Chapter 32

- Induced Electric Fields
- Induced Magnetic Fields
- Maxwell’s Equations
- Magnetic Materials

Induced Magnetic Fields

Faraday's Law gives ***induced electric fields***: namely, "Changing ***B*** creates ***E***."

$$\boxed{\frac{d}{dt}\Phi_M}$$

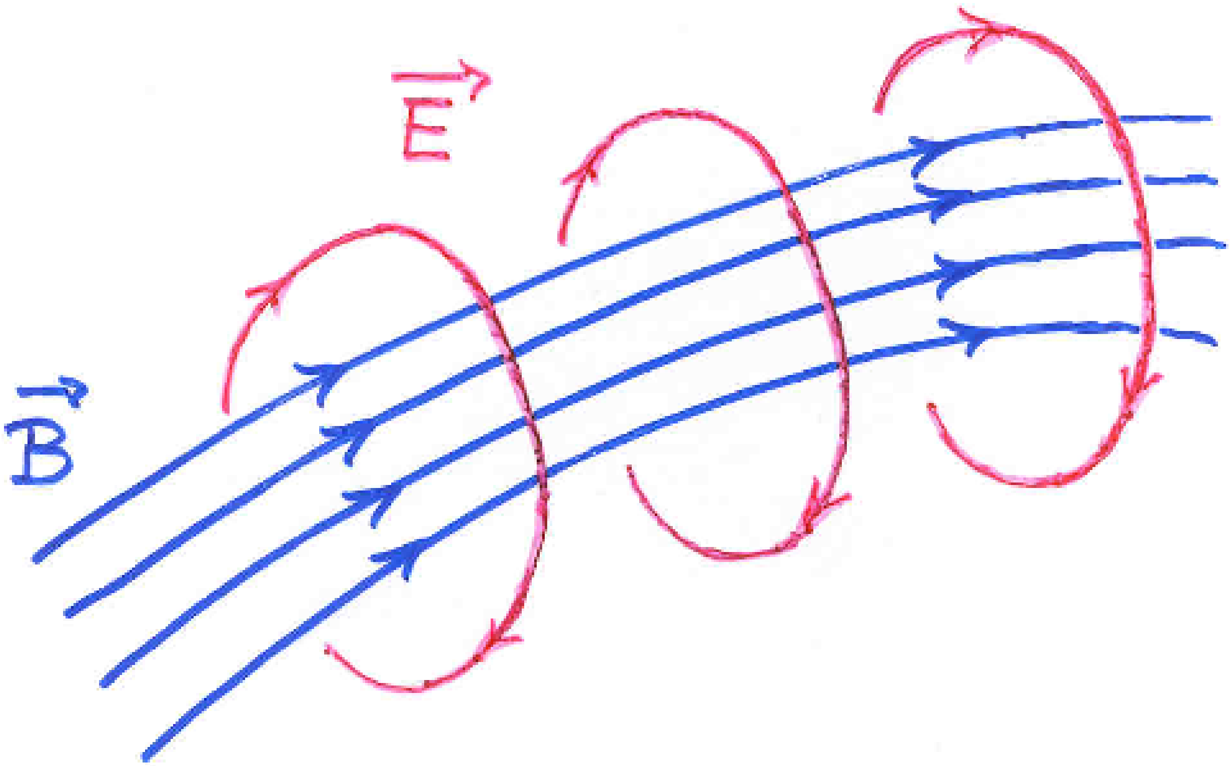
Maxwell discovered ***induced magnetic fields:*** namely, "Changing ***E*** creates ***B***."

$$\boxed{\frac{d}{dt}\Phi_E}$$

This is described by a new term in Ampere's Law (called "displacement current") of form

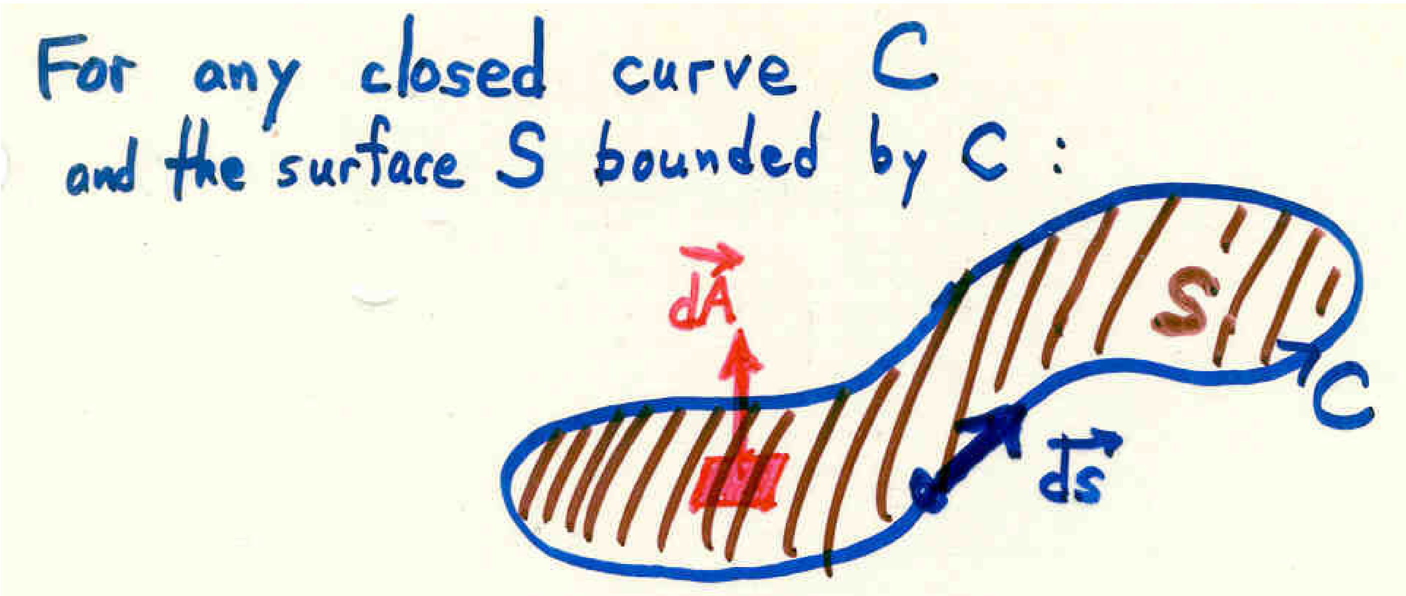
$$i_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Review: Induced E Fields



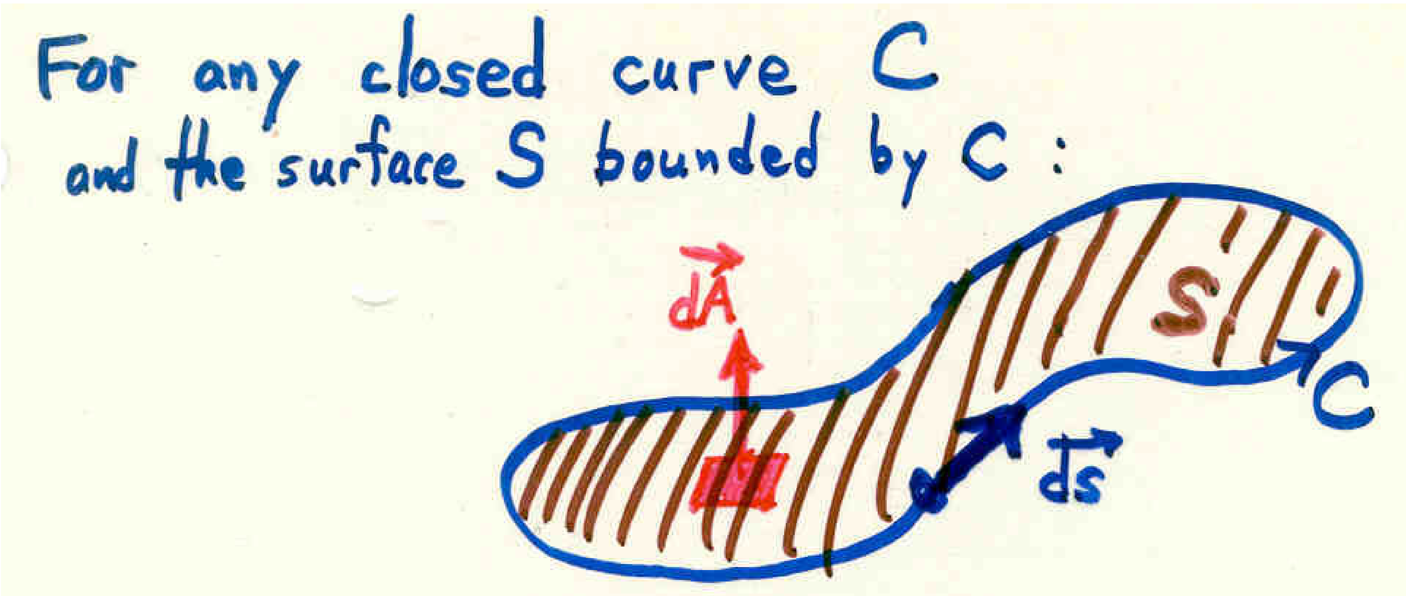
Lines of the *induced E* field make circles around the lines of the *changing B* field, just as lines of *B* circle around a wire.

