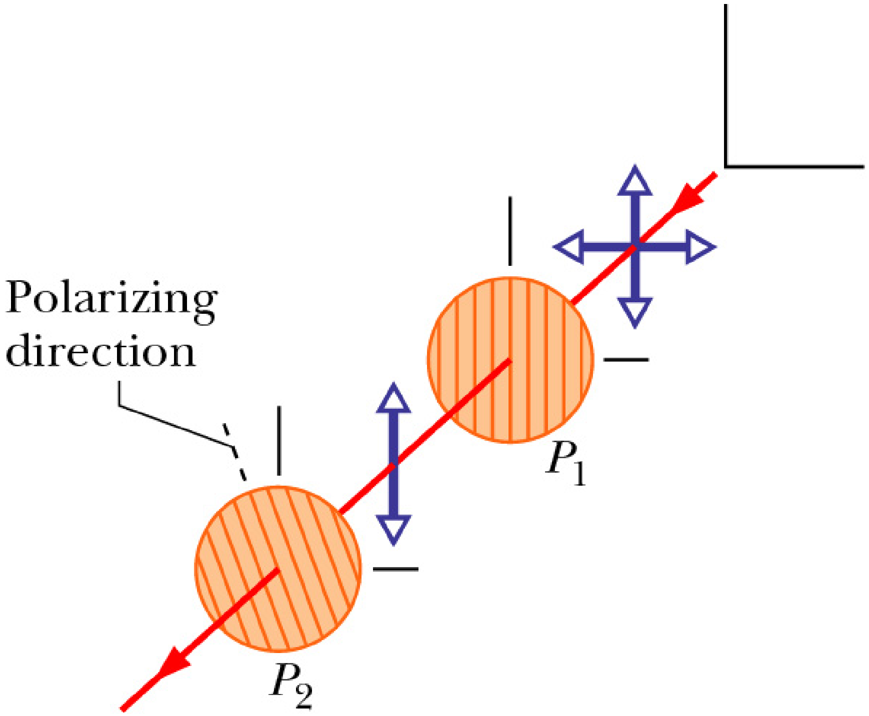


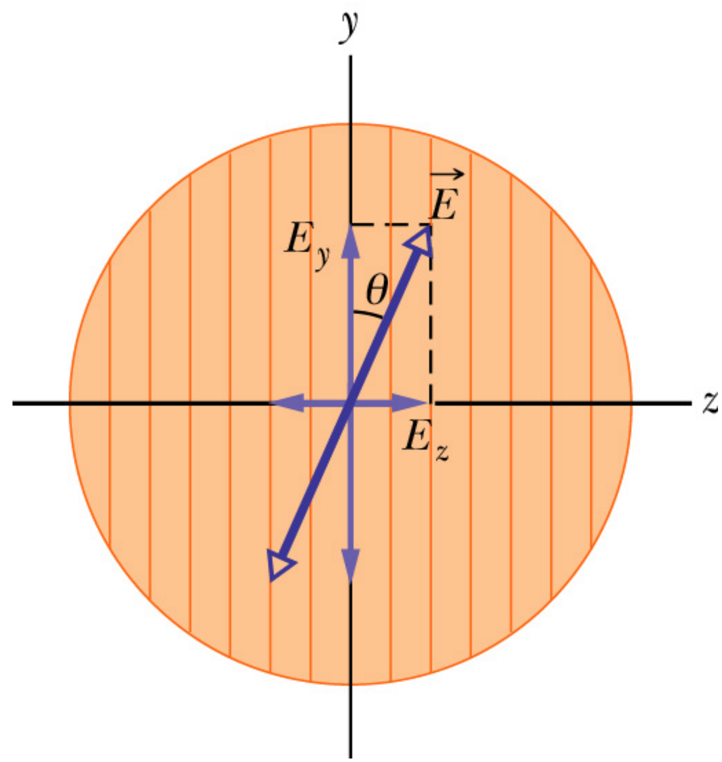
# Polarization

Electromagnetic waves are ***transverse***. For a wave propagating in the ***z*** direction, the polarization can be either in the ***x*** or the ***y*** direction, depending on the direction of the oscillating electric field.



# Polarization




$$E_y = E\cos\theta$$

$$I = I_0\cos^2\theta$$

# Energy

$$I = Intensity \equiv \frac{Power}{Area}$$

$$= \left(\tfrac{1}{2}\varepsilon_0 E^2\right)c = \frac{E^2{}_{rms}}{c\mu_0} \quad \propto E^2$$

**Intensity of a point source**

$$I(r) = \frac{P}{4\pi r^2}$$

**Another inverse square law.**

# Momentum and Radiation Pressure

**Maxwell's Equations show that a light wave carries momentum in addition to energy:**

$$\boxed{momentum = \frac{energy}{c}}$$

NOTE: comes from Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$
For details see text section 33-5

$$force = \frac{momentum}{time} = \frac{power}{c}$$

$$radiation \quad pressure = \frac{force}{area} = \frac{I}{c}$$

# Photons (Chapter 38)

- Quantization
  - Quantize energy and momentum of wave.
- Photon energy: $E = hf$
- Photon momentum: $p = h/\lambda$

Energy-momentum relation for a particle (ch.37): $$E = \sqrt{(pc)^2 + (mc^2)^2}$$

But for a photon we find: $E = hf = hc/\lambda = \underline{pc}$

Equations agree if <u>m=0.</u>

# Light in Matter

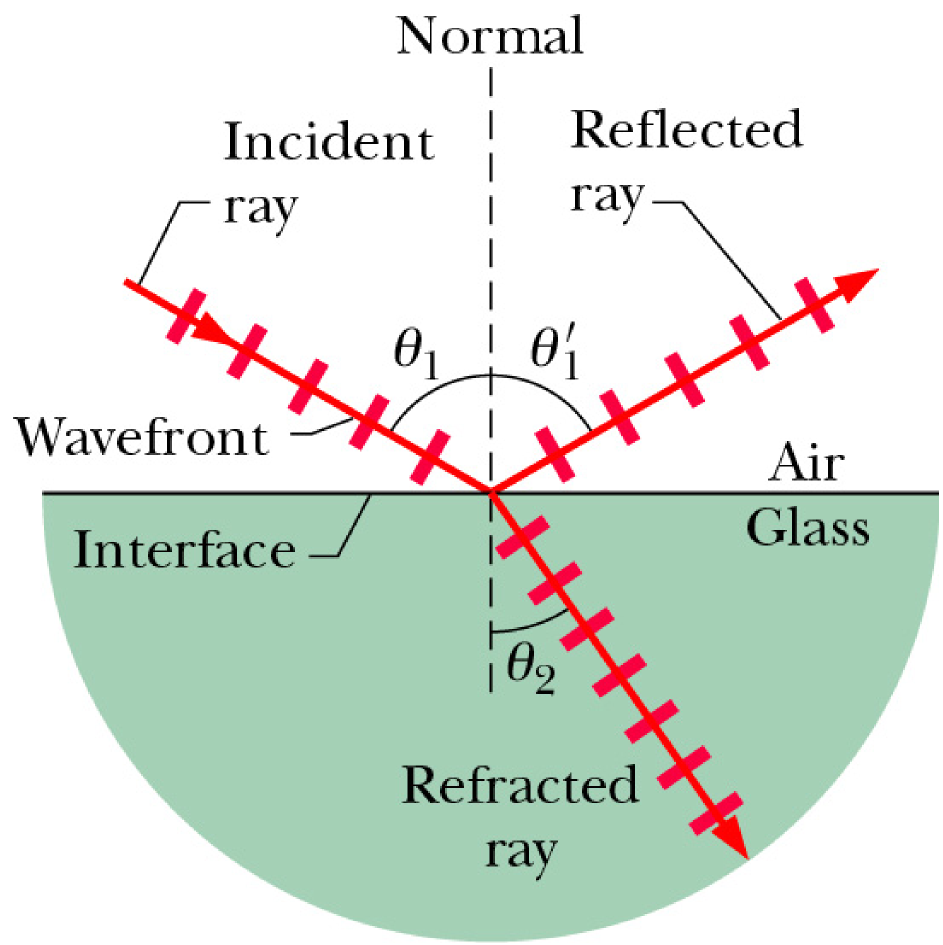
In vacuum light travels with speed c = $3\times10^8$ m/s. But light also travels through many materials such as glass, water, etc. In such a medium, its speed is reduced to $\boldsymbol{v = c/n}$, where $\boldsymbol{n}$ is called the **index of refraction**.

$$\boldsymbol{v = \frac{c}{n} < c}$$

(In fact, no physical object can travel faster than c. There are no "warp drives" on real space ships.)

# Reflection and Refraction

When a light ray passes from one medium to another, it splits into two beams, the ***reflected*** and ***refracted*** rays.



(*b*)

- $\theta_1' = \theta_1$
- $n_1 \sin\theta_1 = n_2 \sin\theta_2$

# Snell's Law

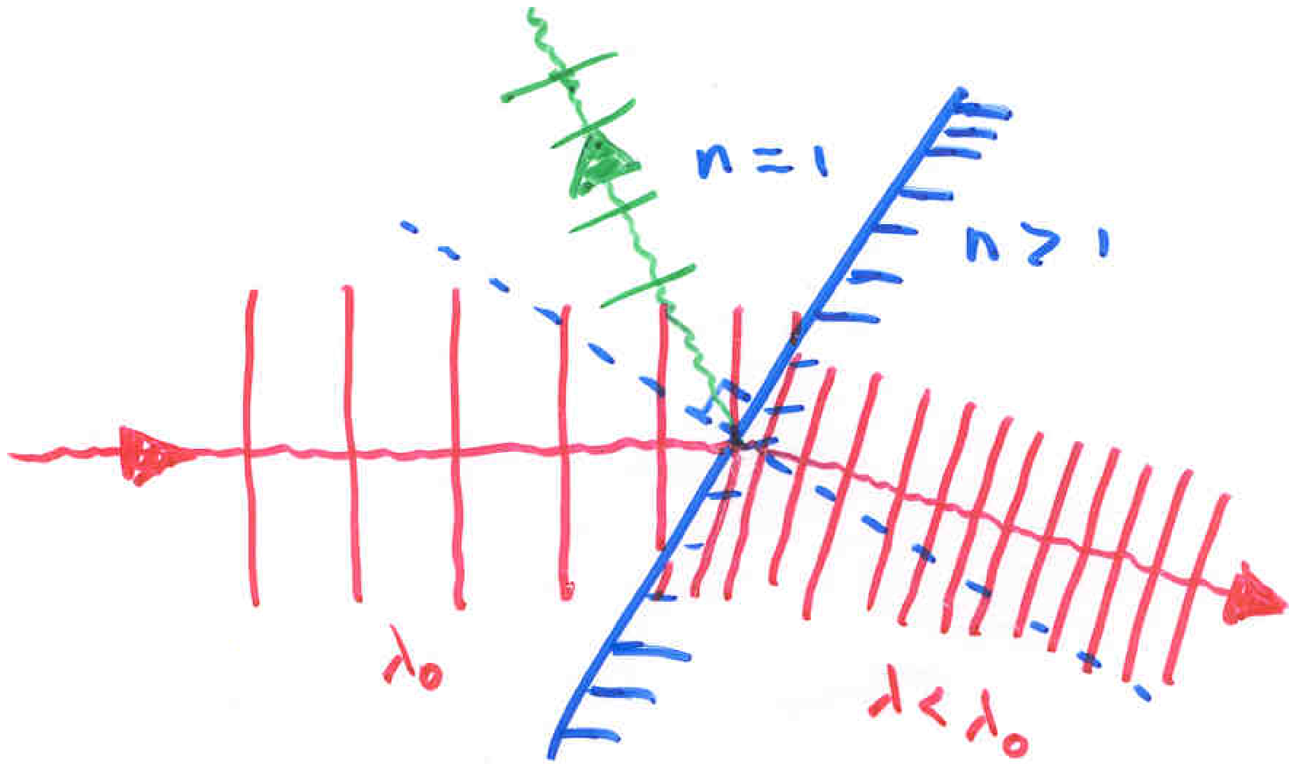
Snell's Law: $$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

What causes this? It's because the wavelength changes. For example, going from vacuum to glass the wavelength decreases along with the speed.

