Electromagnetic Fields

- **Ch.28: The magnetic field: Lorentz Force Law**
- **Ch.29: Electromagnetism:**
  - ***B* field due to a current in a long straight wire**
  - ***B* field due to a current in a short bit of wire**
  - **Ampere's Law: the third of Maxwell's Equations**
- **Ch.30: Induced E Fields: Faraday's Law**

REVIEW:

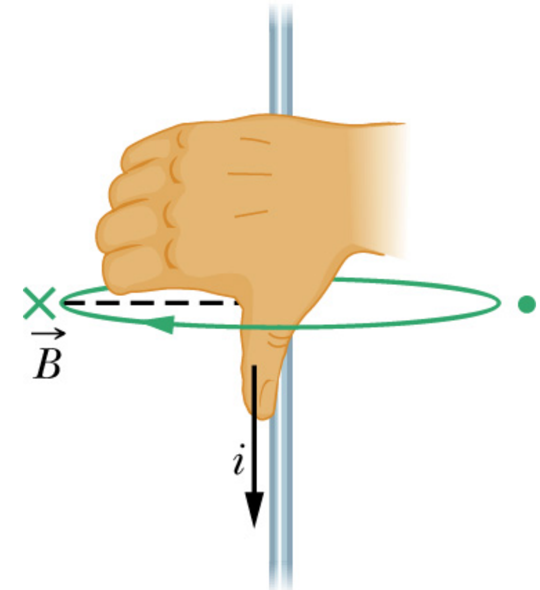
Field of a long straight wire

1. Direction is given by the right-hand rule!

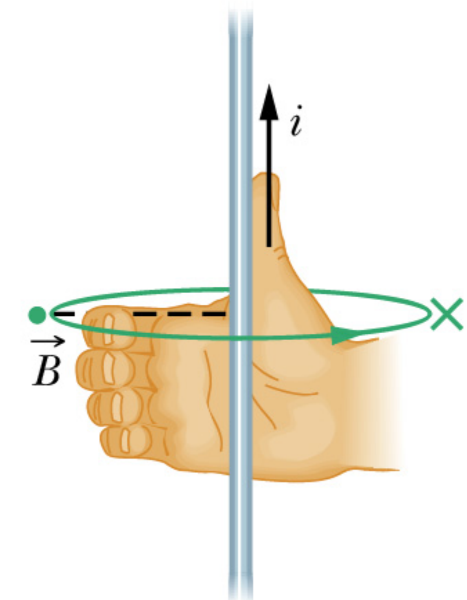
2. Magnitude is $B = \dfrac{\mu_0 i}{2\pi r}$

3. New universal constant:

$$\mu_0 = 4\pi \times 10^{-7}\ Tm/A$$

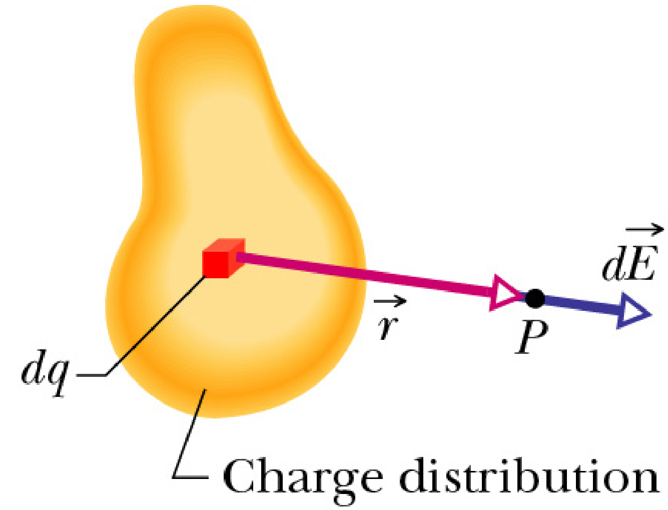


(a)



(b)