Faraday:
Induced Electric Field



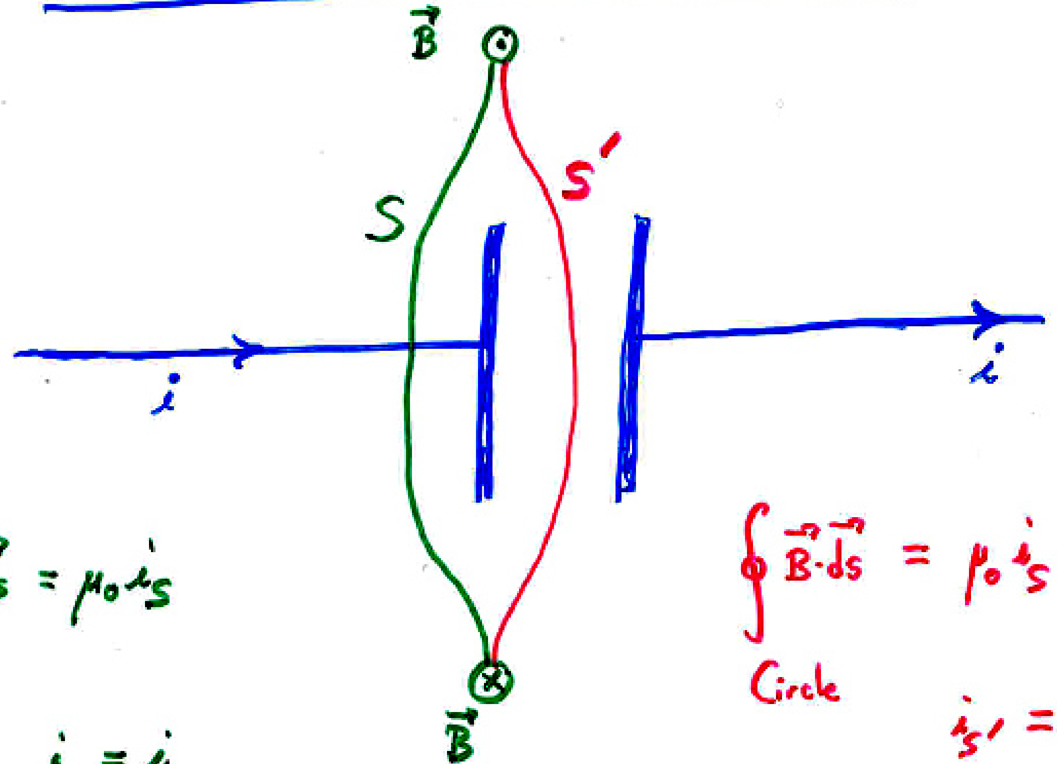
$$\boxed{\oint_C \vec{E} \cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A}}$$

Maxwell:
Induced Magnetic Field



$$\boxed{\oint_C \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}}$$

Problem with Ampere's Law



$$\oint_{\text{Circle}} \vec{B}\cdot d\vec{s} = \mu_0 i_S$$

$i_S = i$

$$\oint_{\text{Circle}} \vec{B}\cdot d\vec{s} = \mu_0 i_{S'}$$

$i_{S'} = 0$

$2\pi r B = \mu_0 i$

✓

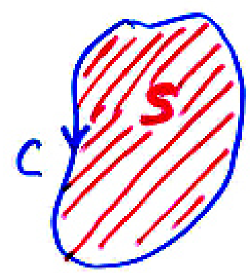
$B = 0$?

"Current linking the loop" is not well-defined when there's a capacitor.

SOLUTION TO PROBLEM

(Maxwell, 1873)

$$\oint_C \vec{B}\cdot\vec{ds} = \mu_0 i_S + \mu_0 \varepsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot\vec{dA}$$



SAME IDEA AS FARADAY:

$$\oint_C \vec{E}\cdot\vec{ds} = -\frac{d}{dt}\int_S \vec{B}\cdot\vec{dA}$$

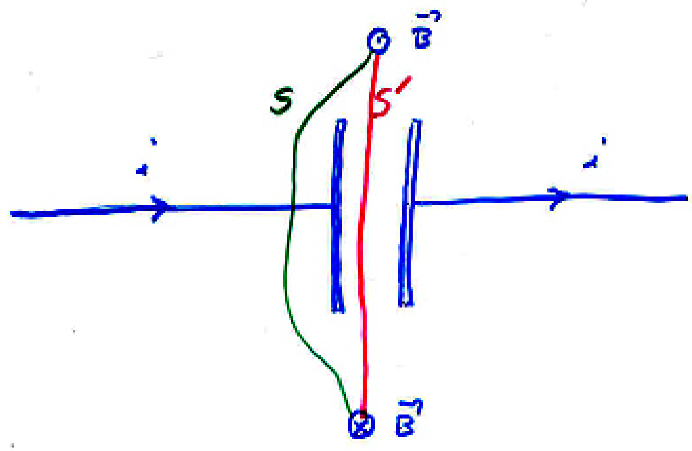
New term called "displacement current"

Define $i_d = \varepsilon_0 \frac{d}{dt}\Phi_E$

Then Ampere becomes $\oint \vec{B}\cdot\vec{ds} = \mu_0 (i + i_d)$

Electric charge on a capacitor periodically varies with frequency ω and amplitude Q_0. The displacement current amplitude trough the capacitor is

1. $Q_0\omega$
2. $2Q_0\omega$
3. Q_0/ω



Choose S:

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i_S + \mu_0 \epsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$2\pi r B \qquad \mu_0 i \qquad 0$$

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

Choose S':

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i_{S'} + \mu_0 \epsilon_0 \frac{d}{dt}\int_{S'} \vec{E}\cdot d\vec{A}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$2\pi r B \qquad 0 \qquad \mu_0 \epsilon_0 \frac{d}{dt}(EA)$$

Inside Capacitor $E = \sigma/\epsilon_0 = \dfrac{Q}{\epsilon_0 A}$

So: $2\pi r B = \mu_0 \epsilon_0 \dfrac{d}{dt}\left(\dfrac{Q}{\epsilon_0}\right) = \mu_0 i$: $\boxed{B = \dfrac{\mu_0 i}{2\pi r}}$ ✓

Maxwell's Equations

Maxwell modified Ampere's Law to account for this new effect of the induced magnetic field, and thereby got the set of four equations which provide the complete theory of the electromagnetic field.

Maxwell's Equations (1873)

$$\oint \vec{E}\cdot d\vec{A} = Q_{in}/\varepsilon_0$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$

Gauss

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\Phi_M$$

Faraday

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{in} + \mu_0 \varepsilon_0 \frac{d}{dt}\Phi_E$$

Ampere - Maxwell

Summary

Gauss for E: **Lines of *E* begin and end on charges.**

Gauss for B: **Lines of *B* never begin or end.**

Faraday: **Changing *B* creates circular *E*.**

Ampere: **Current or changing *E* creates circular *B*.**

We now have the complete theory of the electromagnetic field.

Classical Electrodynamics

This is the most successful theory in all of science. In over 100 years of constant testing, no disagreements with experiment have ever been found. It is the basis for Einstein's theory of relativity and is an essential ingredient in atomic physics and quantum field theory.

Maxwell's Waves: Preview of Chapter 33

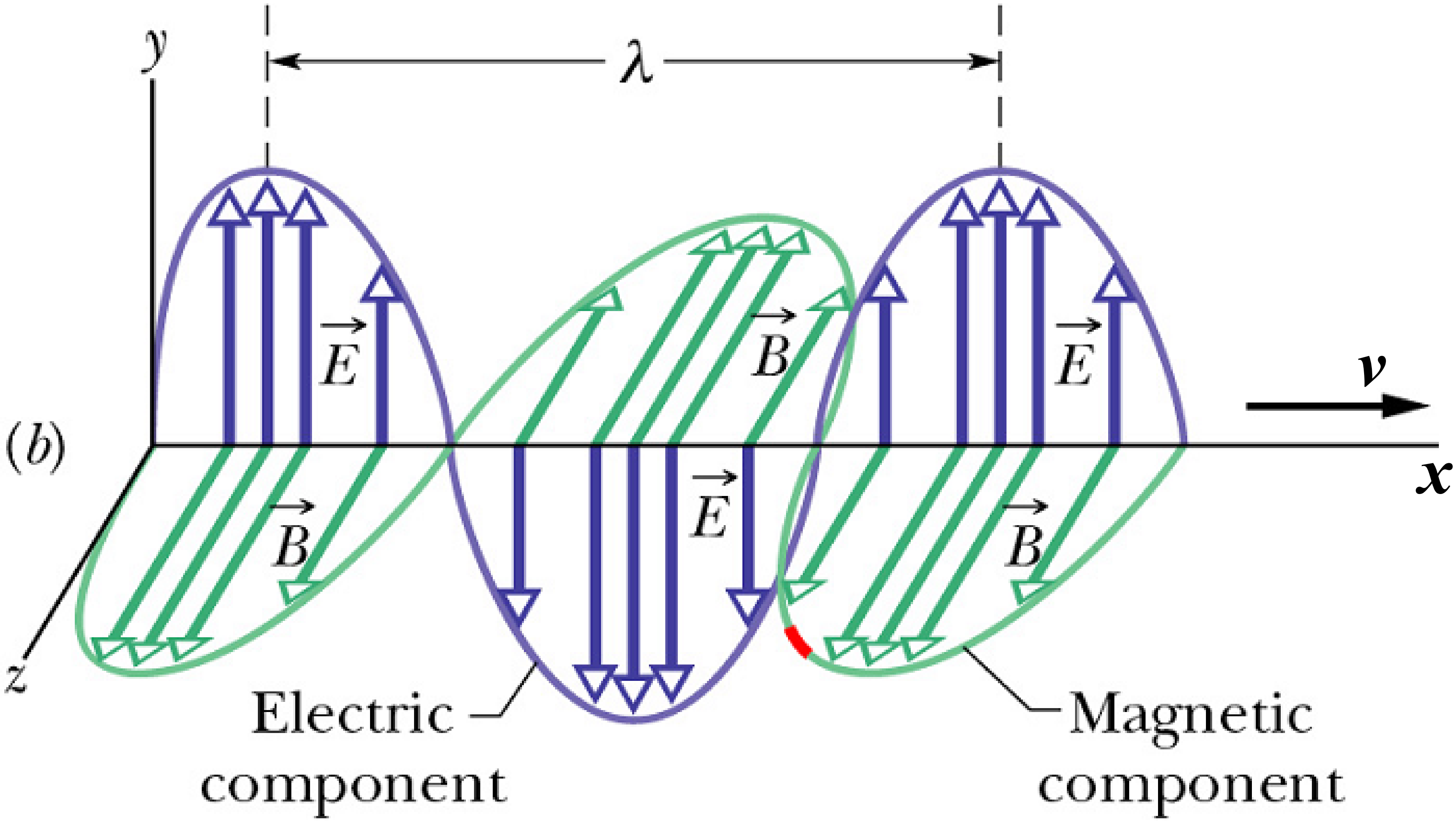
PREDICTION : Electromagnetic <u>waves</u> propagate in a vacuum with speed $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0 \times 10^8$ m/s