Vacuum: $$n = 1, \quad v = c, \quad \lambda_0 = \frac{c}{f}$$

Glass: $$n > 1, \quad v < c, \quad \lambda = \frac{v}{f} < \lambda_0$$

# Refraction

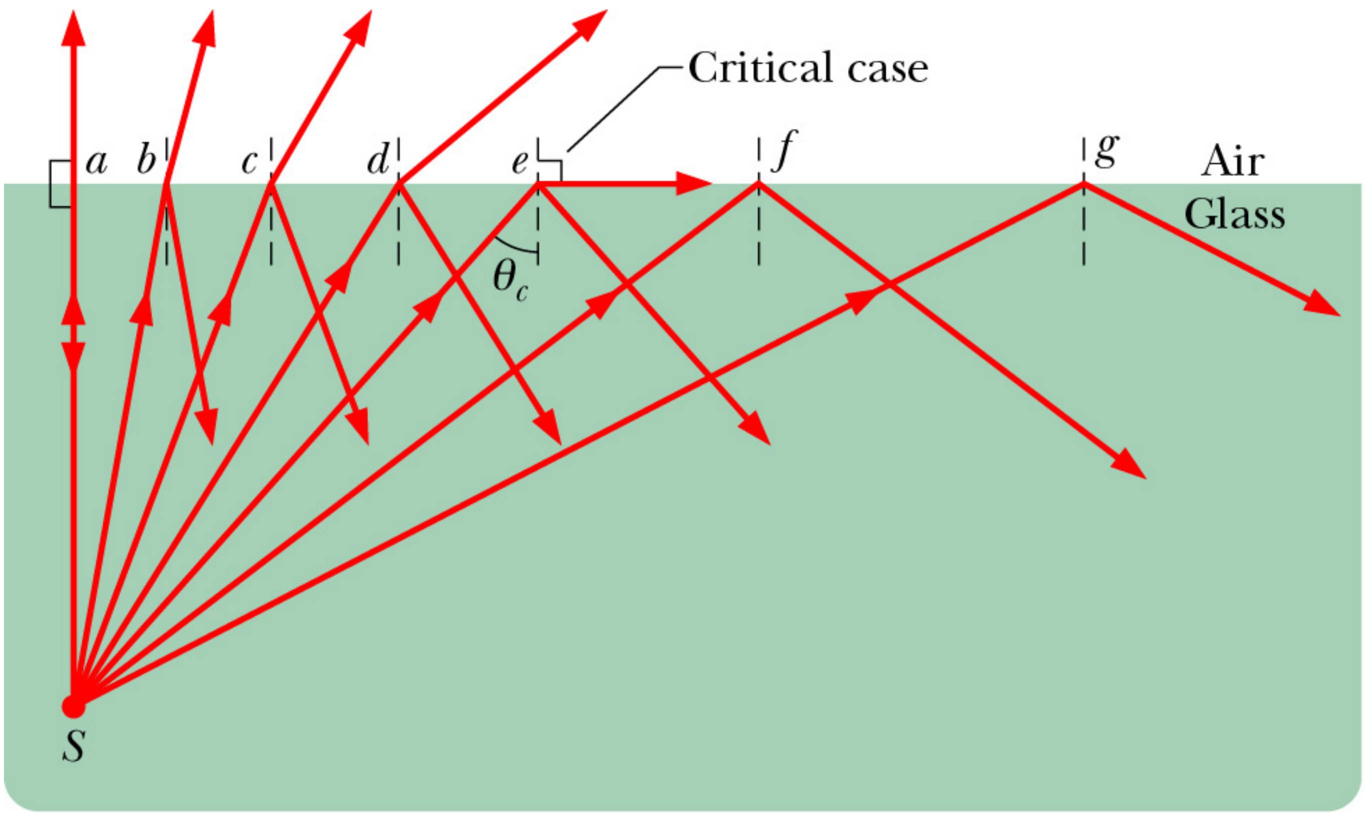



So ray is bent ***toward the normal*** in this case. But if we go from large n to small n, ray is bent ***away from*** the normal.

# Total Internal Reflection

If $n_1 > n_2$ then there may be ***no solution*** of Snell's Law for the angle of refraction.

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 > 1$$



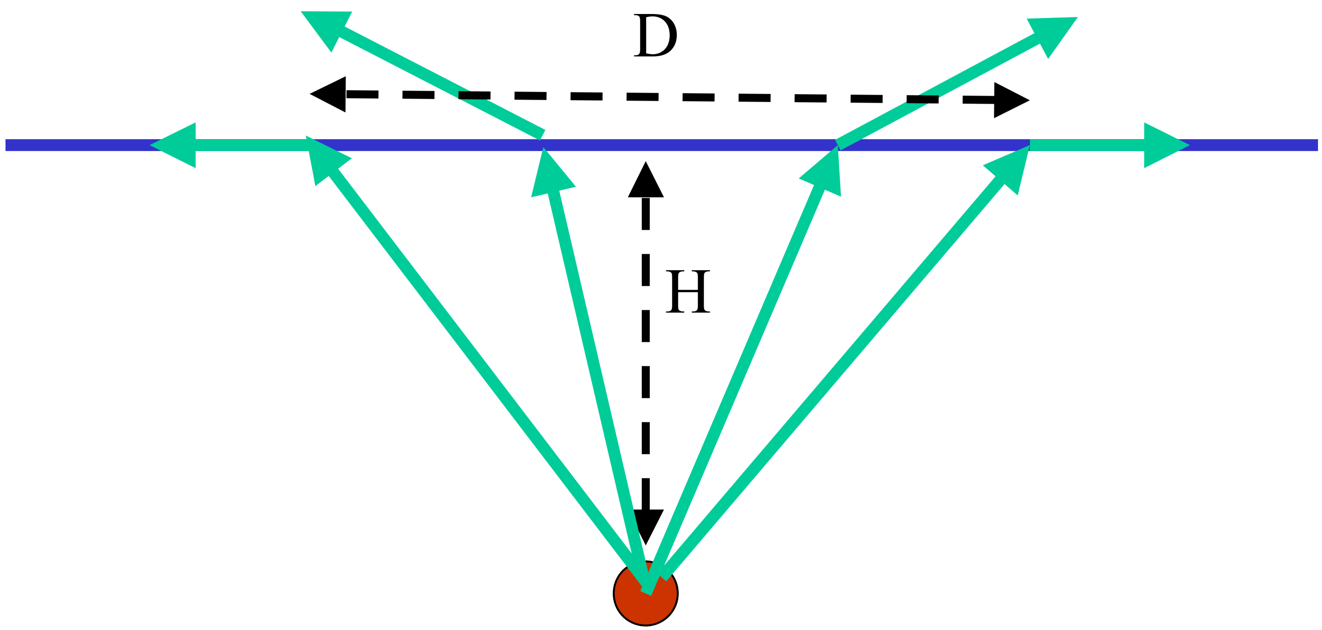

So for $\theta_1 > \theta_c$ there is no refracted ray, only the reflected ray.

# Optical Fibers

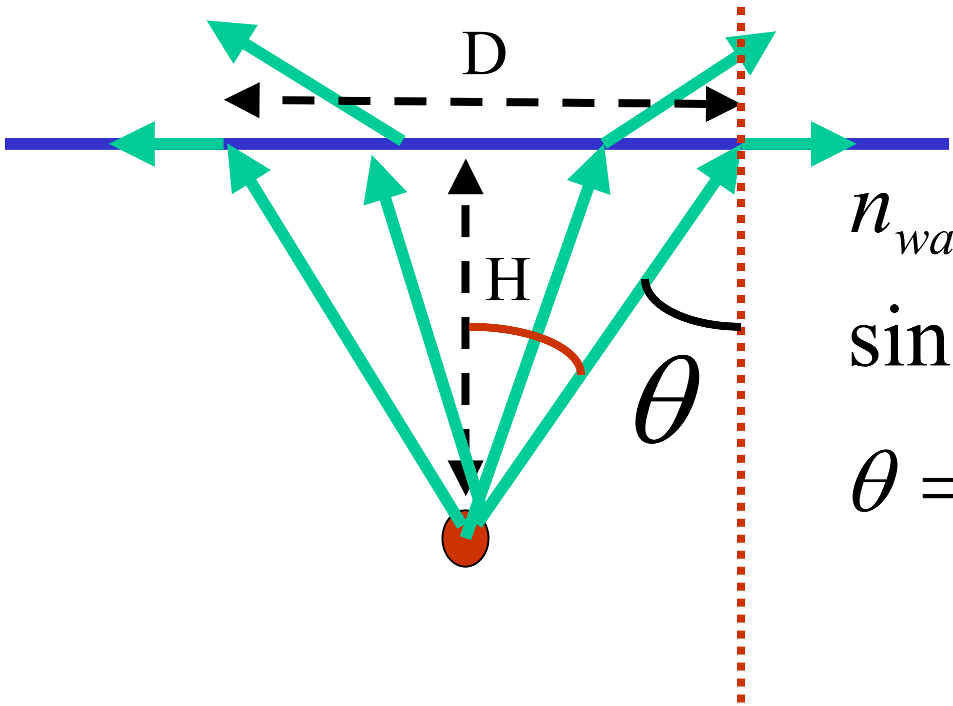
Light is trapped inside tube of transparent material with high ***n***. Kept inside by repeated total internal reflections. Propagates without loss of energy.

