Electrostatics

Exam Tomorrow: Chapters 21-25

- Review lectures.
- Review quizzes.
- Review textbook homework problems.

About the exam

- Format similar to second quiz:
  - 4 multiple choice questions for 50%
  - Two problems for 50%
  - No computer-scanned answer sheet
- All five chapters covered about equally
- Focus on key ideas and fundamental laws
- Quiz and homework problems may reappear.
- Cover page is posted.

Key ideas

- **Electric charge:** conserved and quantized
- **Electric field:**
  - force per unit charge, field lines, adding vectors
- **Flux:** amount of field passing through an area
- **Electric potential:**
  - energy per unit charge, integral of field
  - new unit: electron volt (eV)
- **Dipole moment:** paired + and - charges
- **Capacitance:** device to store charge and energy
- **Dielectrics:** polarization, dielectric constant

Fundamental Laws

Coulomb's Law:

1. $F = kQq/r^2$
2. $E = kQ/r^2$
3. $V = kQ/r$

Gauss's Law:

The outward flux of the electric field through any closed surface equals the net enclosed charge divided by ε_0.

Relations between potential and field:

The potential difference between A and B is the work required to carry a unit positive charge from A to B.

$$\Delta V = -\int E_x dx \qquad E_x = -\frac{dV}{dx} \quad etc.$$

Terminology

Words whose precise definitions you must know:

Field
Flux
Potential
Potential difference
Dipole moment
Capacitance
Dielectric constant

And of course the SI units for all these things.

Q. 25 - 3

$$C = \frac{2\pi\varepsilon_0}{\log(b/a)}$$

In the text, this formula is derived for the capacitance per unit length of a long cylindrical capacitor, such as a coaxial cable.

In this derivation, the potential difference was calculated by means of an integral over the electric field. What was that integral?

(1) $\int r\,dr$ (2) $\int r^2\,dr$ (3) $\int \frac{dr}{r}$ (4) $\int \frac{dr}{r^2}$

Q.25-3

What was the integral needed for the capacitance of a coaxial cable?

1) ∫ r dr
2) ∫ r² dr
3) ∫ (1/r) dr
4) ∫ (1/r²) dr

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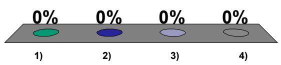
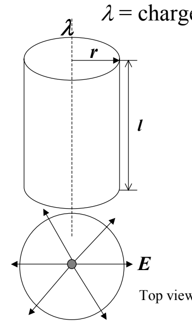


Long Line of Charge

λ = charge/length = linear charge density



$$\Phi = \frac{Q}{\varepsilon_0}$$ **Gauss' law**

$$Q = l\lambda$$

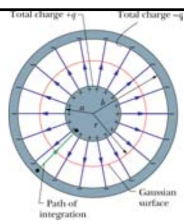
$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = EA$$

$$A = 2\pi rl$$

$$E = \frac{\lambda}{2\pi r\varepsilon_0}$$

Q. 25 - 3

Fig. 25-6:



Gauss's Law: $E = 2k\lambda / r$

Potential difference: $V = \int_a^b E dr = 2k\lambda \int_a^b \frac{dr}{r}$

(1) $\int r\, dr$ (2) $\int r^2\, dr$ **(3) $\int \frac{dr}{r}$** (4) $\int \frac{dr}{r^2}$

Review: The electron-volt

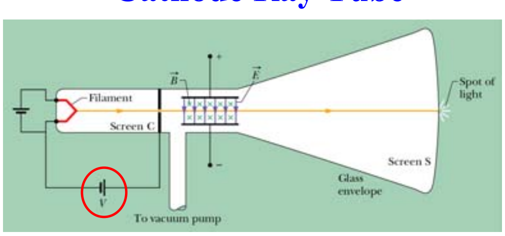
- **One eV is the energy to move an electron through a potential difference of one volt.**
- **Note this is a unit of energy, not potential.**
- **This is not an SI unit but is used in all processes involving electrons, atoms, etc.**

$U = qV$ **So if V=1 and q=e then:**

$$U = 1\, eV = e \times V$$

$$= (1.6 \times 10^{-19}\ C) \times (1\ V) = 1.6 \times 10^{-19}\ J$$

Cathode Ray Tube



Electron gun: potential V gives electron energy in eV.

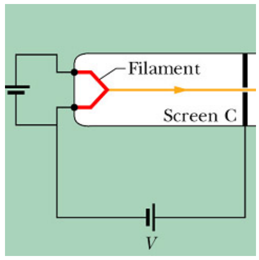
$$K = \frac{1}{2} mv^2 = qV$$

So if V = 500 volts, electron energy is K = 500 eV

High energies

- **For X-ray machines, accelerate electrons with potentials of thousands of volts, so we speak of kinetic energies in *keV*.** $10^3\ eV$
- **In nuclear physics, accelerators produce beams of particles with energies in *MeV*.** $10^6\ eV$
- **In elementary-particle physics, high-energy particles beams have energies measured in giga-volts: *GeV*.** $10^9\ eV$

Q. 25-4



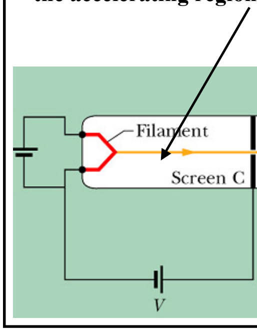
Which direction will the electric field lines point in this electron gun?