Electromagnetic Waves

- Maxwell's Equations have wave solutions.
- Wave must have both $\boldsymbol{E}$ and $\boldsymbol{B}$ fields.
- $\boldsymbol{E}$ and $\boldsymbol{B}$ have same wavelength and velocity.
- $\boldsymbol{E}$, $\boldsymbol{B}$, $\boldsymbol{v}$ are three perpendicular vectors
- Their magnitudes are related by $E/B = v$.
- The speed must be

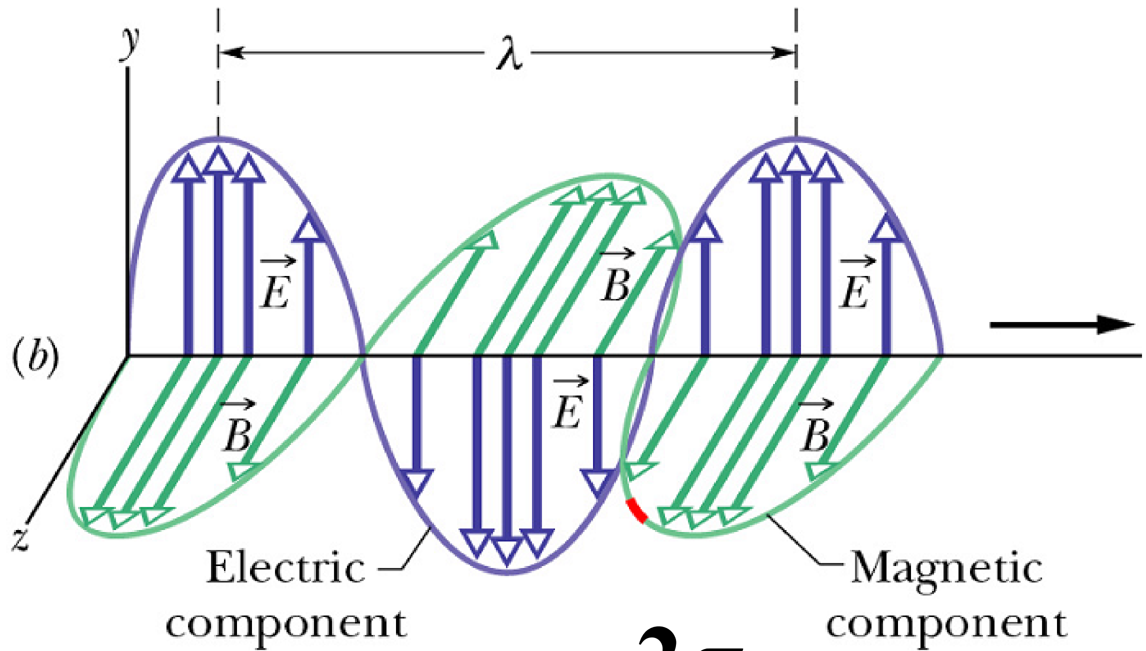
$$\boldsymbol{v} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0 \times 10^8 \; m/s$$

Electromagnetic Waves



Wave Equations

Recall equations for waves from Ch. 16:



$$f(x,t) = f_m \sin(kx - \omega t) \qquad k = \frac{2\pi}{\lambda}$$

with the wave velocity $\underline{v = \omega / k}$

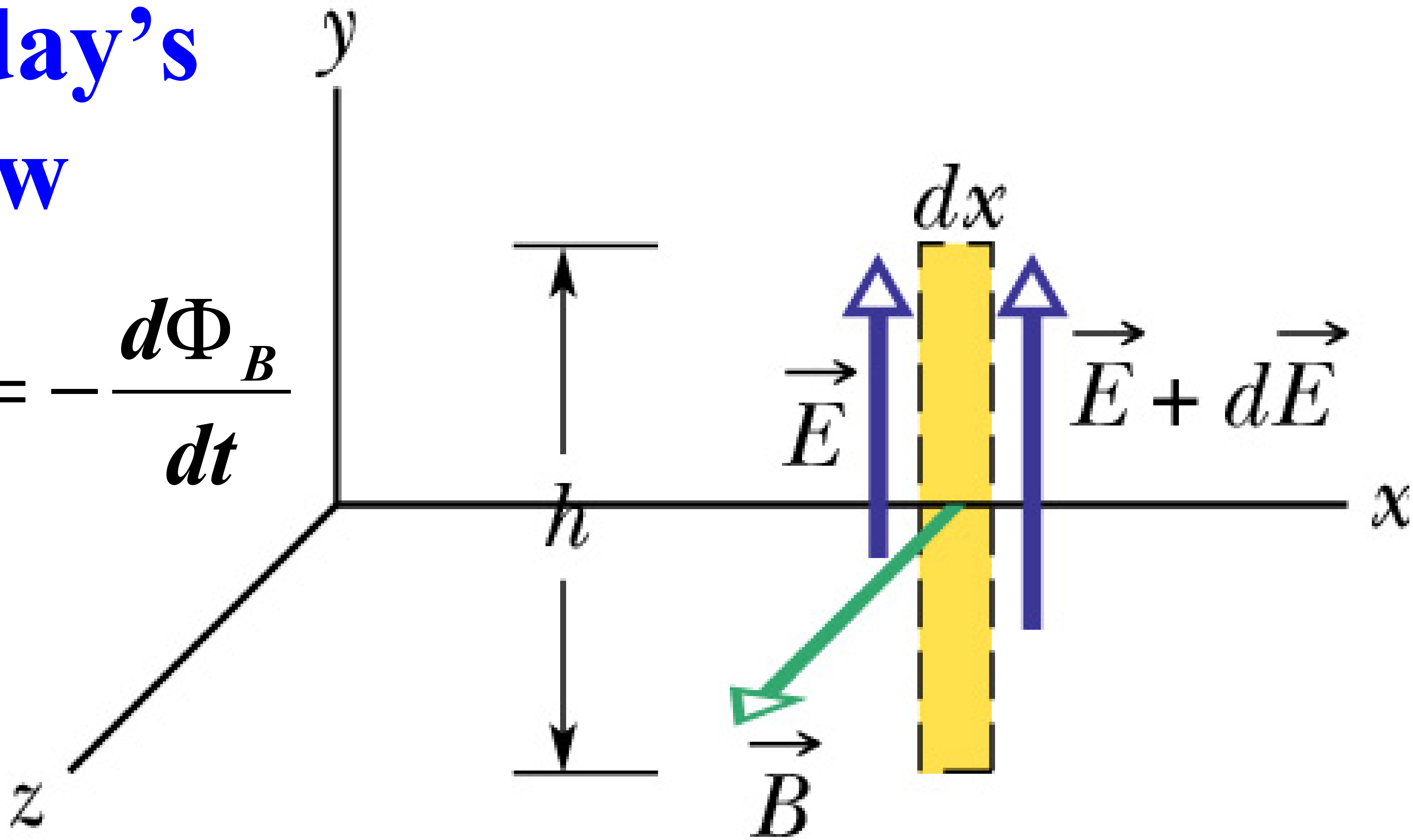
Could ***E*** and ***B*** both have this same form?

$$E = E_m \sin(kx - \omega t) \quad \textit{and} \quad B = B_m \sin(kx - \omega t)$$

YES! These are solutions if: $\boxed{v = 1/\sqrt{\varepsilon_0 \mu_0}}$

Faraday's Law

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$



$$h\, dE = -h\, dx \frac{dB}{dt}$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

Apply Faraday to Wave

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

$$E = E_m \sin(kx - \omega t) \qquad B = B_m \sin(kx - \omega t)$$

$$\frac{dE}{dx} = E_m k \cos(kx - \omega t) \qquad \frac{dB}{dt} = -\omega B_m \cos(kx - \omega t)$$