# Example: Problem 33-55

**A point source of light is 80 cm below the surface of a pool. Find the diameter of the circle at the surface through which light emerges from the water.**



# Problem 33-55 (cont'd)




$$n_{water}\sin\theta = n_{air}\sin 90°$$

$$\sin\theta = 1/n_{water} = 1/1.33 = 0.75$$

$$\theta = \sin^{-1}(0.75) = 48.6°$$

$$\frac{D/2}{H} = \tan\theta$$

$$D = 2H\tan\theta = 2(80cm)\tan(48.6°) = \underline{181cm}$$

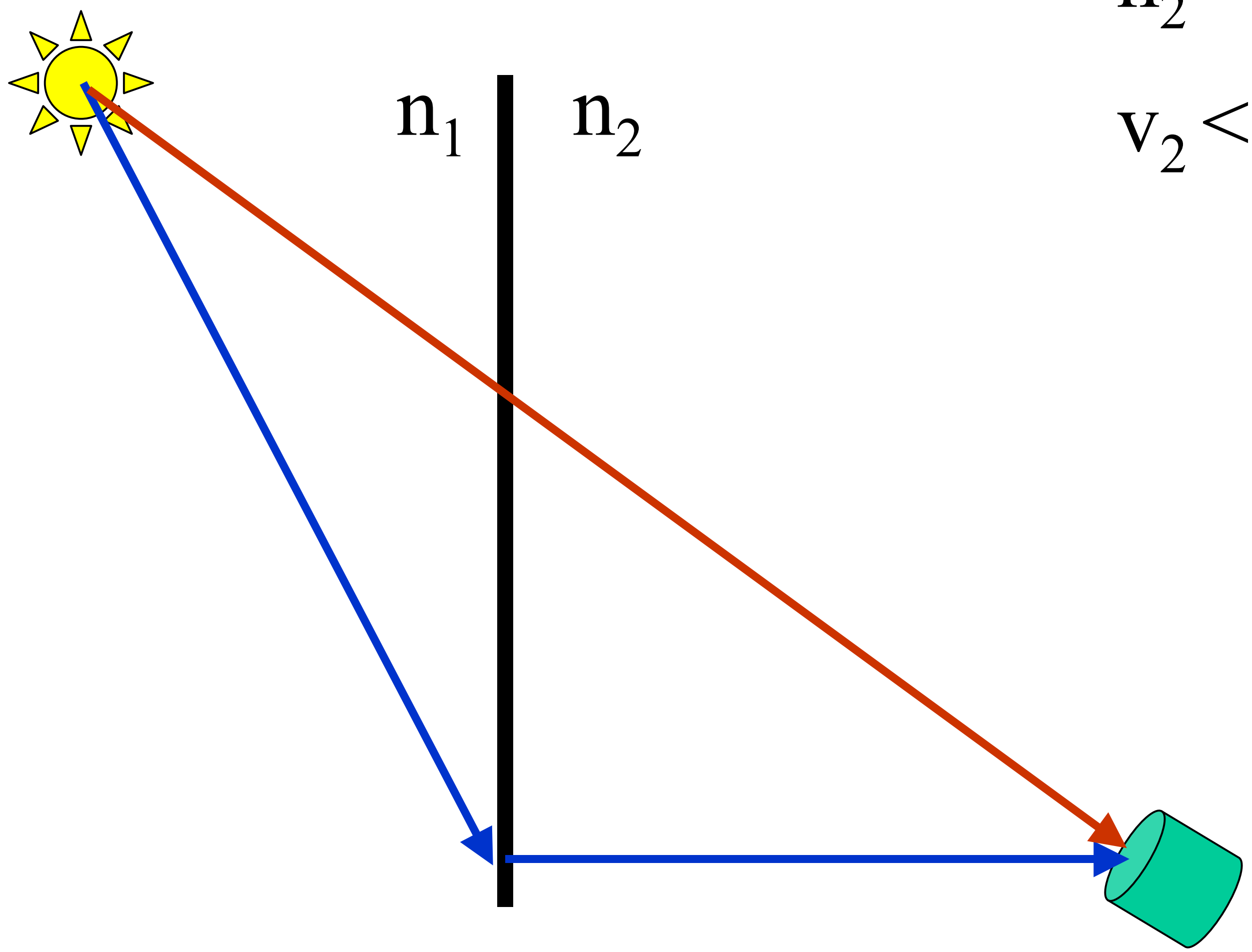
# Fermat's Principle

The path chosen by a light ray will be the one which ***minimizes the time***.

$v = c/n$

$n_2 > n_1$

$v_2 < v_1$



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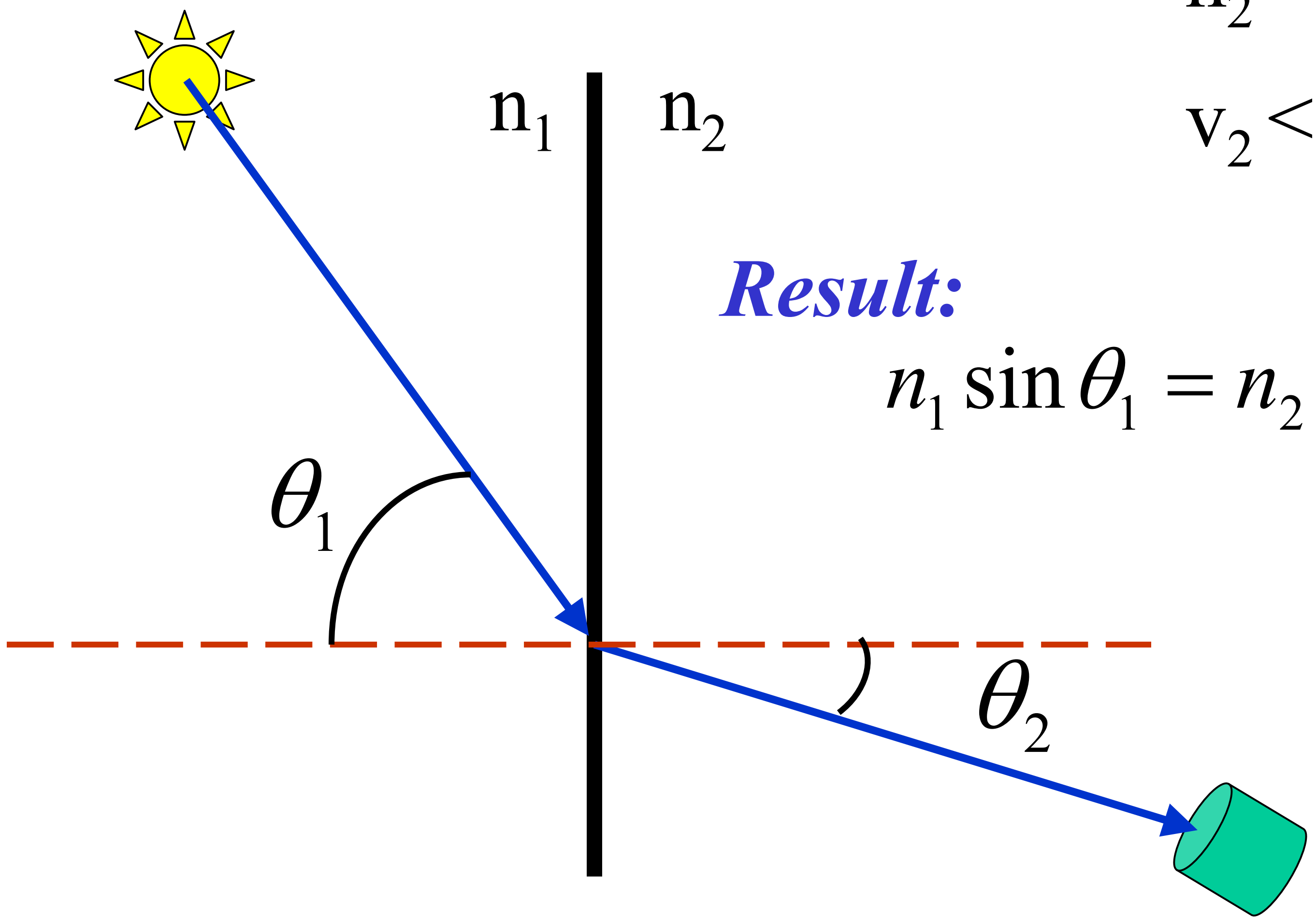



***Result:***

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

# Optics

- **Last time:** **Reflection and refraction**
- **Today:** **Image formation by lenses and mirrors**

# IMAGES

**REAL IMAGE:** **The light is *really* concentrated, such as when you start a fire using sunlight and a lens.**

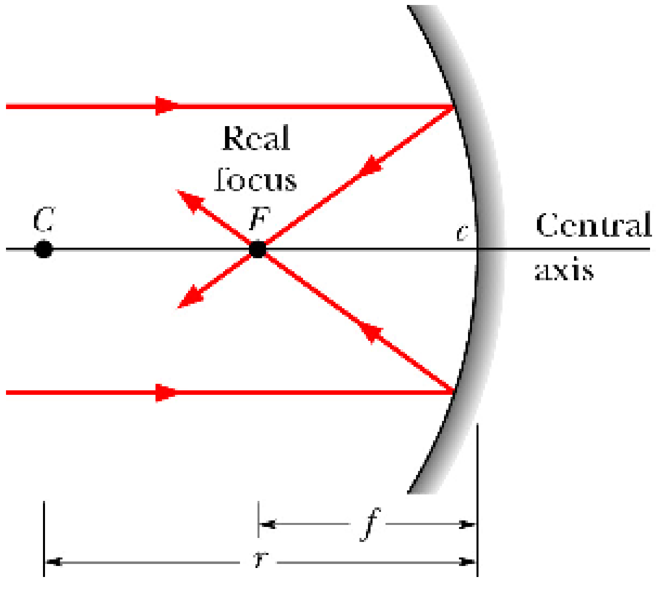
**In formulas the image distance *i* is *positive*.**

**VIRTUAL IMAGE:** **The light only *appears* to come from it, as when you seem to see your face behind the bathroom mirror.**

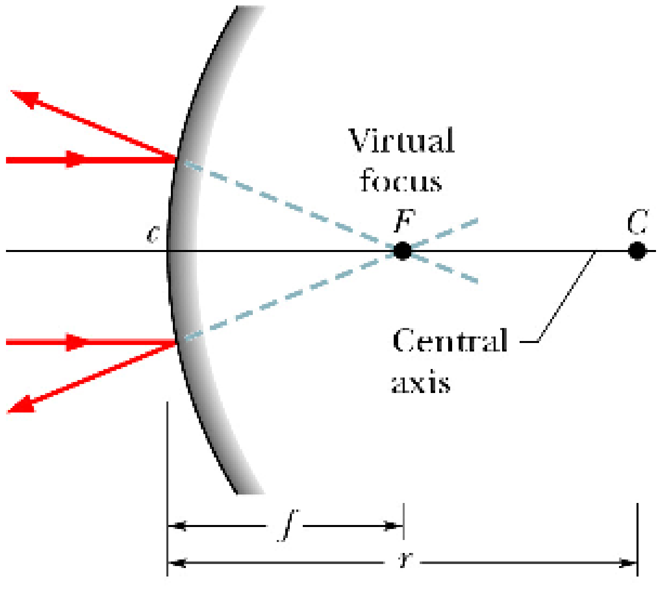
**In formulas the image distance *i* is *negative*.**

# Spherical Mirrors

- **Focal point**
- **Focal distance**
- **Concave mirror is *converging*: f > 0.**
- **Convex mirror is *diverging*: f < 0.**

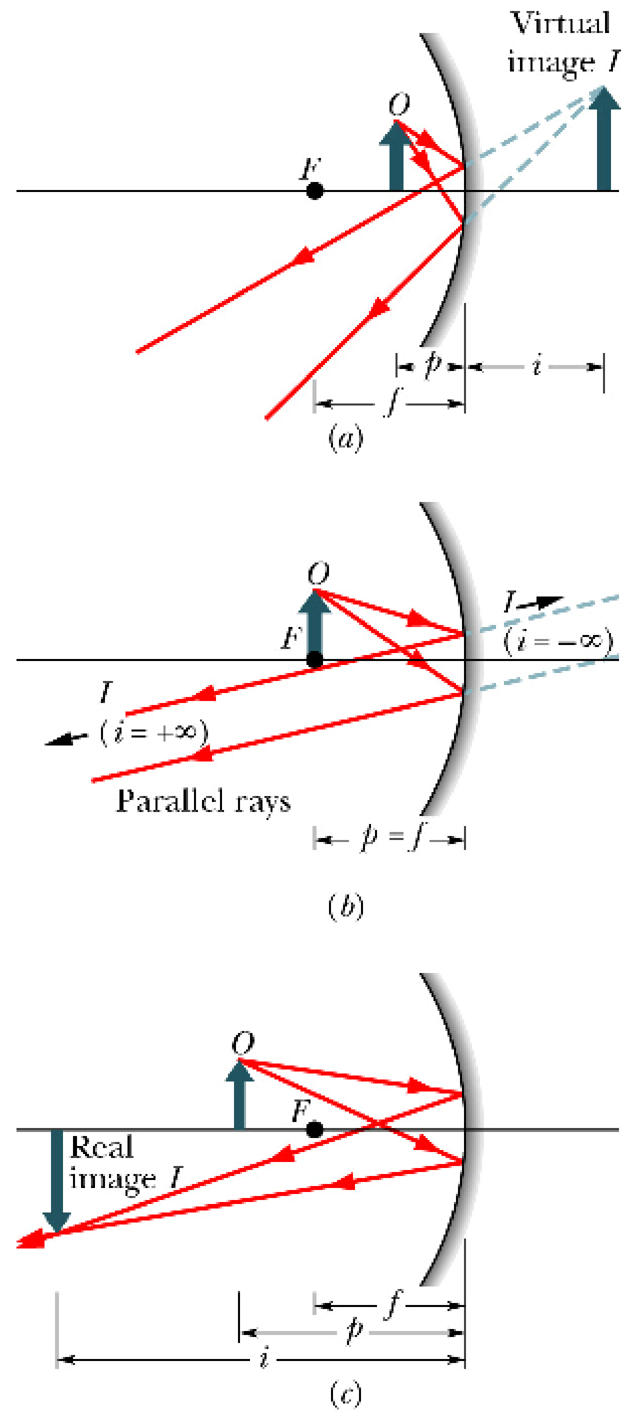


(a)



