Current and Resistance

Chapters 26, 27

TODAY:

- Ohm's Law
- Ideal Meters
- Sources of Voltage and Power
- Resistors in series and parallel

HOMEWORK

- **Read Chapters 26 and 27**
- **Do Chapter 26 Questions 1, 3, 10**
- **Do Chapter 26 Problems 1, 17, 18, 35, 77**

Current is rate of flow of charge

- **If you watch closely at a fixed point as current is flowing along a wire, the current is the amount of charge that passes by per unit time.**
- **SI unit is the Ampere: 1A = 1C/s.**
- **Example: In a cathode-ray tube, n = 3×10^{15} electrons leave the electron gun per second. What is the current of this electron beam?**
- **Solution:**

$$ne = 3\times10^{15}\times1.6\times10^{-19} = 5\times10^{-4}C/s = 0.5\ mA$$

Current and Resistance

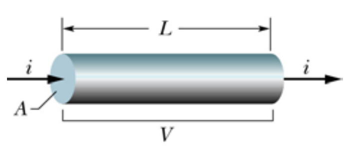
- **When a current flows through a *perfect* conductor (*e.g.* copper wire) there is no change in potential.**
- **When current flows through an *imperfect* conductor (*resistor*) there must be a field E to make it flow and thus a change in potential V = Ed for a distance d.**
- **If the current I is proportional to the field E which is causing it, then it will be proportional to the potential change V. This is Ohm's "Law" and the proportionality constant is called the *resistance* R.**

$$V = IR$$

Notes on Ohm's Law

- **This is not a fundamental law of nature like Gauss's Law. It's just a proportionality which is approximately true for some materials under some conditions.**
- **The entire electronics industry is based on materials which *violate* Ohm's Law.**
- **Ohm's Law should really be written as a *potential drop* $\Delta V = -IR$ because if you follow the current the potential decreases.**

Microscopic Form



$$E = V/L = IR/L = JRA/L = J\rho$$

So we define ***resistivity*** ρ: $R = \rho\ L/A$

and ***current density j:*** $J = I/A$

So the ***microscopic form*** of Ohm's Law is

$$E = J\rho$$

Resistance

- **So for a given object (resistor) we can measure its resistance R = V/I.**
- **The SI unit of resistance is the ohm (Ω).**
- **Clearly 1Ω = 1V/A (ohm = volt/amp).**
- **Thus resistivity ρ has units Ω-m.**
- **But remember Ohm's Law is only an approximation.** *For example, resistance normally changes with temperature.*

Example: Problem 26-15

- A wire is 2 m long, with a diameter of 1 mm.
- If its resistance is 50 mΩ, what is the resistivity of the material?

$$R = \rho l / A$$

$$\rho = RA / l = \frac{\left(50 \times 10^{-3}\ \Omega\right) \times \pi \times \left(0.5 \times 10^{-3}\ m\right)^2}{2}$$

$$= 2 \times 10^{-8}\ \Omega m$$

(Note this is consistent with table on page 689.)

Voltage and Power Sources

When a battery or any other voltage source delivers a current i at a potential difference V it is supplying *power* $P = iV$.

$$P = \frac{U}{t} = \frac{qV}{t} = iV$$

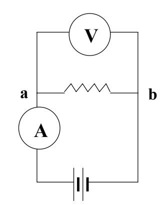
When a current i flows through a resistor with a voltage drop V, a friction-like process called *Joule heating* converts this power $P=iV$ into heat.

Note on P = I V

This is the easiest equation in the world to remember, IF you know what the three quantities mean:

$$voltage \times current = \frac{energy}{charge} \times \frac{charge}{time} = \frac{energy}{time} = power$$

Ideal Meters



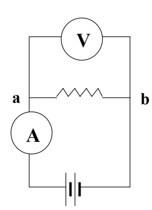
"Ideal ammeter"
Measures current and has zero potential drop (zero R)

"Ideal voltmeter"
Measures $V_{ab}=V_a-V_b$ but draws zero current (infinite R)

"Ideal wires"
V is constant along any wire (zero R)

Example

Meter A reads i = 2A flowing upward as shown, and meter V reads V = 6V.



(a) What is resistance R?

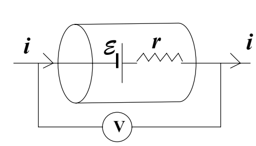
$$R = V / i = 6V / 2A = 3\Omega$$

(b) Which point is at the higher potential, ***A*** or ***B***?

Point ***A*** because potential ***drops*** by 6 volts.