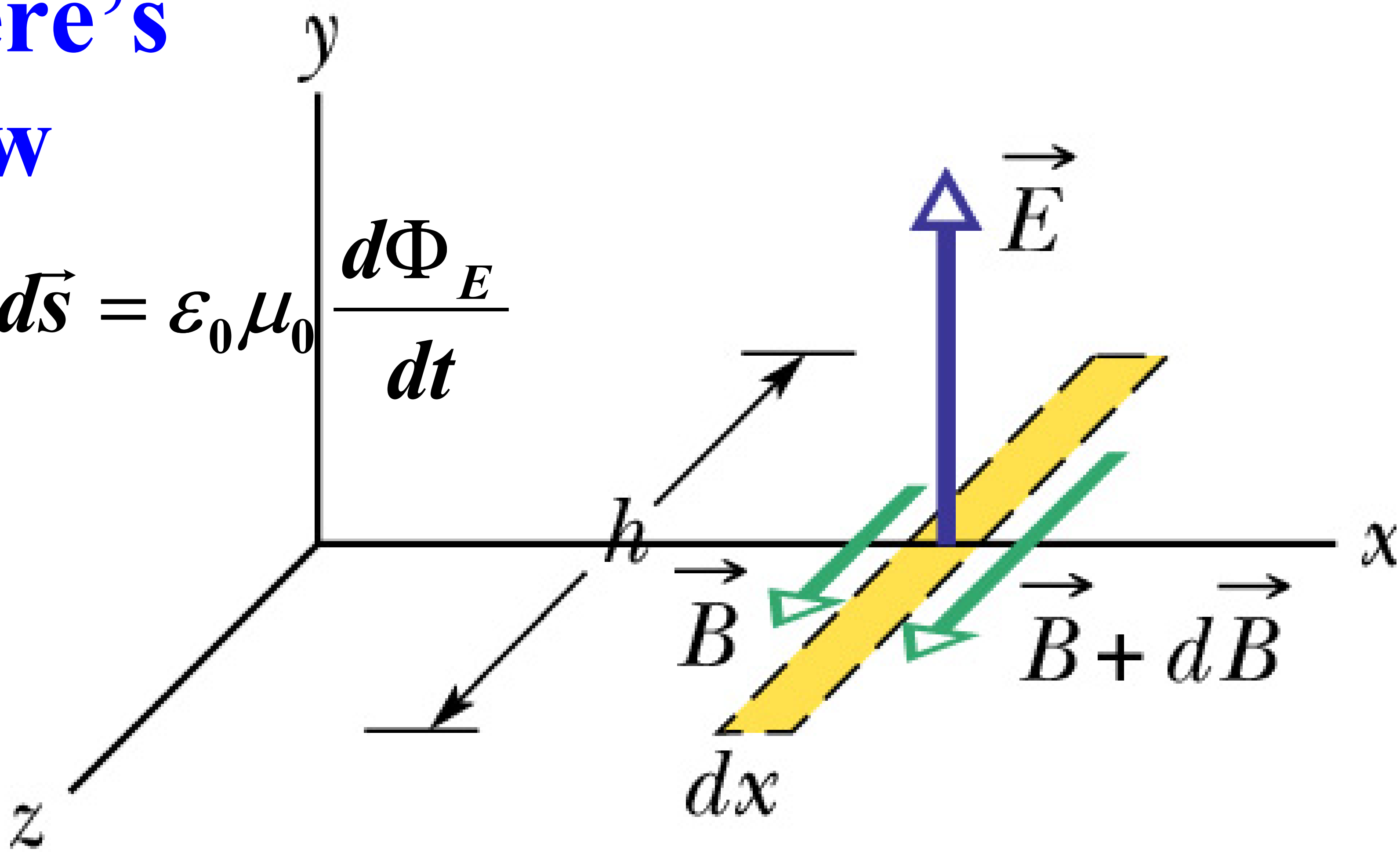
So Faraday gives

$$E_m k = -\left(-\omega B_m\right) \qquad \frac{E_m}{B_m} = \frac{\omega}{k} = v$$

Or finally $\boxed{E / B = v}$

Ampere's Law

$$\oint \vec{B}\cdot d\vec{s} = \varepsilon_0\mu_0 \frac{d\Phi_E}{dt}$$



$$h\,dB = \varepsilon_0\mu_0\left(-h\,dx\,\frac{dE}{dt}\right)$$

$$\frac{dB}{dx} = -\varepsilon_0\mu_0\frac{dE}{dt}$$

Apply Ampere to Wave $\frac{dB}{dx} = -\varepsilon_0 \mu_0 \frac{dE}{dt}$

$$E = E_m \sin(kx - \omega t) \qquad B = B_m \sin(kx - \omega t)$$

$$\frac{dB}{dx} = B_m k \cos(kx - \omega t) \qquad \frac{dE}{dt} = -\omega E_m \cos(kx - \omega t)$$

$$B_m k = -\varepsilon_0 \mu_0 \left(-\omega E_m\right) = \varepsilon_0 \mu_0 \omega E_m$$

$$\frac{B_m}{E_m} = \frac{\omega}{k} \varepsilon_0 \mu_0 \qquad \frac{1}{v} = v\, \varepsilon_0 \mu_0 \qquad \boxed{v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$$

In electromagnetic wave

1. Vectors B and E perpendicular to each other and equal
2. Parallel to each other and equal
3. Perpendicular to each other with different magnitudes in SI
4. Perpendicular to each other with different magnitudes in any system

Introduction to Magnetic Materials

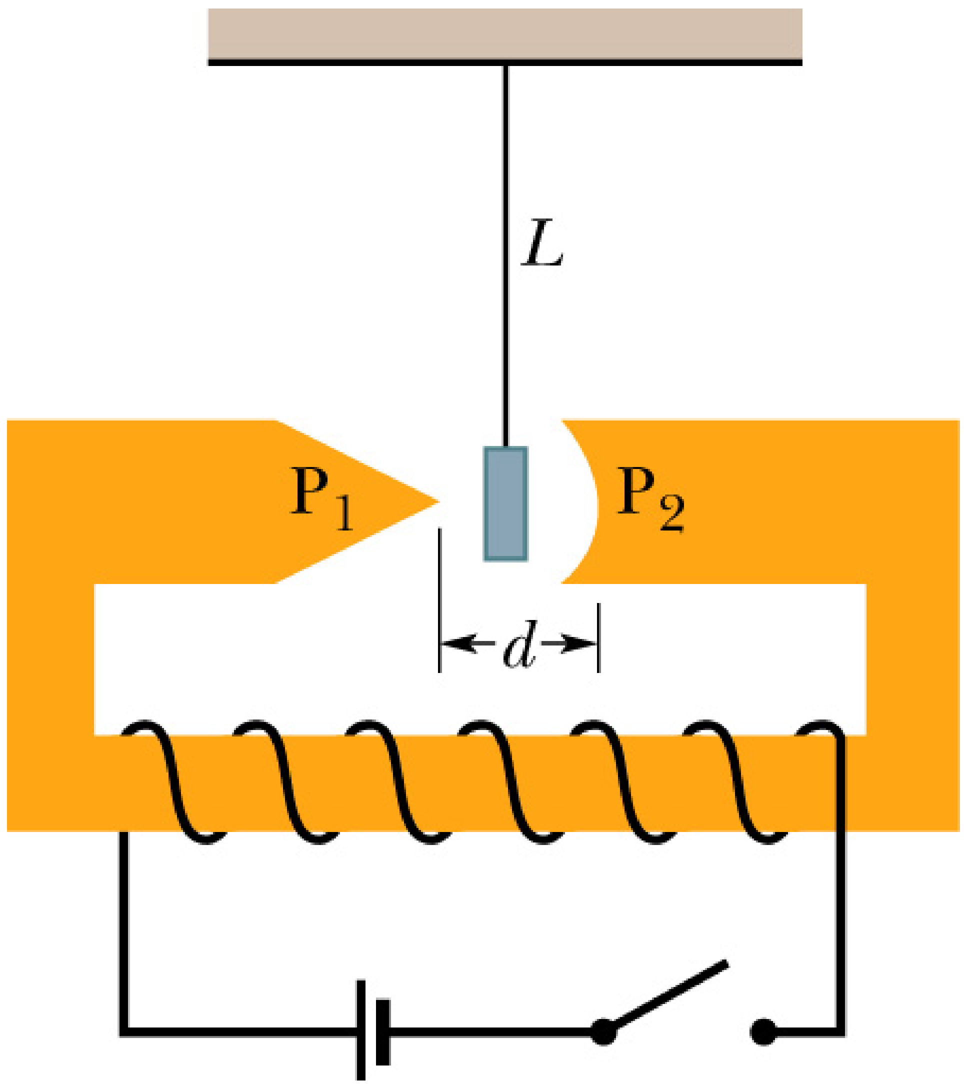
- **Three main classes:**
  - **Ferromagnetic**
  - **Paramagnetic**
  - **Diamagnetic**

$$\vec{M} = \vec{\mu}/(Volume)$$

- **Define *magnetization*, the *density of dipole moments.***

Magnetic effects occur when dipoles *line up* with an applied field, or with each other, to produce large magnetization.

Materials in an External Field



Place sample in external magnetic field. Measure its magnetization.