(b)

# Images in Concave Mirrors

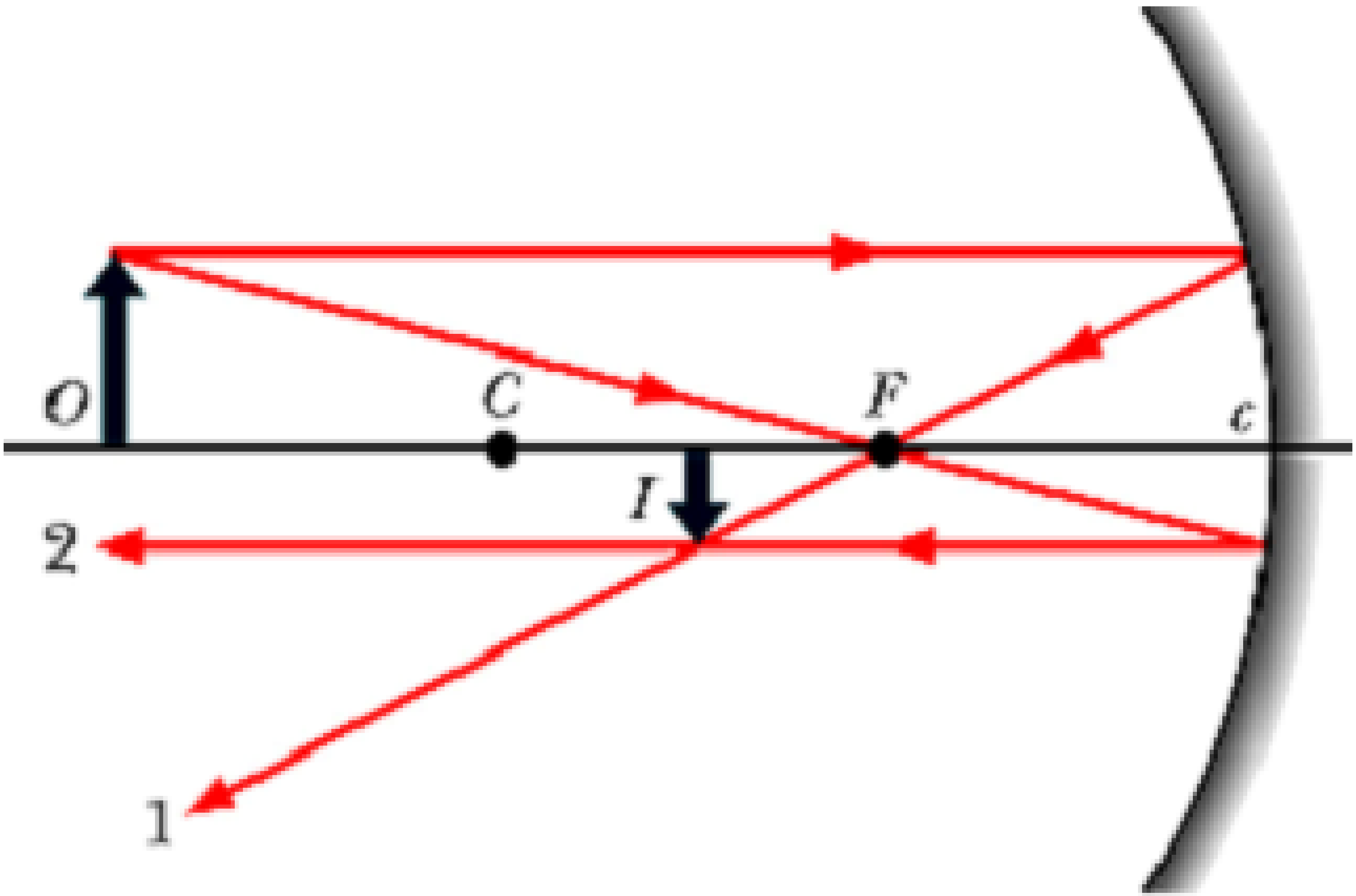



**Let $p$ = object distance, let $i$ = image distance.**

**Let $p,i$ be positive for real, negative for virtual.**

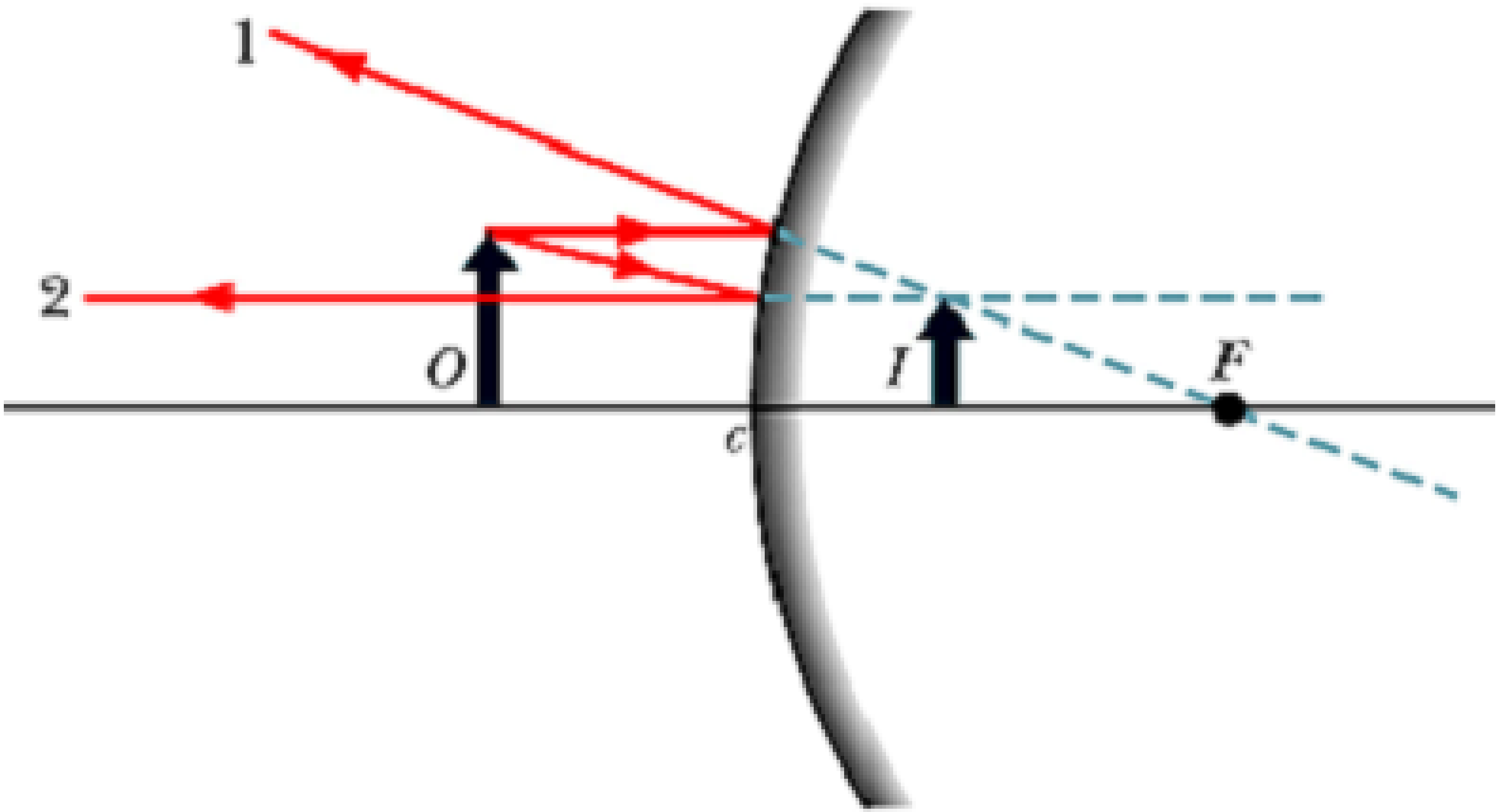
# Locating the Images

## Principal Rays for concave mirror



# Locating the Images

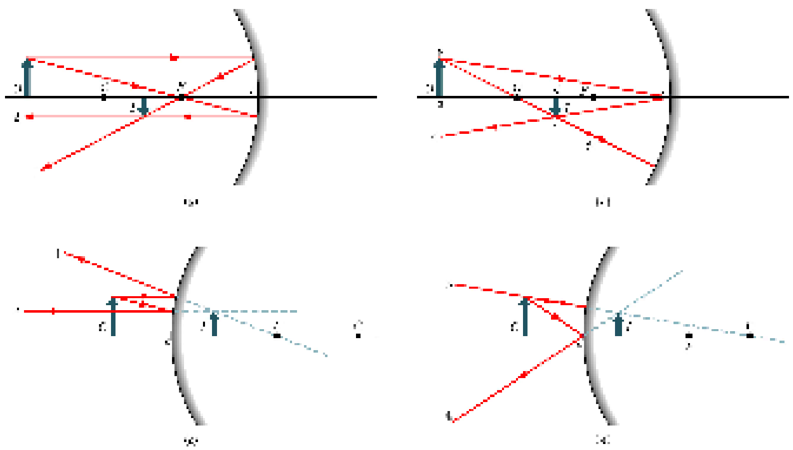
## Principal Rays for convex mirror



# Equations for the Images

## ** Small-angle Approximation **

### Summarize Results:



$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}$$

Also: let ***magnification m*** be defined as ***image size divided by object size***. Then:

$$m=-\frac{i}{p}$$

Negative ***m*** means ***inverted image***!