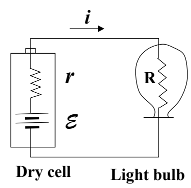
EMF

- Electromotive force (emf) is not a force: it's the potential difference provided by a power supply.
- For a battery providing a current i, the terminal voltage V is less than the emf $\mathcal{E}$ because there is an internal resistance r:



$$V = \mathcal{E} - ir$$

Power in a Simple Circuit



$$\Delta V_{loop} = -ir + \mathcal{E} - iR = 0$$

$$\therefore \quad \mathcal{E} = i(r+R)$$

$$P_{chem} = i\mathcal{E} = i^2 r + i^2 R$$

$$= P_{heating\ battery} + P_{heating\ filament}$$

Example

For a battery with internal resistance r = 25 Ω, what load resistor R will get maximum power?

$$P_{load} = i^2 R = \left(\frac{\mathcal{E}}{r+R}\right)^2 R = \mathcal{E}^2 \left(\frac{R}{(r+R)^2}\right)$$

$$\frac{d}{dR}\left(\frac{R}{(r+R)^2}\right) = \frac{(r+R)^2 - 2R(r+R)}{(r+R)^4} = 0$$

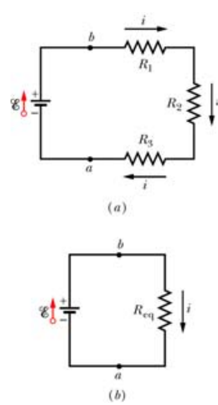
$$(r+R)^2 = 2R(r+R)$$

$$r + R = 2R$$

$$R = r = 25\Omega$$

Resistors in Series

- Voltage drops add
- Currents are equal.



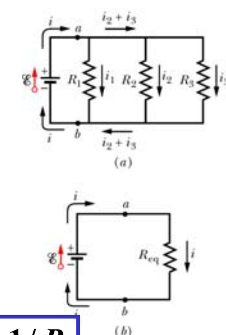
$$V_b - V_a = \mathcal{E} = iR_1 + iR_2 + iR_3$$

But we want $\mathcal{E} = iR_{eq}$

$$\therefore \quad R_{eq} = R_1 + R_2 + R_3$$

Resistors in Parallel

- Currents add
- Voltage drops are equal.



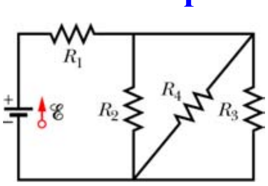
$$i = i_1 + i_2 + i_3$$

$$\frac{\mathcal{E}}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

But $\mathcal{E} = V_1 = V_2 = V_3$

$$\therefore \quad 1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$$

Example: Problem 27-30



$$\mathcal{E} = 6.0V \quad R_1 = 100\Omega$$

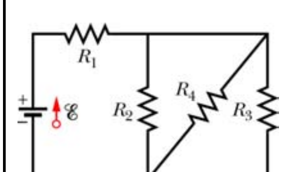
$$R_2 = R_3 = 50\Omega$$

$$R_4 = 75\Omega$$

(a) Find the equivalent resistance of the network.

(b) Find the current in each resistor.

Problem 27-30 (part a)



$$\mathcal{E} = 6.0V \quad R_1 = 100\Omega$$

$$R_2 = R_3 = 50\Omega$$

$$R_4 = 75\Omega$$

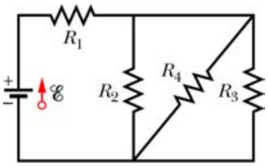
(a) Find the equivalent resistance of the network.

$$1/R_{234} = 1/R_2 + 1/R_3 + 1/R_4$$

$$= 1/50 + 1/50 + 1/75 = 16/300$$

So $R_{234} = 300/16 = 19\Omega$

Problem 27-30 (part a cont'd)



$\mathcal{E} = 6.0\,V \quad R_1 = 100\,\Omega$

$R_2 = R_3 = 50\,\Omega$

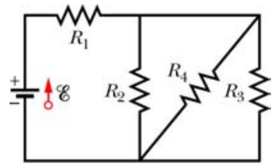
$R_4 = 75\,\Omega$

(a) Find the equivalent resistance of the network.

Now R_1 and R_{234} are in series so

$$R_{eq} = R_1 + R_{234} = 100\,\Omega + 19\,\Omega = \boxed{119\,\Omega}$$

Problem 27-30 (part b)



$\mathcal{E} = 6.0\,V \quad R_1 = 100\,\Omega$

$R_2 = R_3 = 50\,\Omega$

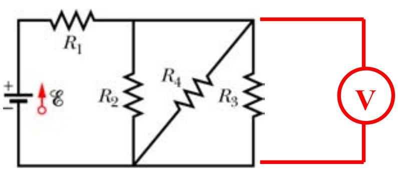
$R_4 = 75\,\Omega$

(b) Find the current in each resistor.

First get the total current from the battery, which is also the current through R_1:

$$i_1 = \mathcal{E} / R_{eq} = 6.0 / 119 = .050\,A = 50\,mA$$

Problem 27-30 (part b cont'd)



$V = iR_{234}$

$= .050\,A \times 19\,\Omega$

$= 0.95\,V$

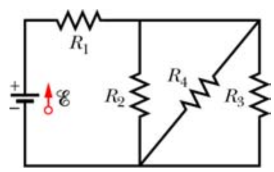
(b) Find the current in each resistor.

$$i_2 = V / R_2 = .95 / 50 = 19\,mA$$

$$i_3 = V / R_3 = .95 / 50 = 19\,mA$$

$$i_4 = V / R_4 = .95 / 75 = 12\,mA$$

Problem 27-30 (check)



$i = i_1 = 50\,mA$

$i_2 = i_3 = 19\,mA$

$i_4 = 12\,mA$

Check by adding the currents in the three branches:

$$i_2 + i_3 + i_4 = 19 + 19 + 12 = 50\,mA$$

$$= i_1 \quad \checkmark$$

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