Magnetic Materials

Chapter 32:

- Induced Electric Fields
- Maxwell's Equations
- Magnetic Materials

Magnetic Materials

- **Three main classes of magnetic materials:**
  - **Ferromagnetic (includes permanent magnets)**
  - **Paramagnetic**
  - **Diamagnetic**
- **Key quantity is the *magnetization*, defined as the *density of dipole moments*.**

$$\vec{M} = \vec{\mu}\,/(Volume)$$

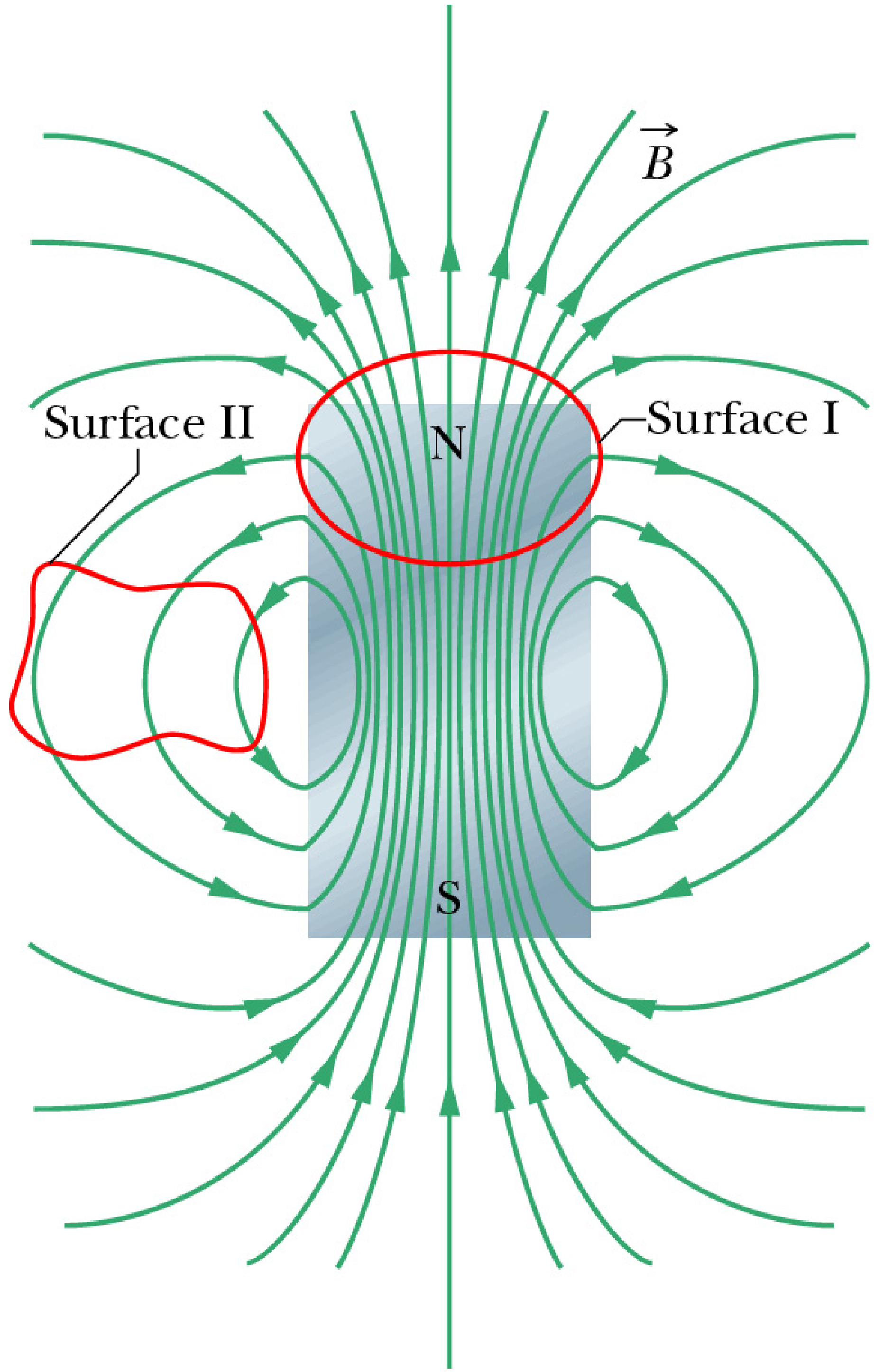
Gauss's Law for Magnetism

- Given any ***closed*** surface
- Outward ***electric*** flux = enclosed charge
- Outward ***magnetic*** flux = ***zero.***
- **"There are no magnetic monopoles."**
- **This is Maxwell Equation #2:**

$$\oint \vec{B} \cdot d\vec{A} = 0$$

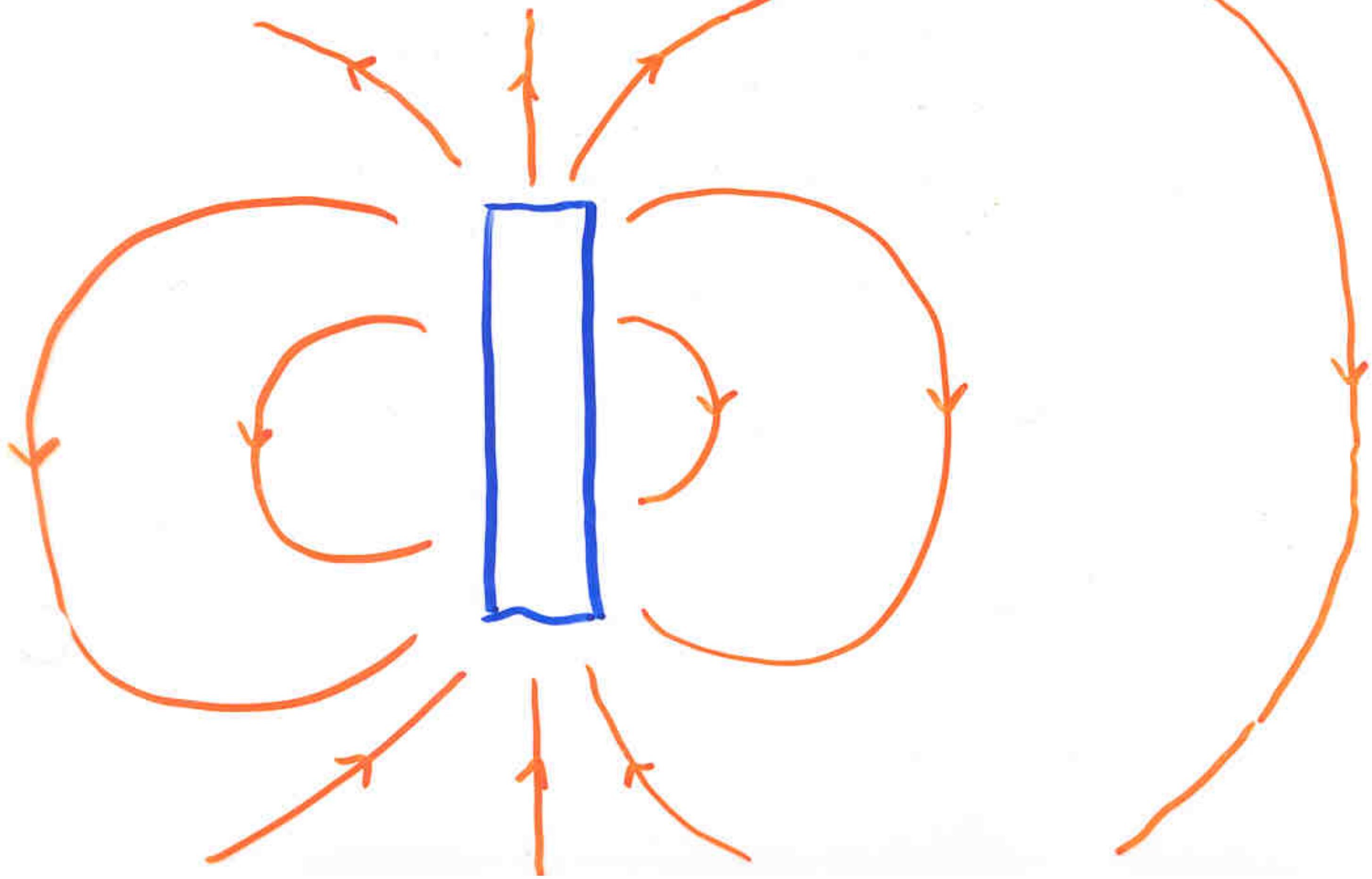
Bar magnet

Possible closed Gaussian surfaces shown in red. Zero net outward flux in both cases.