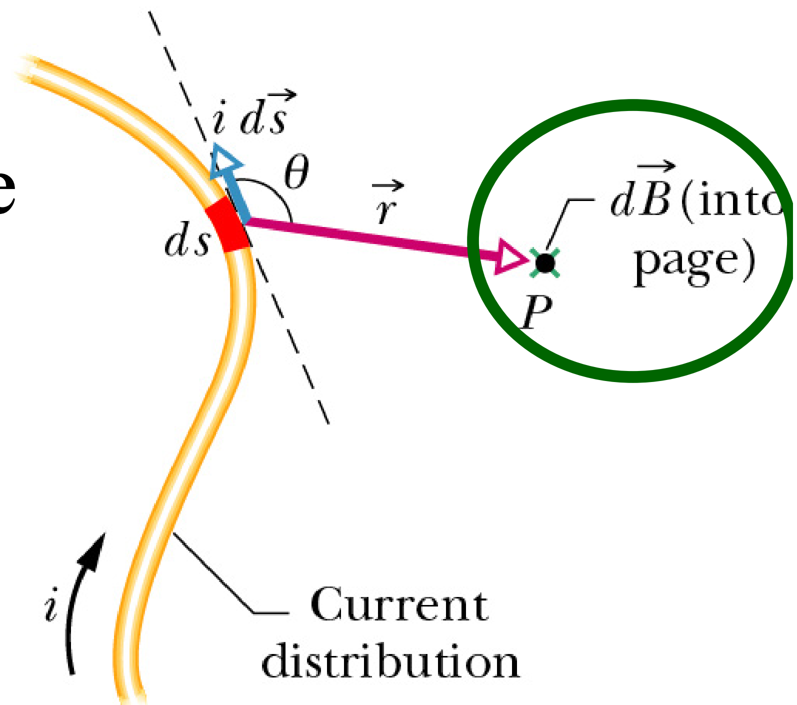
REVIEW: Field due to a *short bit* of wire



(*a*)

Recall Coulomb:
E is **parallel** to ***r*.**

But as usual for magnetism, we find ***B*** is **perpendicular** to ***r***!



(*b*)

$$d\vec{B} \propto i\, d\vec{s} \times \vec{r}$$

Another right-hand rule!

REVIEW: Ampere's Law

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

C = Any closed path

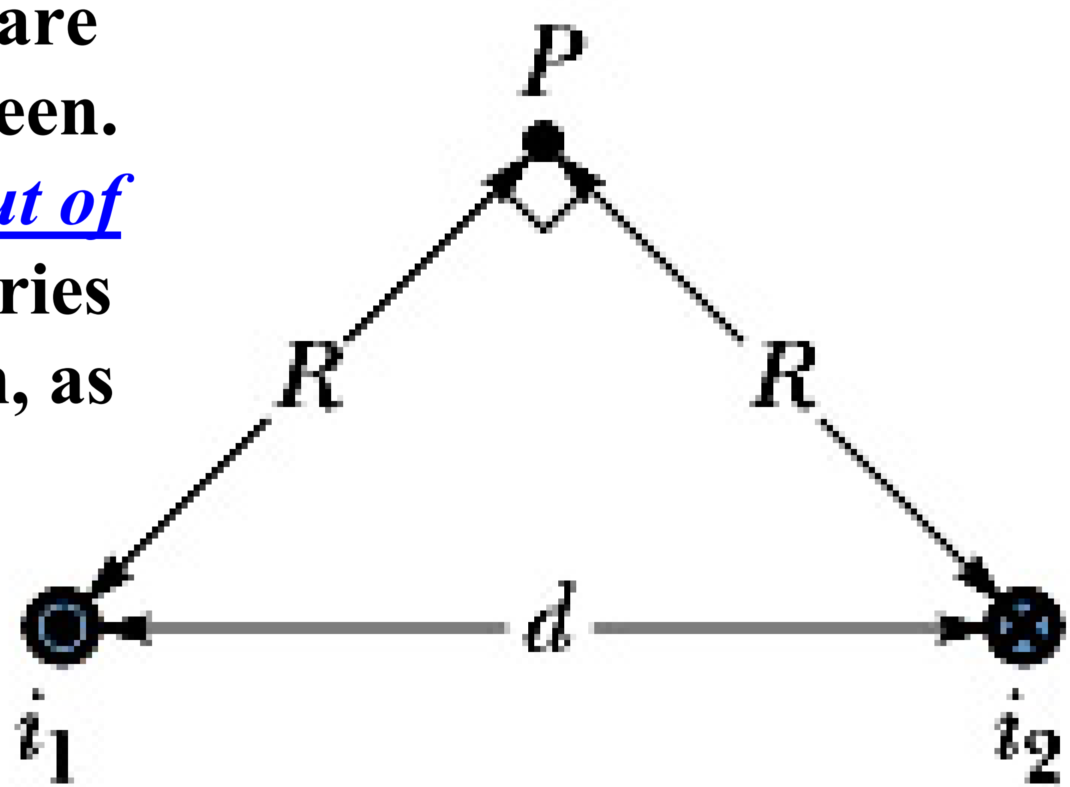
i_{enc} = Net current linking C (*Right-hand rule)*