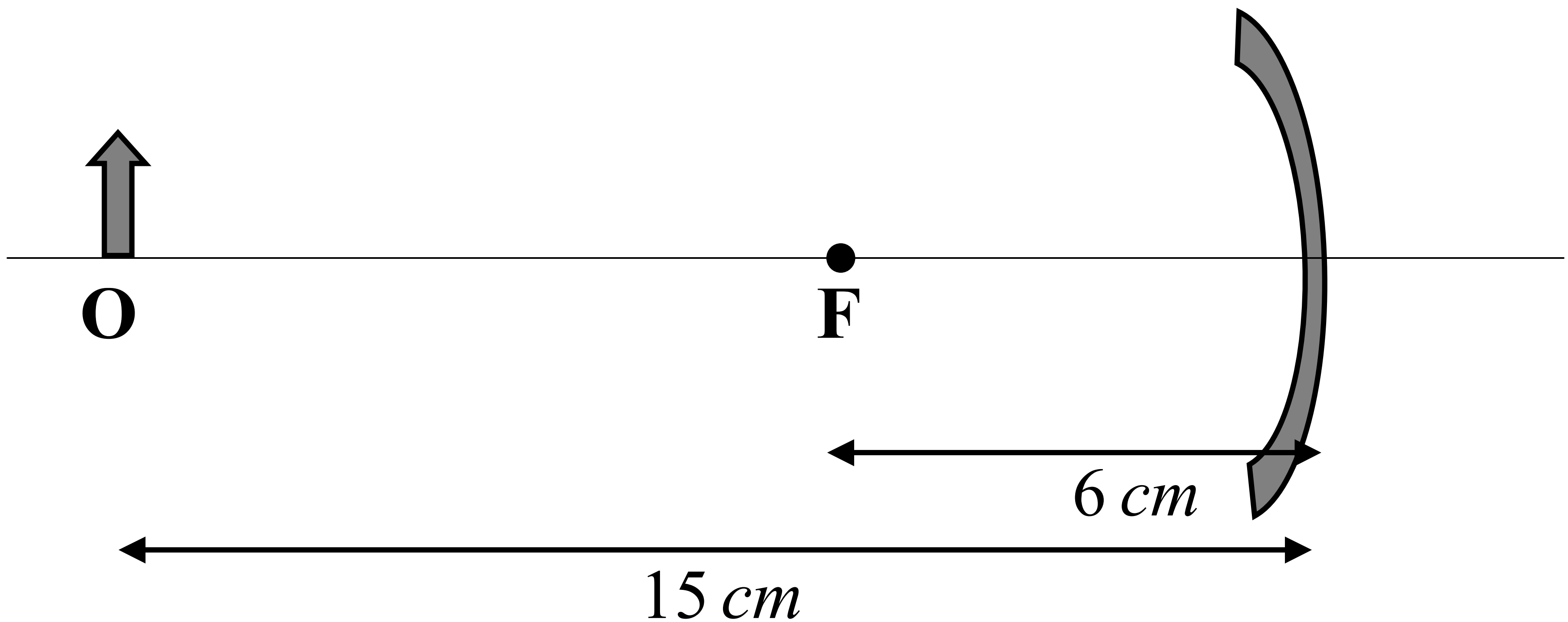
# Spherical mirror example 1

**An object is placed 15 cm in front of a concave mirror with focal length 6 cm. Describe the resulting image.**

$$1/f = 1/p + 1/i$$

$$f = +6\ cm$$

$$p = +15\ cm$$

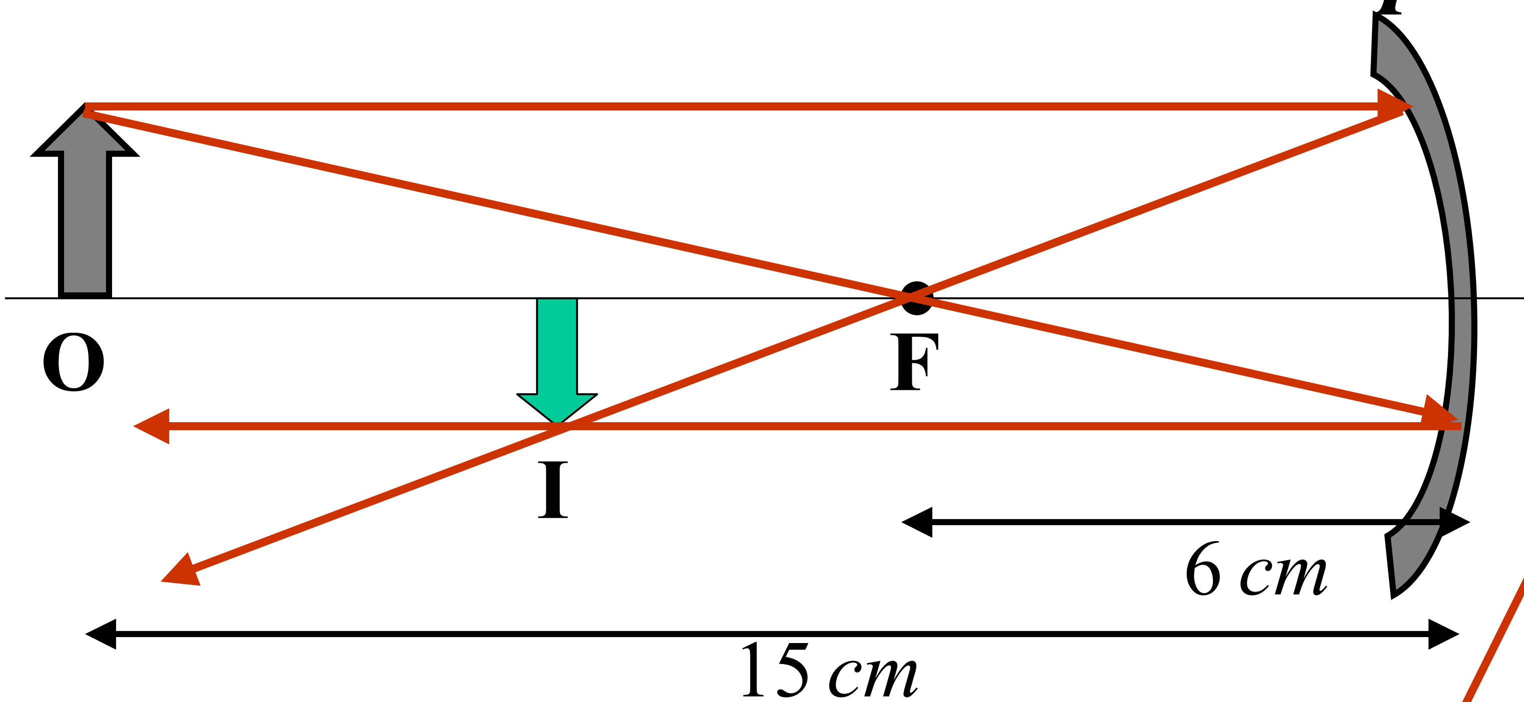


# Example 1 solution

$f = +6\ cm$

$p = +15\ cm$

Method 1: Principal Rays




- Real
- Reduced
- Inverted

Method 2: $\frac{1}{i} = \frac{1}{6} - \frac{1}{15} = \frac{5}{30} - \frac{2}{30} = \frac{1}{10}$ $so\ \ i = +10\ cm$