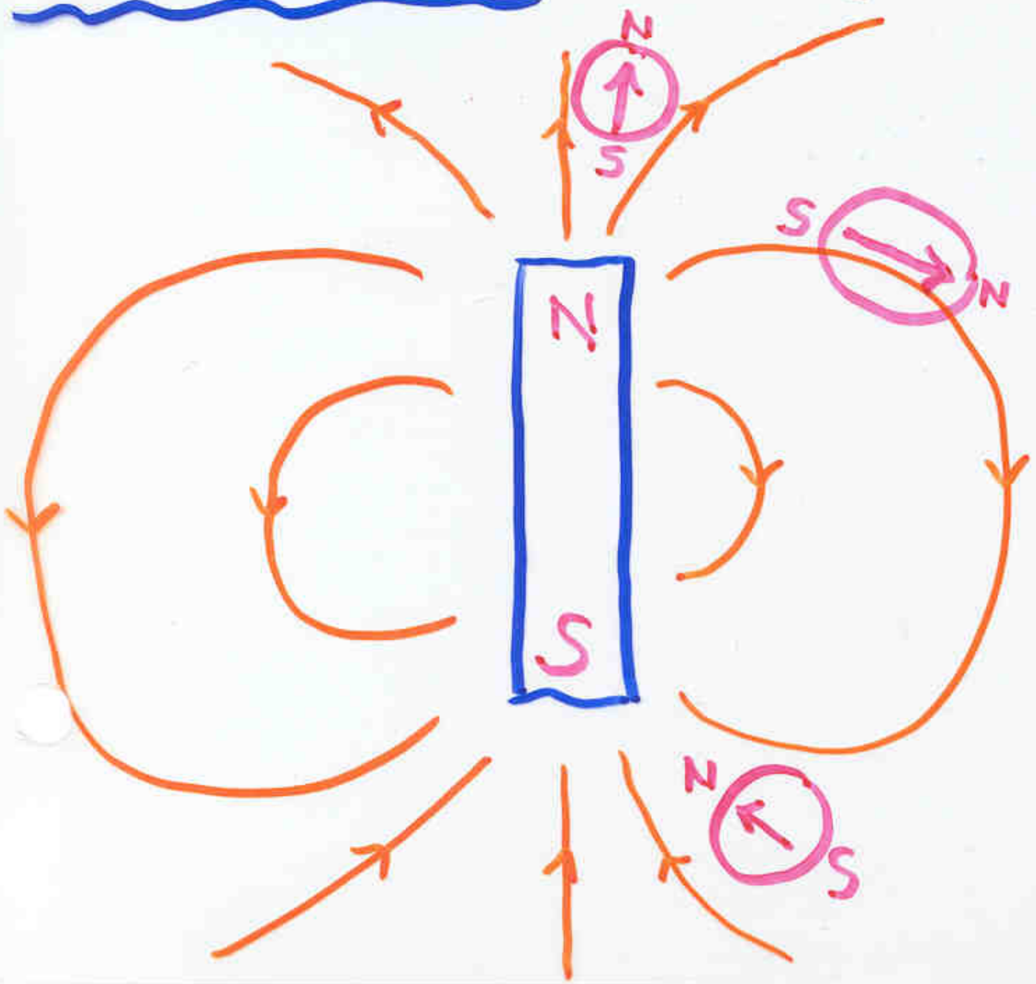
Bar magnets

[DIPOLE FIELDS]



Bar magnets

[DIPOLE FIELDS]



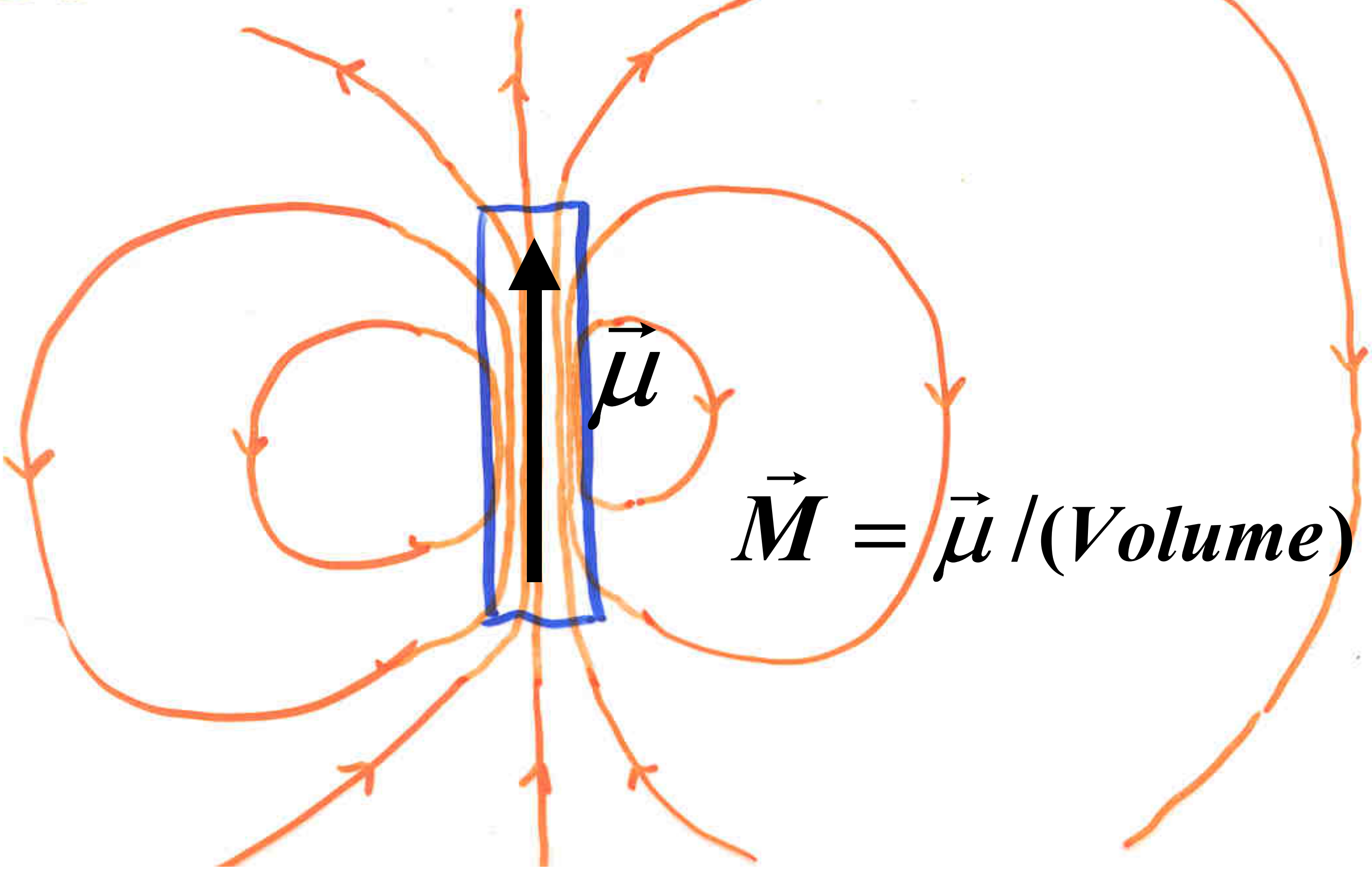
N + S Poles
Like poles repel
Unlike poles attract

HOWEVER:

Monopoles do not exist! Only DIPOLES!

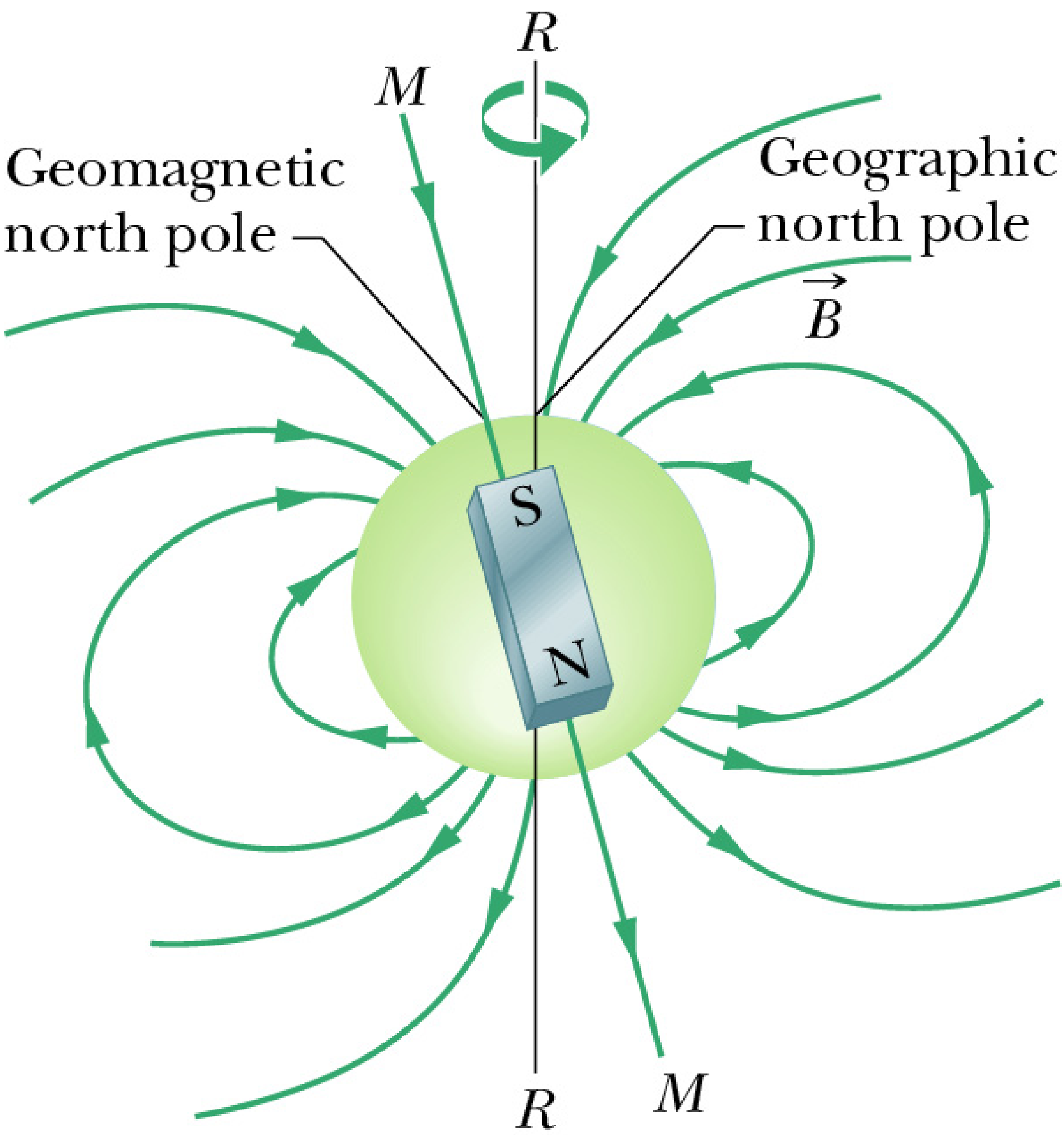
Bar magnets

[DIPOLE FIELDS]



$$\vec{M} = \vec{\mu}\,/(Volume)$$

Magnetic Field of Earth



Approximately a dipole field.