***B* = The total magnetic field**

***ds* = A short step along the path**

Example: Sample Problem 29-2

Two long parallel wires are perpendicular to the screen. One carries current i_1 ***out of*** the screen, the other carries current i_2 ***into*** the screen, as shown.



$i_1 = 15$ A

$i_2 = 32$ A

d = 5.3 cm

What is the magnetic field at point P?

(Notice the right angle at P.)

Example continued

What is the magnetic field at point P?

$i_1 = 15$ A

$i_2 = 32$ A

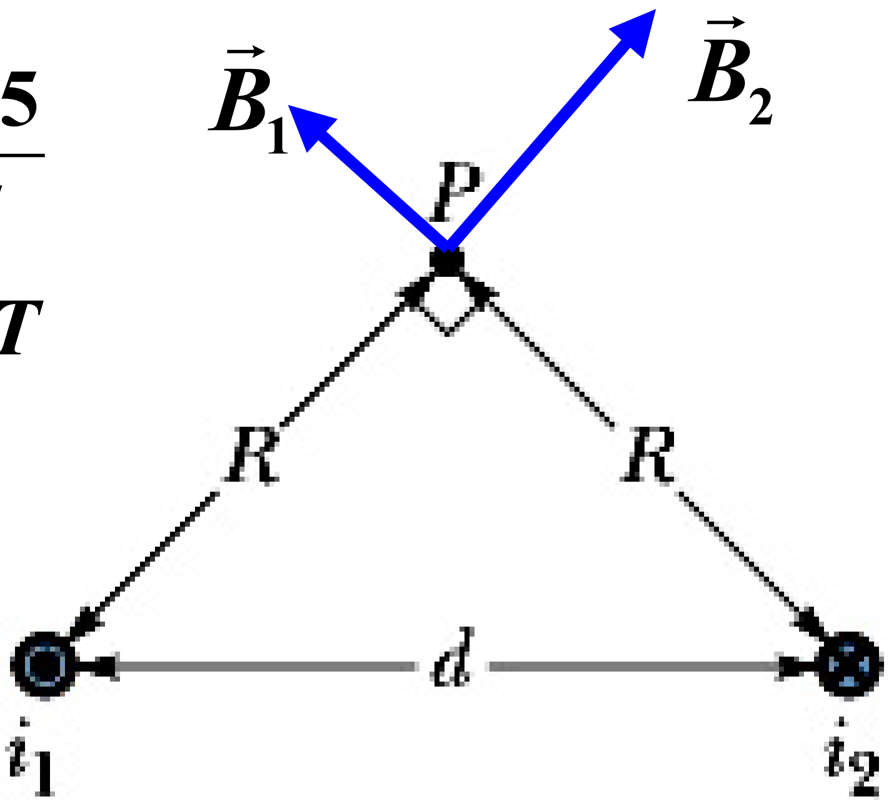
$d = 5.3$ cm

$$R = d\cos 45° = .037\ m$$