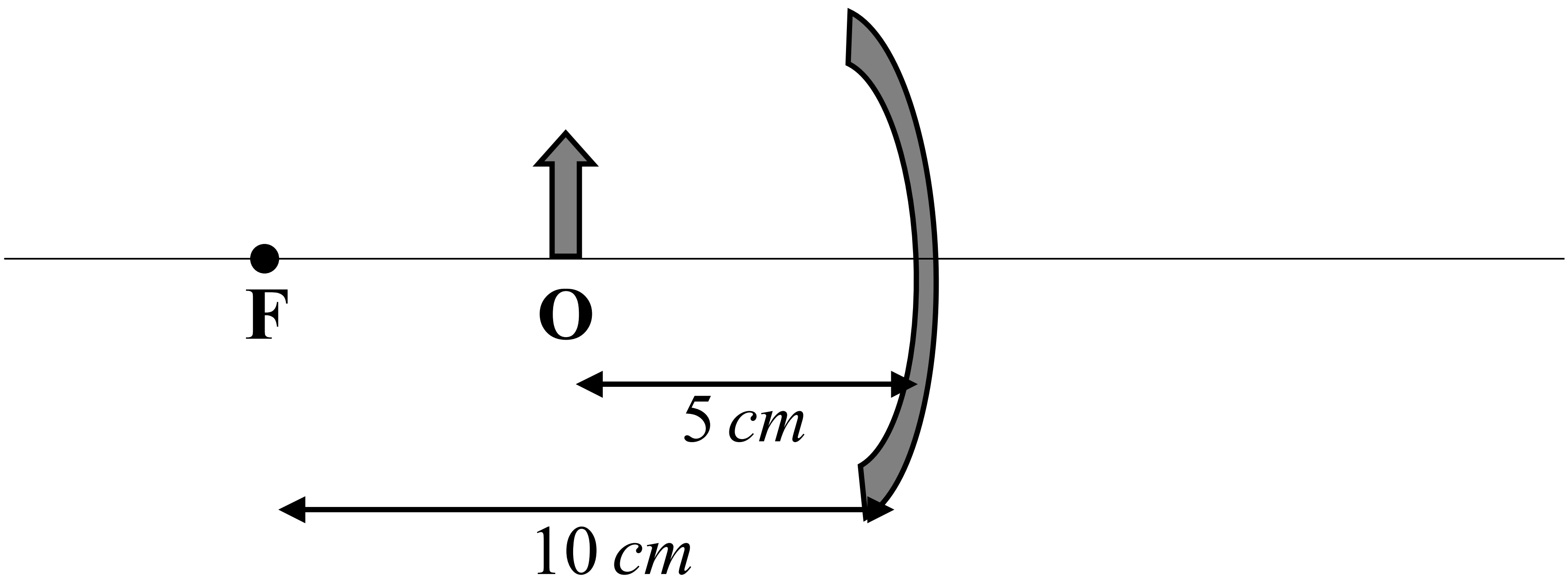
$$m = -i / p = -10/15 = -2/3$$

# Spherical mirror example 2

**An object is placed 5 cm in front of a concave mirror with focal length 10 cm. Describe the resulting image.**

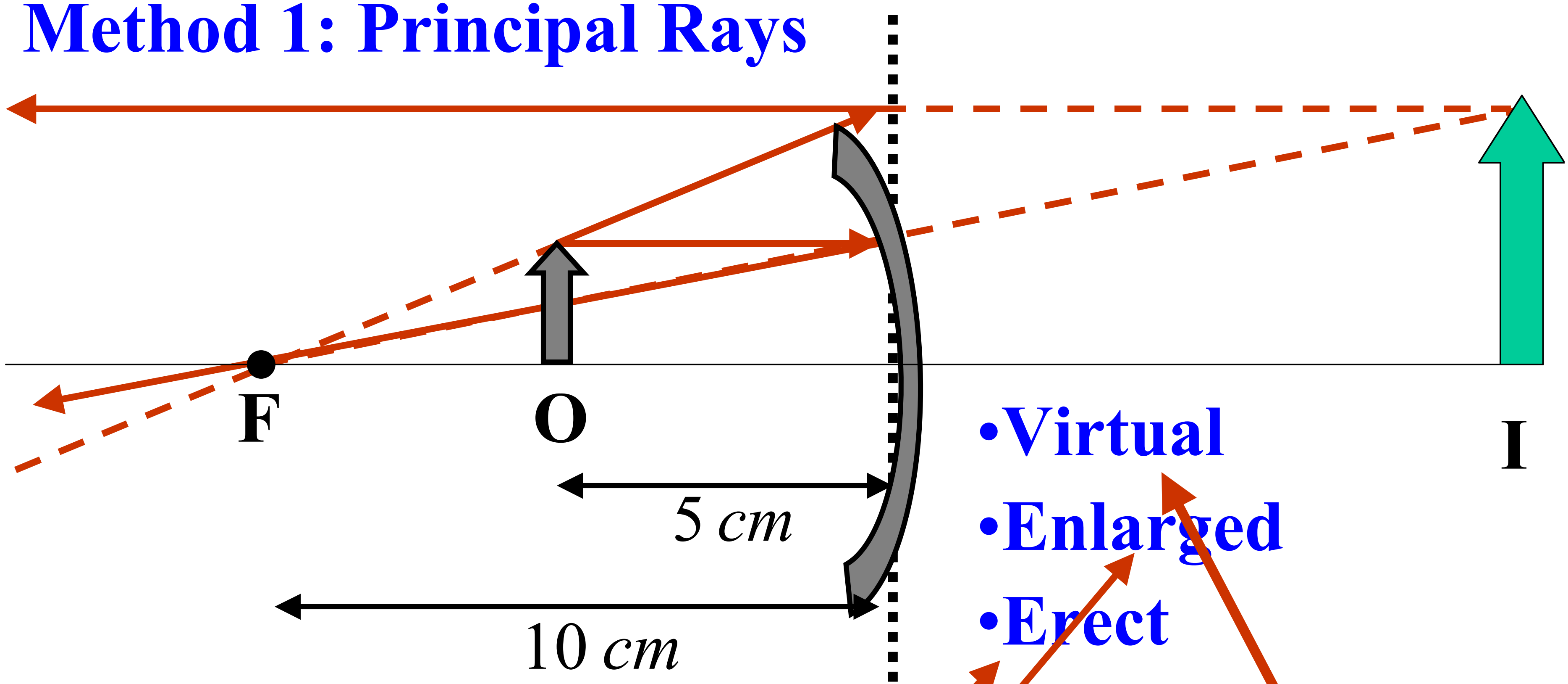
$$f = +10\ cm$$

$$p = +5\ cm$$



# Example 2 Solution

## Method 1: Principal Rays



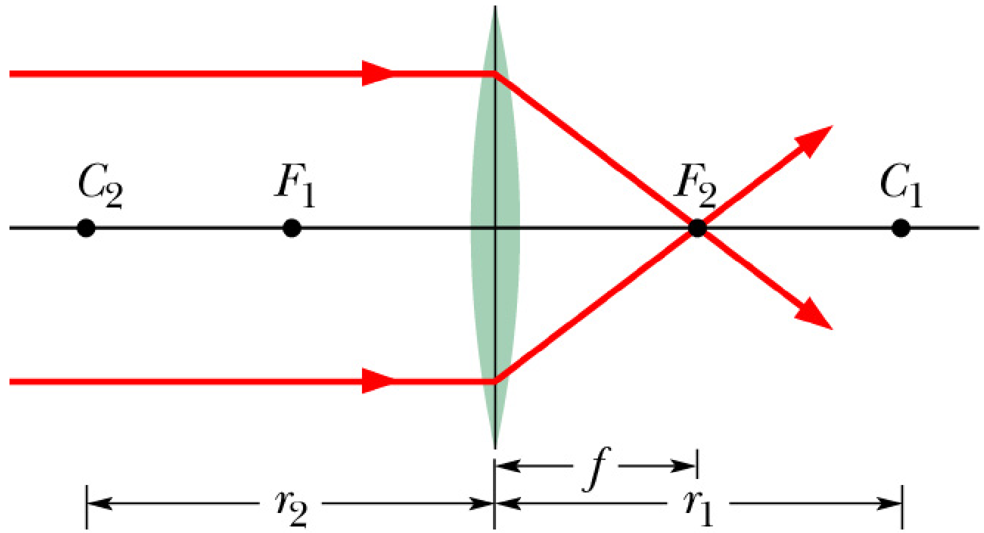

- Virtual
- Enlarged
- Erect

**Method 2:** $$\frac{1}{i}=\frac{1}{f}-\frac{1}{p}=\frac{1}{10}-\frac{1}{5}=-\frac{1}{10} \quad so \quad i=-10\ cm$$

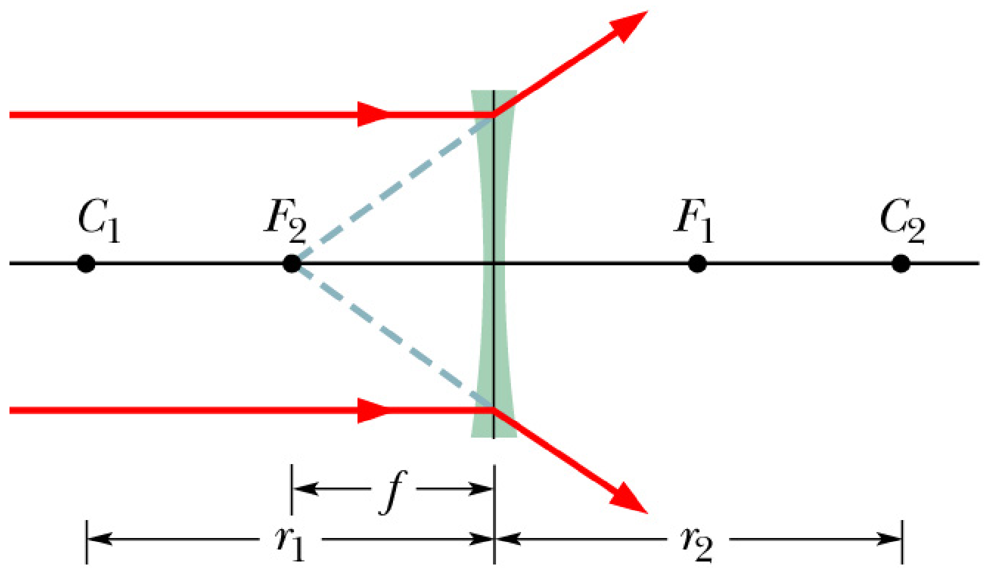
$$m=-i/p=-(-10)/5=+2$$

# Thin Lenses

- **Focal point**
- **Focal distance**
- **Convex lens is *converging*: f > 0.**
- **Concave lens is *diverging*: f < 0.**

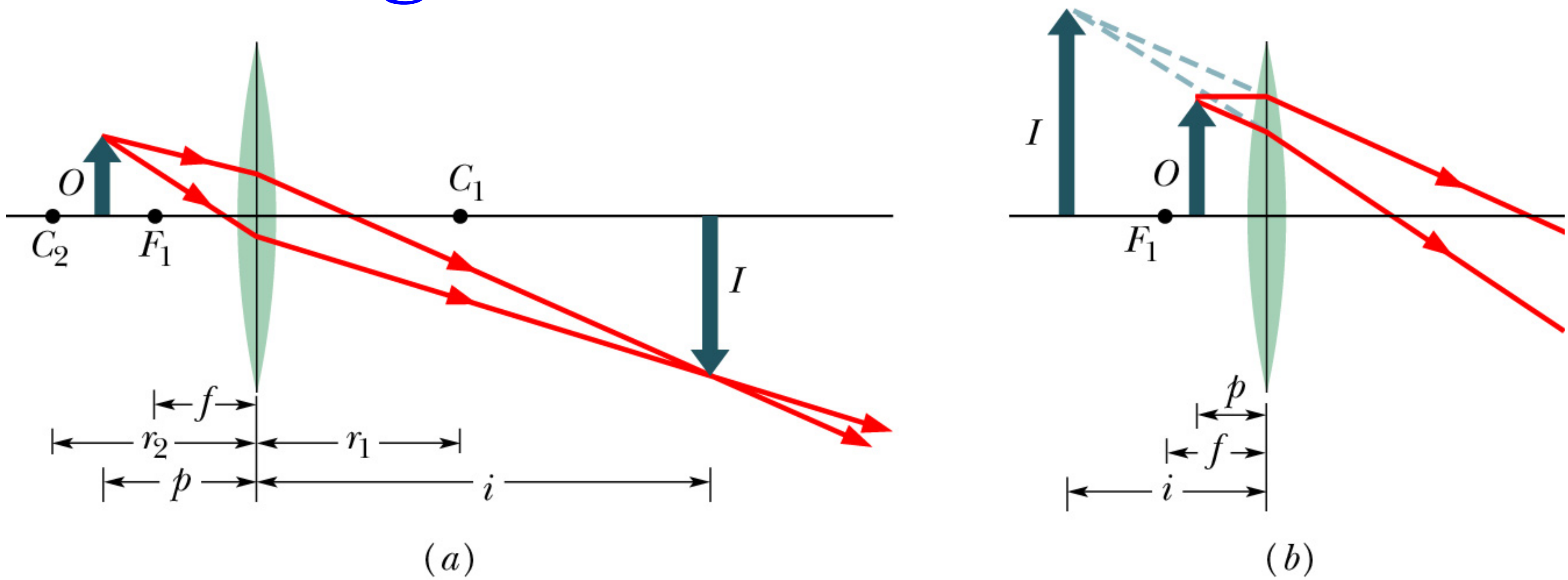


(a)



(c)

# Images in Thin Lenses

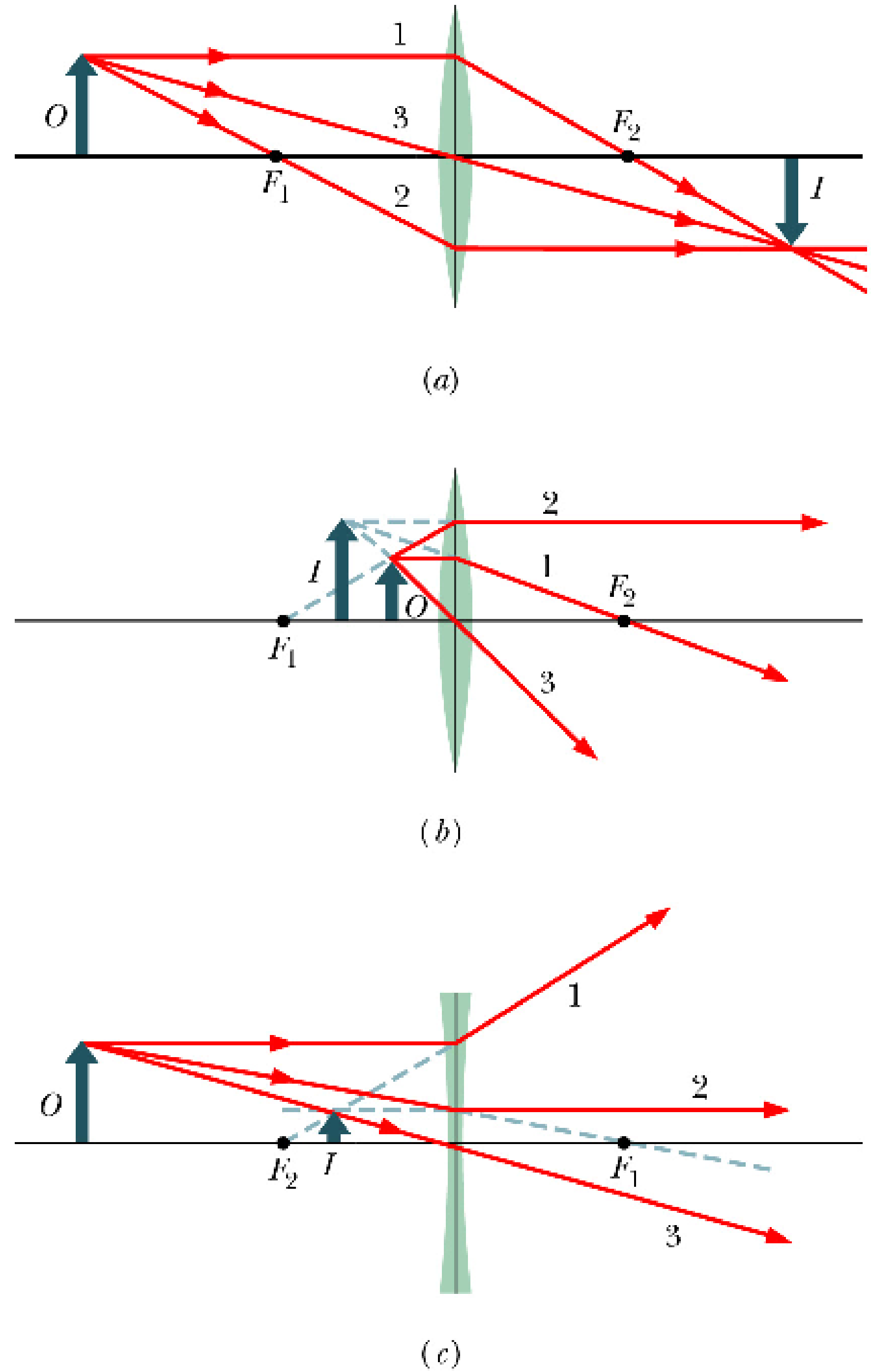



Let ***p*** = object distance,
let ***i*** = image distance.