Q.32-1

What can you say about the vertical component of the earth's magnetic field in North America?

(1) It is upward (2) It is downward (3) It is zero

Q.32-1

What can you say about the vertical component of the earth's magnetic field in North America?

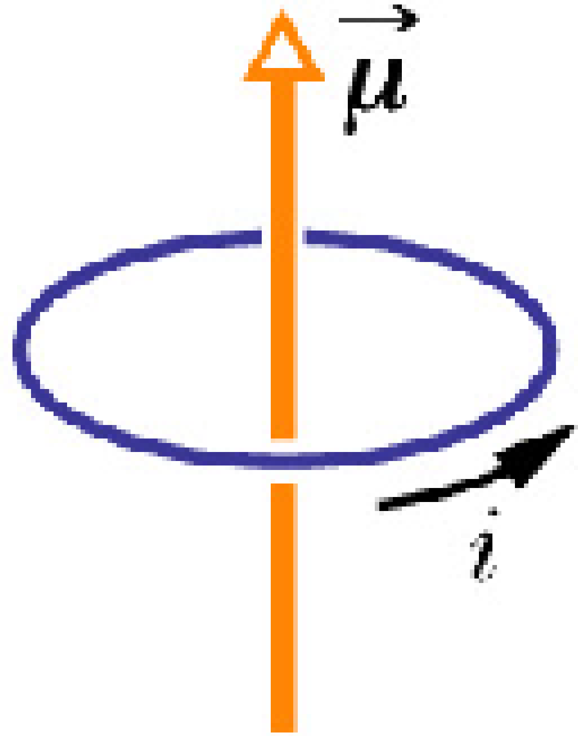
(1) It is upward

(2) It is downward

(3) It is zero

Review: Dipole Moment of Current Loop

Definition: ***Magnetic dipole moment vector:*** $\boxed{\vec{\mu}}$

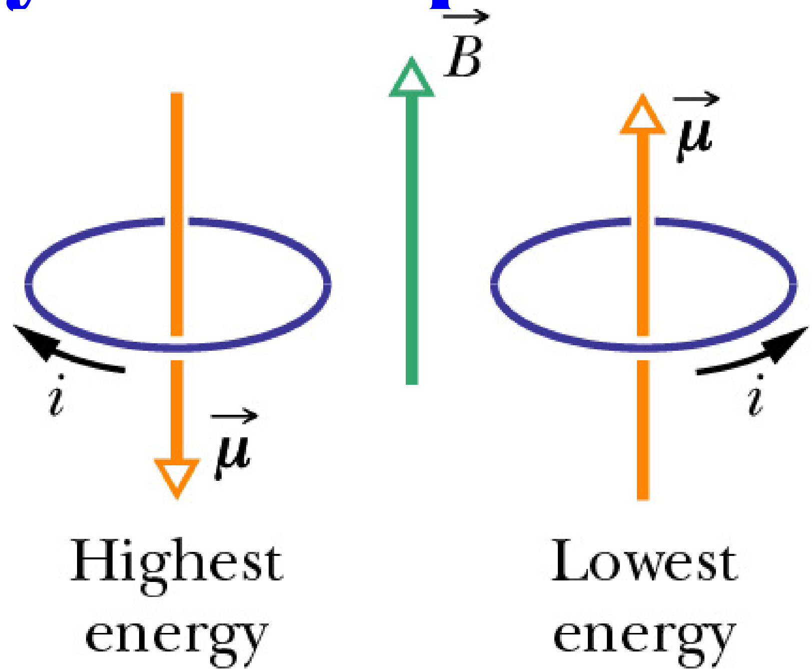


- Direction: ***RH rule***
- Magnitude: $\mu = iA$

Analogous to electric dipole moment vector $\vec{p}$

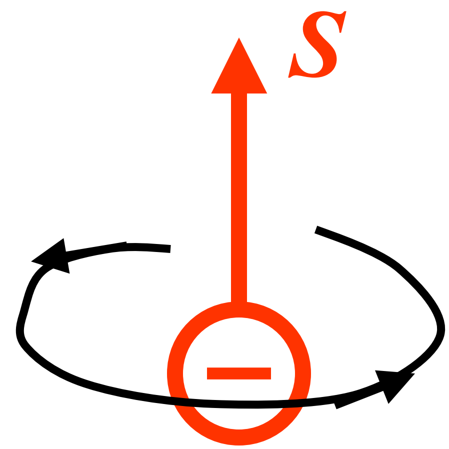
Review:
Potential Energy of a Dipole

Work required to turn dipole moment ***against*** the field.