(1) To the right **(2) To the left**

(3) Upward **(4) Downward**

Q.25-4

Which way does E point in the accelerating region?



1) **To the right.**
2) **To the left.**
3) **Upward.**
4) **Downward.**

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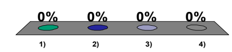
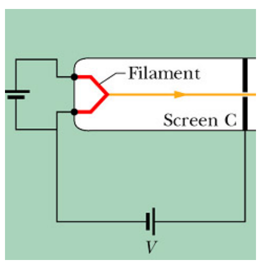


Q. 25-4



Which direction will the electric field lines point in this electron gun?

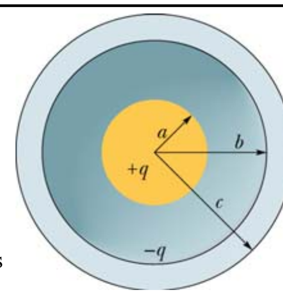
$$\vec{F} = q\vec{E} = -e\vec{E}$$

So for force to the right, we need field to the left!

(1) To the right **(2) To the left**

(3) Upward **(4) Downard**

Text problem 23-49

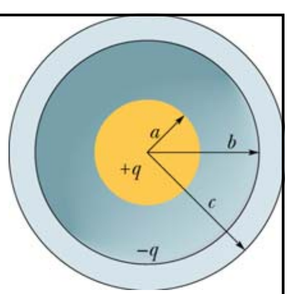


- Nonconducting sphere of radius a with charge q uniformly distributed
- Concentric metal shell of inner radius b, outer radius c, with total charge –q.

(a) Find field everywhere.

(b) How is charge distributed on shell?

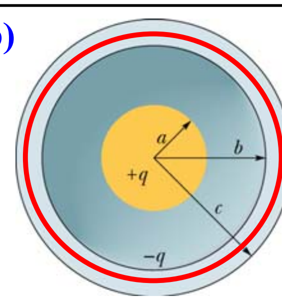
Text problem 49(a)



- For r<a, have previous solution: $E = \dfrac{\rho r}{3\varepsilon_0}$
- For a<r<b, shell theorem gives: $E = \dfrac{kq}{r^2}$
- For b<r<c, we're inside metal, so **E = 0.**
- For c<r, shell theorem gives **E = 0.**

In all cases, field is radially outward.

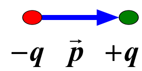
Text problem 49(b)



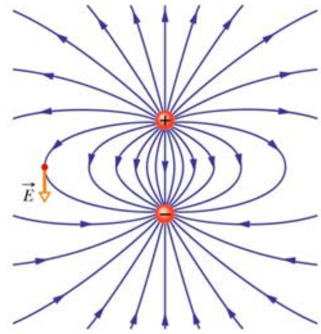
- Draw ***gaussian sphere of radius r*** with b<r<c.
- Because we're inside a metal, E = 0.
- **Therefore** flux = 0.
- **Therefore** enclosed charge = 0.
- **Therefore** there is **–q** on inner surface of shell.
- **Therefore** there is **no** charge on outer surface.

Electric Dipole

- The combination of two charges of equal but opposite sign is called a dipole.
- If the charges +q and –q are separated by a distance d, then the *dipole moment* $\vec{p}$ is defined as a vector pointing from –q to +q of magnitude $p = qd$.



Electric Field Due to a Dipole



Potential due to a dipole

Exact potential at P:

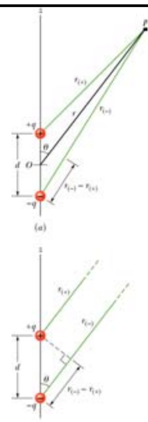
$$V = kq / r_+ - kq / r_-$$

$$r_+ = \sqrt{r^2 + (d/2)^2 - rd\cos\theta}$$

$$r_- = \sqrt{r^2 + (d/2)^2 + rd\cos\theta}$$

Approx. potential at P, r>>d:

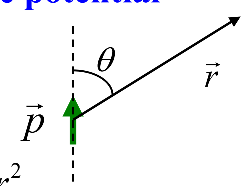
$$V \approx kp\cos\theta / r^2$$



Result: the dipole potential

So we have found that for large r, the ***potential produced by a dipole*** is:

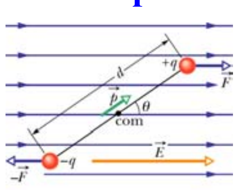
$$V(\vec{r}) = kp\cos\theta / r^2$$



Note this can also be written:

$$V(\vec{r}) = k\frac{\vec{r}\cdot\vec{p}}{r^3}$$

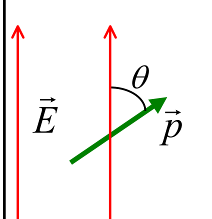
Torque on a Dipole in a Field



$$\tau = 2 \times F \times (\frac{d}{2}\sin\theta) = qE \times d\sin\theta = pE\sin\theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

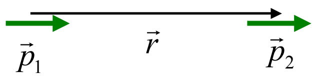
Energy of dipole in given field



$$U = -2Q(E\cos\theta)(d/2)$$
$$= -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

So dipole tends to ***align with*** an applied field.

Interaction of two dipoles



$$U = qV(r + d/2) - qV(r - d/2)$$
$$= \frac{kqp}{(r + d/2)^2} - \frac{kqp}{(r - d/2)^2} = -k\frac{p_1 p_2}{r^3}$$

Attractive potential: work required to pull apart.

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