Let ***p*,*i*** be positive for
real, negative for virtual.

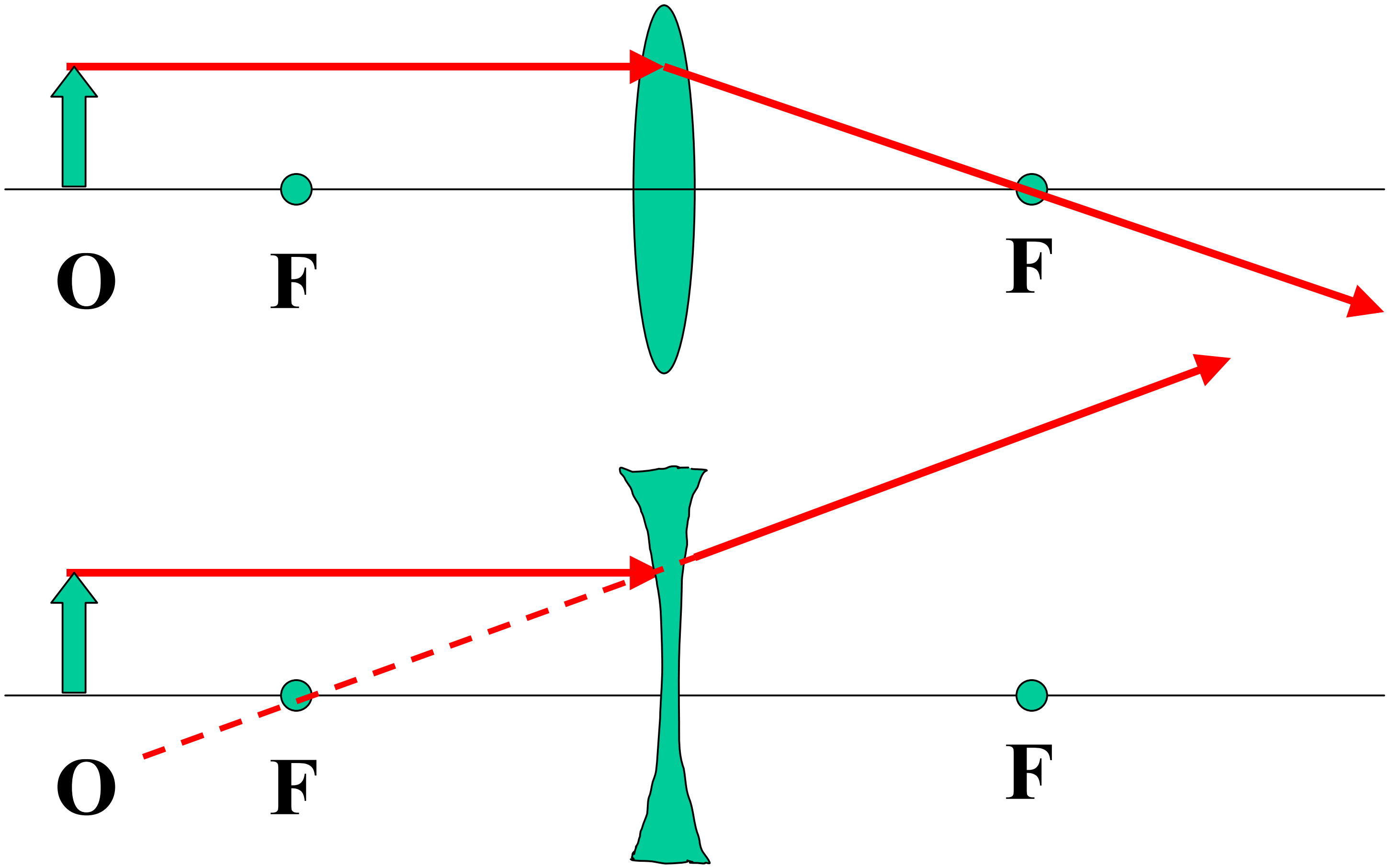
# Locating the Images

**Just as with the mirrors, follow the *principal rays* to locate image graphically.**



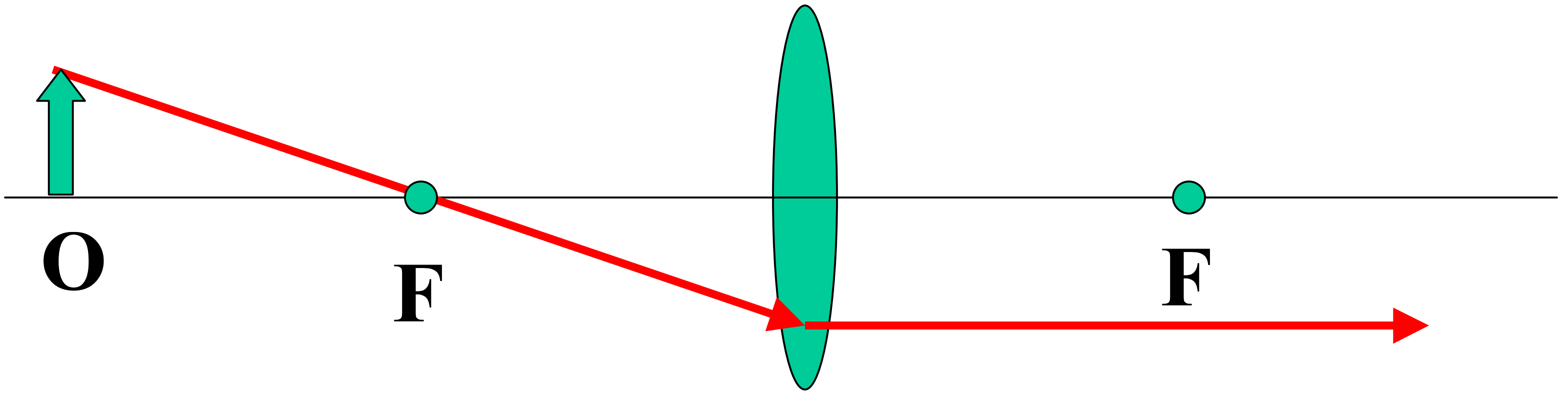
# Finding Images: Principal Rays

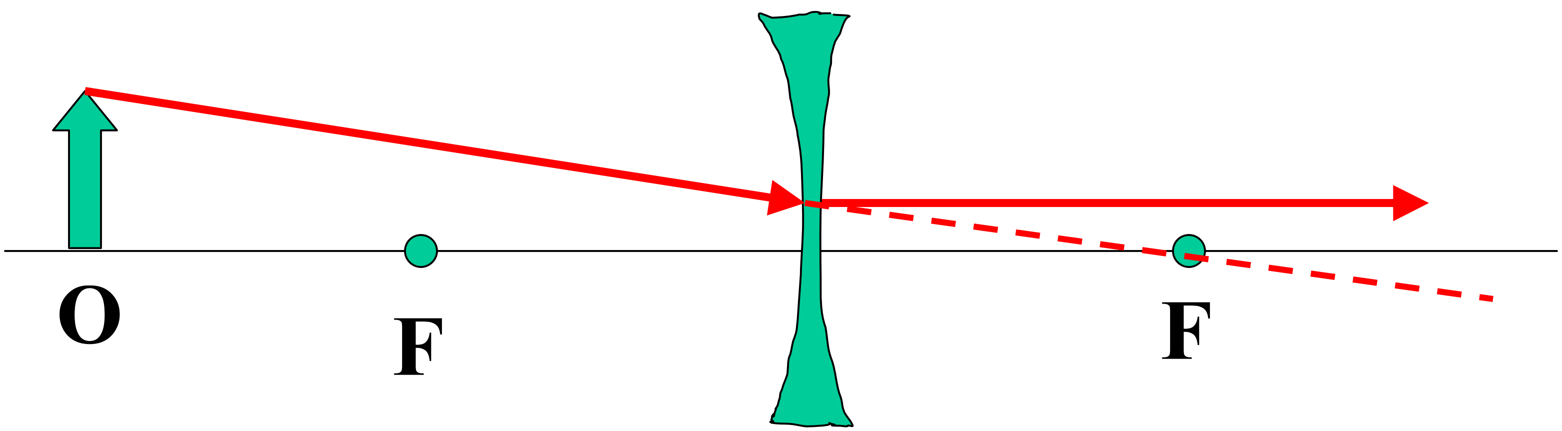
**1.** In parallel to axis: out through **F**.