$$U = -\vec{\mu} \cdot \vec{B}$$

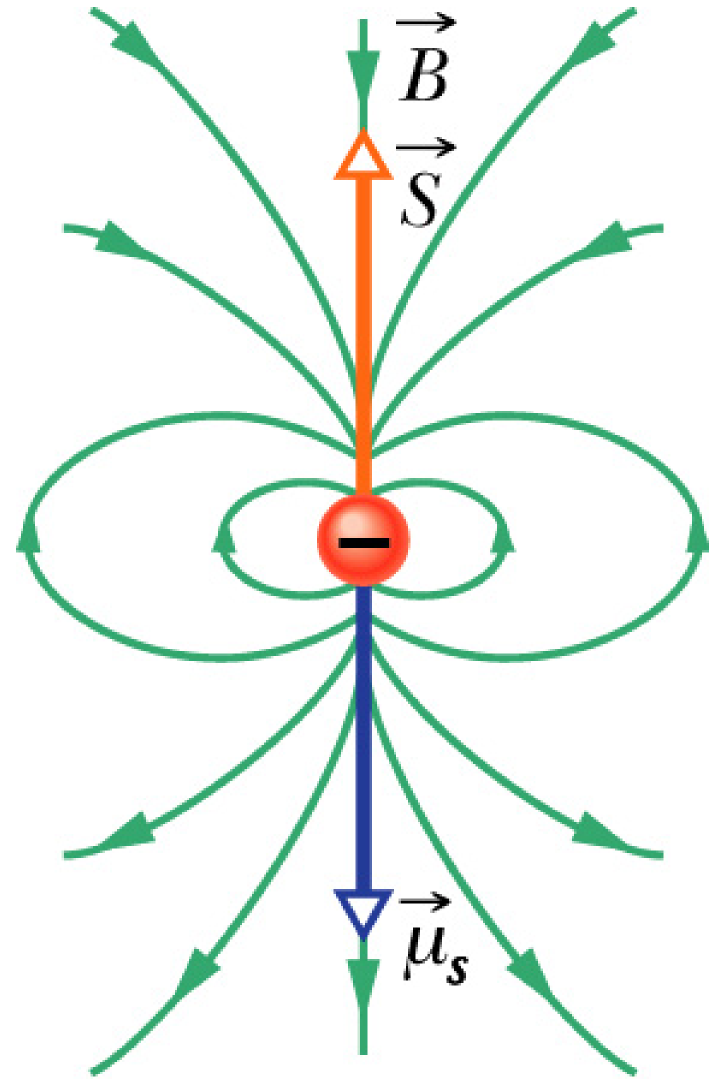
Electron spin



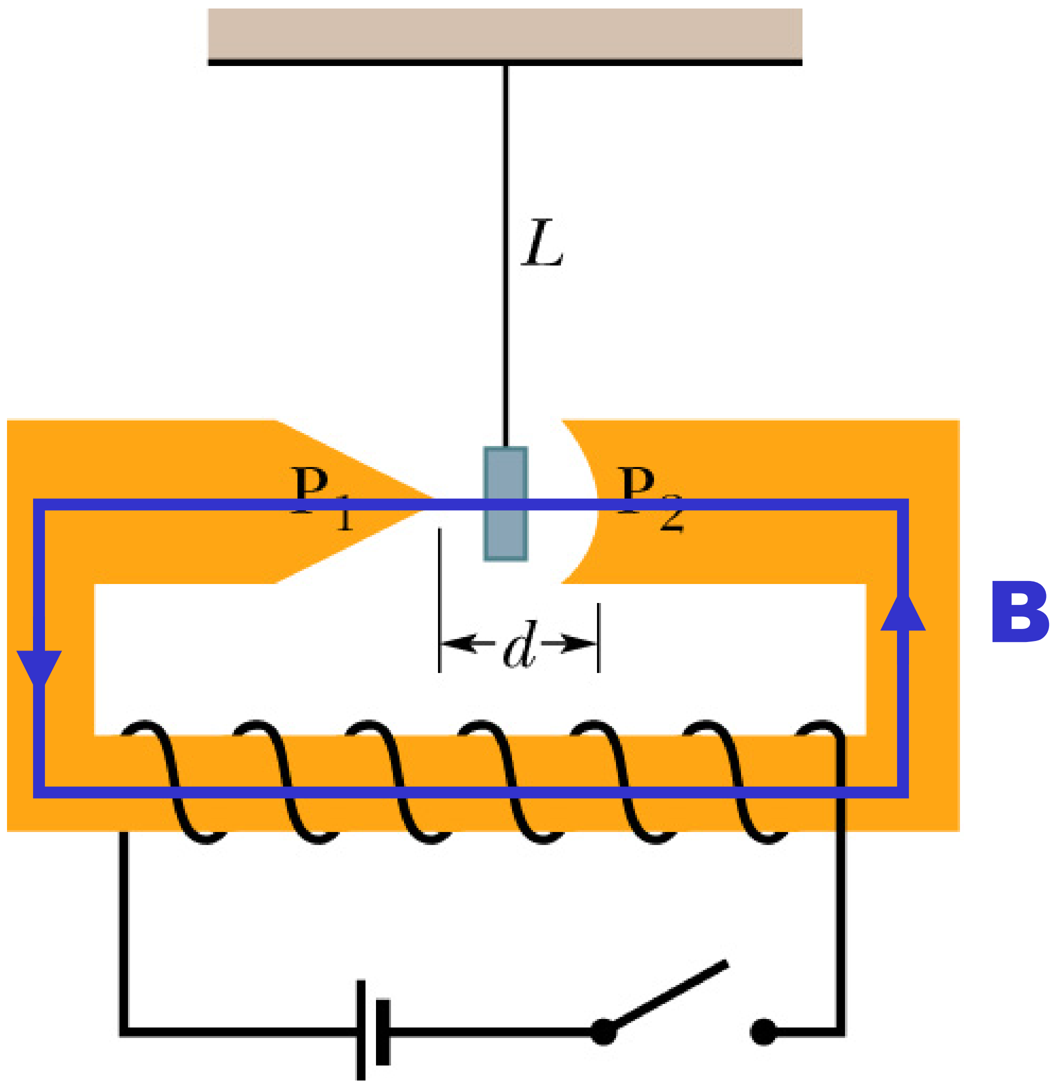
- Electrons have spin as an intrinsic quality. There are no non-spinning electrons.
- This gives an angular momentum vector $\vec{S}$
- This corresponds to right-hand rotation about $\boldsymbol{S}$ as shown.

Electron Magnetic Dipole

- Electron spin creates a magnetic dipole just as if it were a tiny current loop.
- Negative charge means dipole momentum vector points opposite to the spin angular momentum vector.
- Text figure shows the resulting magnetic field.



Materials in an External Field



Place sample in external magnetic field. Measure its magnetization.

Paramagnetism

$$\vec{M} = \left(C/T\right)\vec{B}_{ext}$$

1. The magnetization is ***parallel*** to an applied field. Interior field is ***strengthened***.
2. Curie Law: Thermal and magnetic forces compete. As temperature goes up, magnetization goes down.

Diamagnetism

$$\vec{M} \propto -\vec{B}_{ext}$$

Dipoles align themselves ***opposite*** to the applied field. Field inside is ***weakened.*** (Like a dielectric material.)

Text gives explanations in terms of electron currents inside atoms. We won't worry about that.

Ferromagnetism

$$\vec{M} \not\propto \vec{B}_{ext}$$

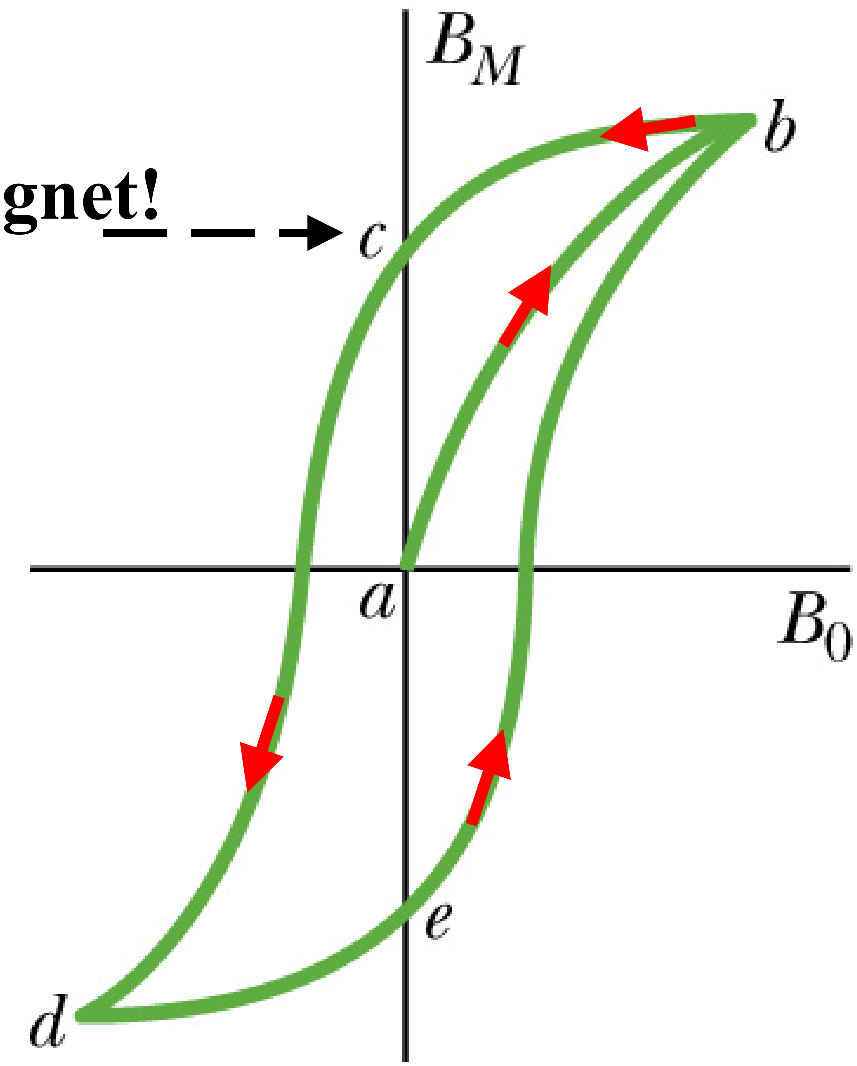
• The basic dipole moments responsible for ferromagnetism are the ***electron spins.***

• In a ***permanent magnet,*** dipoles align themselves ***spontaneously,*** due to their interactions with each other.

• In an ***applied field***, dipoles tend to line up ***with*** the applied field, but a ferromagnet shows the phenomenon of ***hysteresis:*** $\boldsymbol{M}$ is not determined just by the present value of $\boldsymbol{B}_{ext}$, but by the history of $\boldsymbol{B}_{ext}$.

Hysteresis Loop

Permanent magnet!



Q.32-2

- **In techniques such as MRI and NMR, the magnetic properties of spinning *protons* may be detected.**
- **How should the proton's magnetic dipole moment vector be related to its angular momentum vector?**

(1) Parallel (2) Anti-parallel (3) Perpendicular

Q.32-2

How should the proton's magnetic dipole moment vector be related to its angular momentum vector?

Because the proton has a positive charge, its spin angular momentum and magnetic dipole moment are parallel.

$\boxed{\text{(1) Parallel}}$ (2) Anti-parallel (3) Perpendicular

Example: Problem 32-45

The saturation magnetization M_{max} of nickel is 4.7×10^5 A/m. Calculate the magnetic moment of a single nickel atom.

Remember the definition of M:

$$M = \frac{dipole}{volume} = \frac{atoms}{volume}\frac{dipole}{atom}$$

First find the number of atoms per unit volume.

Example (cont'd)

$$\frac{atoms}{volume} = \frac{mass/volume}{mass/atom}$$

$$\frac{mass}{volume} = (density) = 8.90\ g/cm^3$$

$$\frac{mass}{atom} = \frac{mass}{nucleon}\frac{nucleons}{atom} = \left(1g/6\times10^{23}\right)(59) = 9.8\times10^{-23}\ g$$

So: $$\frac{atoms}{volume} = \frac{8.9\ g/cm^3}{9.8\times10^{-23}\,g/atom} = 0.91\times10^{23}\ atoms/cm^3$$

$$= 0.91\times10^{23}\ \frac{atoms}{cm^3}\,10^6\,\frac{cm^3}{m^3} = 0.91\times10^{29}\ \frac{atoms}{m^3}$$

Example (cont'd)

The saturation magnetization M_{max} of nickel is 4.7×10^5 A/m. Calculate the magnetic moment of a nickel atom.

$$\mu = \frac{dipole}{atom} = \frac{dipole/volume}{atoms/volume}$$

$$\mu = \frac{4.7\times10^{5}\ A/m}{0.91\times10^{29}\ /m^{3}} = \underline{5.2\times10^{-24}\ Am^{2}}$$

Summary: 3 kinds of materials

- **Paramagnetic:** $\vec{M} = +(const)\,\vec{B}_{ext}$
- **Diamagnetic:** $\vec{M} = -(const)\,\vec{B}_{ext}$
- **Ferromagnetic:** $\vec{M} \neq (const)\,\vec{B}_{ext}$

$\vec{M}$ $\left\{ \begin{array}{l} \textbf{Depends on history.} \\ \textbf{Spins align with each other.} \\ \textbf{May be very strong.} \end{array} \right.$