# Finding Images: Principal Rays

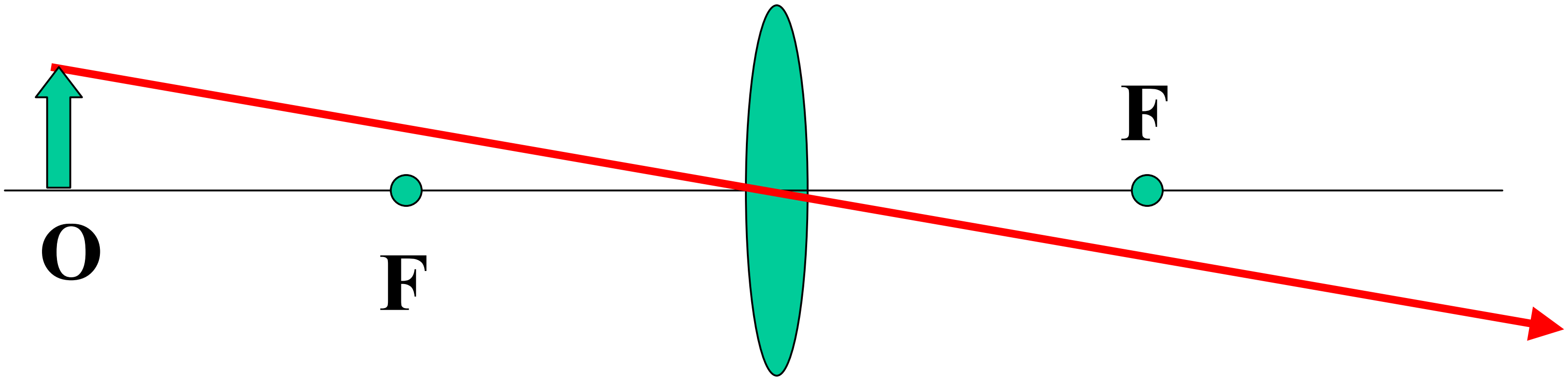
**2.** In through **F**, out parallel

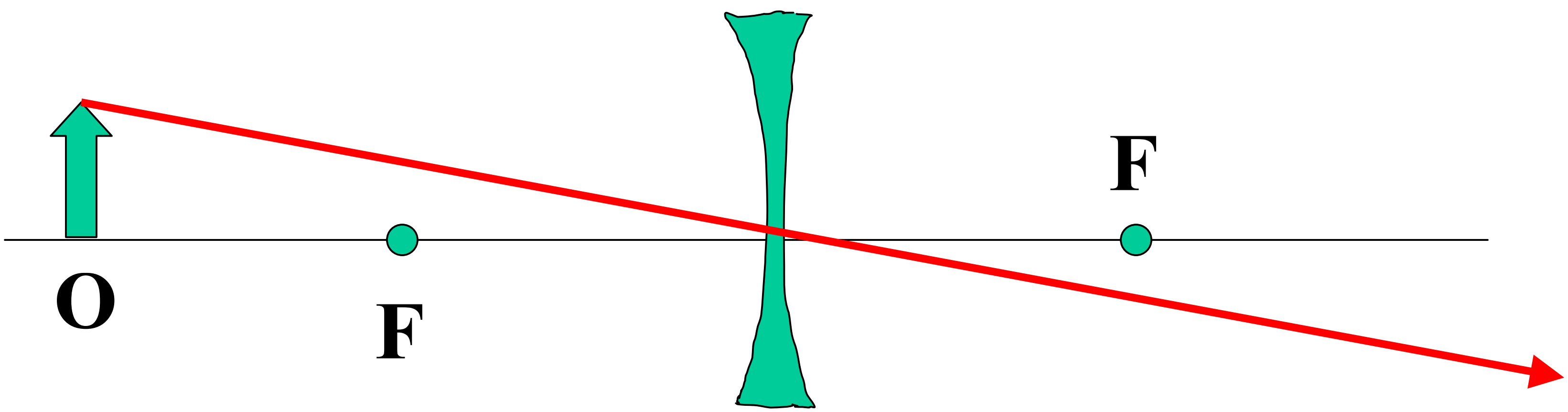


# Finding Images: Principal Rays

**3.** Straight through center



# Equations for the Images

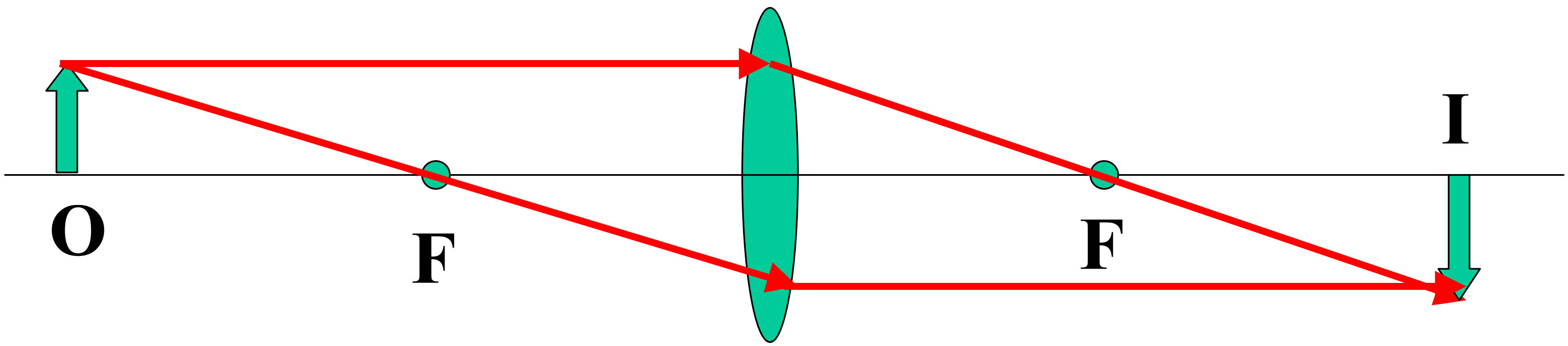
## Results in small-angle approximation:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \qquad m = -\frac{i}{p}$$

**Same equations as for spherical mirrors!**

# Thin-Lens Example 1

Given $f$=+20cm, $p$=+40cm, find $i$, $m$




$$\frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40} \quad so \quad i = +40cm$$

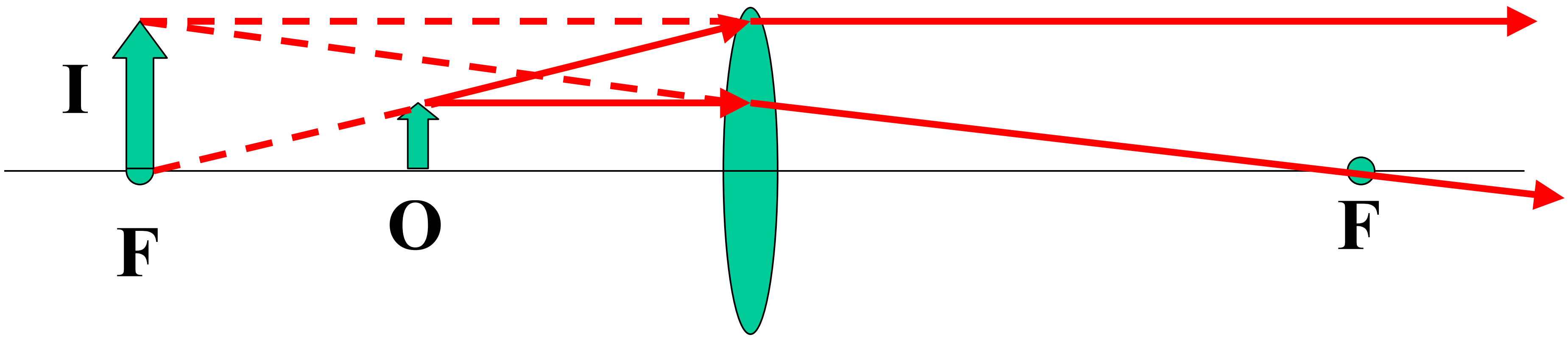
$$m = -\frac{i}{p} = -\frac{40}{40} = -1$$

**Image is:**

- **Real**
- **Inverted**
- **Same size as object**

# Example 2

Given $\boldsymbol{f}$=+20cm, $\boldsymbol{p}$=+10cm, find $\boldsymbol{i}$, $\boldsymbol{m}$




$$\frac{1}{i}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20}-\frac{1}{10}=-\frac{1}{20} \quad so \quad i=-20cm$$

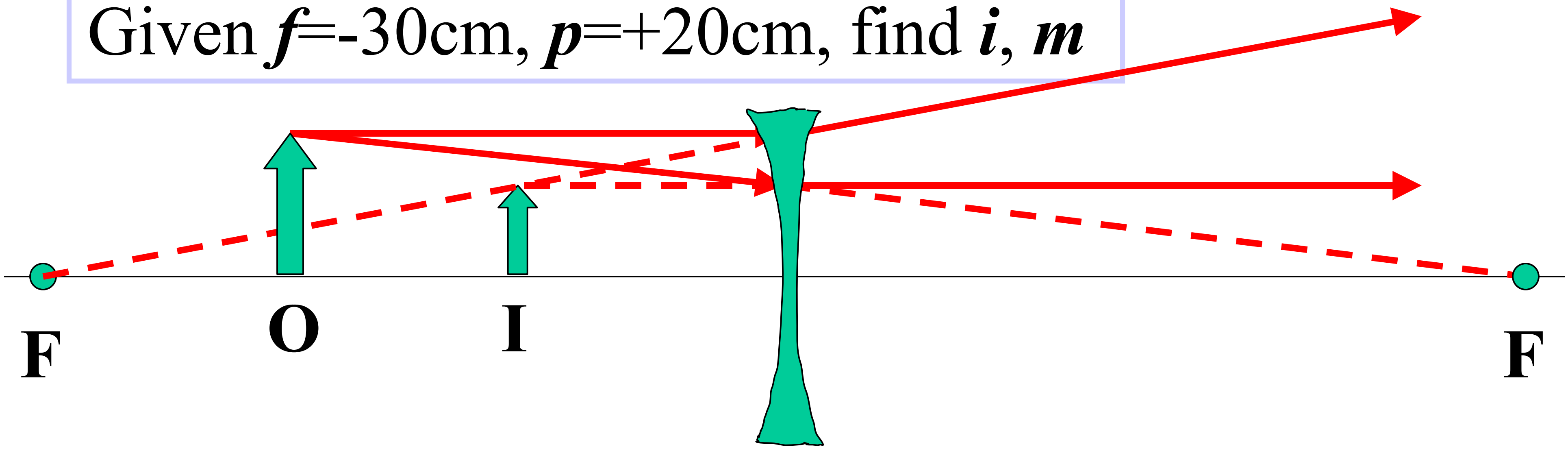
$$m=-\frac{i}{p}=-\frac{(-20)}{10}=+2$$

**Image is:**

- **Virtual**
- **Erect**
- **Enlarged**

# Example 3

Given ***f***=-30cm, ***p***=+20cm, find ***i***, ***m***




$$\frac{1}{i}=\frac{1}{f}-\frac{1}{p}=\frac{1}{(-30)}-\frac{1}{20}=-\frac{5}{60} \quad so \quad i=-12cm$$

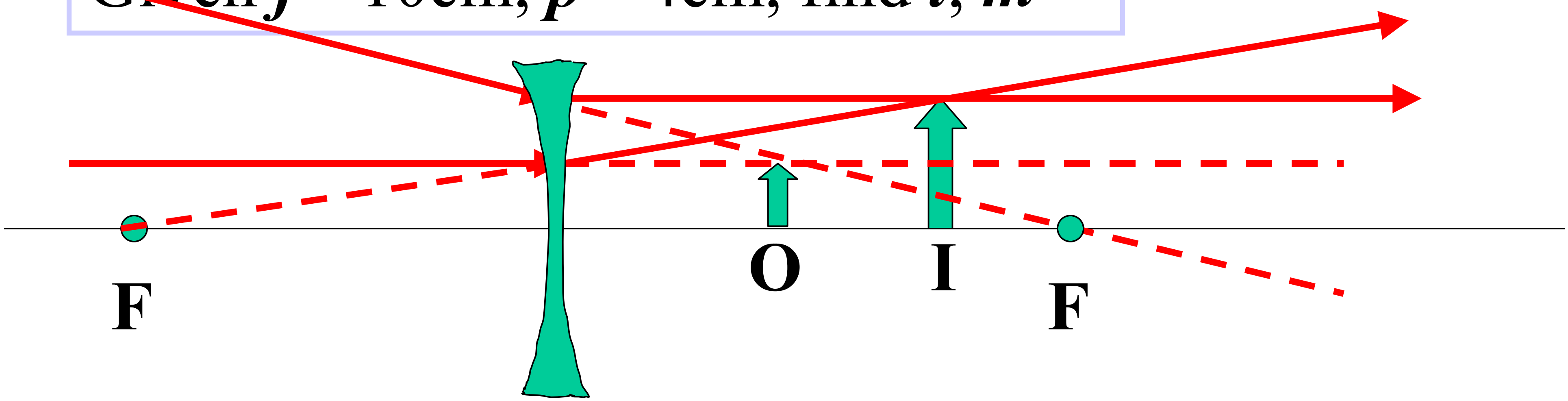
$$m=-\frac{i}{p}=-\frac{(-12)}{20}=+0.6$$

**Image is:**

- **Virtual**
- **Erect**
- **Reduced**

# Example 4

Given ***f***=-10cm, ***p***=-4cm, find ***i***, ***m***




$$\frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{(-10)} - \frac{1}{(-4)} = +\frac{3}{20} \qquad so \quad i = +6.7cm$$

$$m = -\frac{i}{p} = -\frac{20/3}{(-4)} = +\frac{20}{12} = +1.7$$

**Image is:**

- **Real**
- **Erect**
- **Enlarged**

## Recap: Formulas for spherical mirrors and thin lenses in the small angle approximation

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p}$$

## Definitions and sign conventions:

- $f$ = focal length: + = converging, – = diverging
- $p$ = object distance: + = real, – = virtual
- $i$ = image distance: + = real, – = virtual
- $m$ = magnification: + = erect, – = inverted