$$B_1 = \frac{\mu_0 i_1}{2\pi R}$$

$$= \frac{4\pi 10^{-7} 15}{2\pi .037}$$

$$= 8 \times 10^{-5}\ T$$



$$B_2 = \frac{\mu_0 i_2}{2\pi R}$$

$$= \frac{4\pi 10^{-7} 32}{2\pi .037}$$

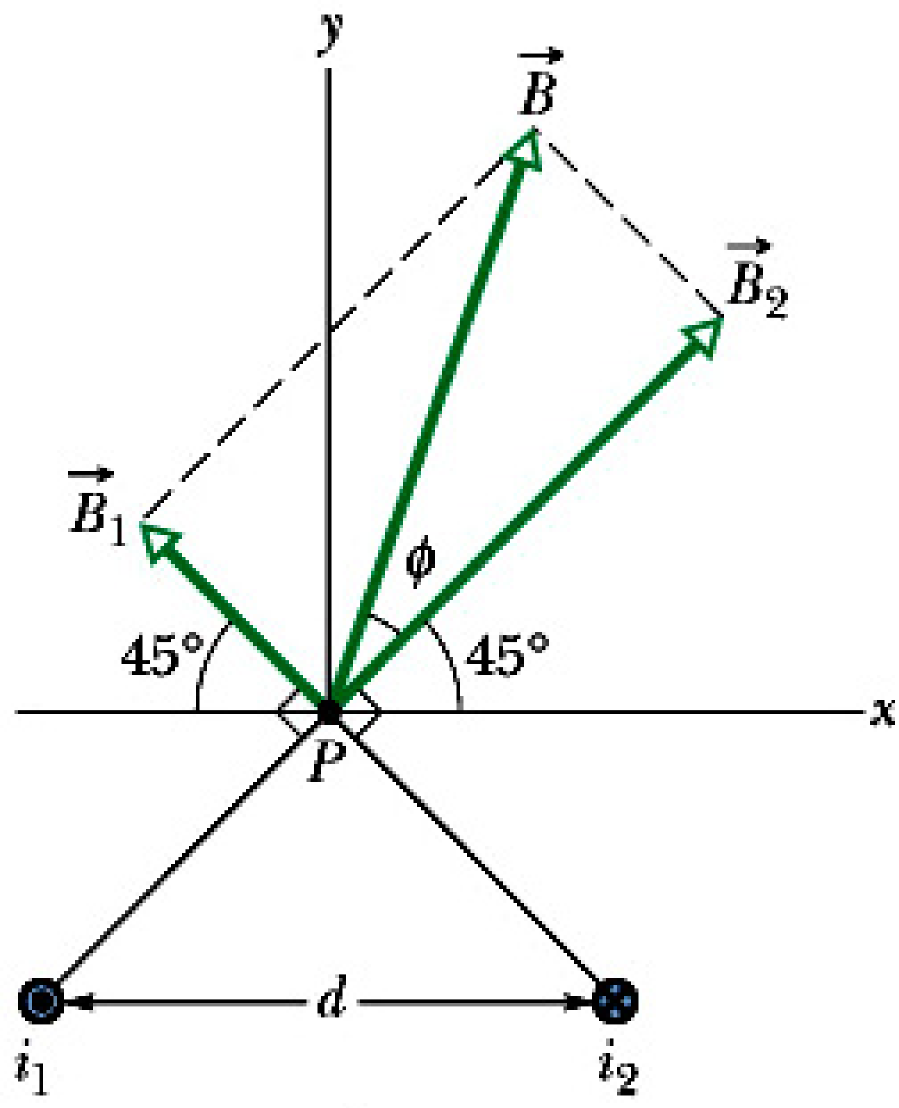
$$= 17 \times 10^{-5}\ T$$

Example continued

What is the magnetic field at point P?

$$B_1 = 8 \times 10^{-5}\ T$$

$$B_2 = 17 \times 10^{-5}\ T$$



$$B_x = B_{1x} + B_{2x}$$

$$= B_2 \cos 45° - B_1 \cos 45°$$

$$= (17 - 8) \times 10^{-5} / \sqrt{2} = 64\ \mu T$$

$$B_y = B_{1y} + B_{2y}$$

$$= B_1 \sin 45° + B_2 \sin 45°$$

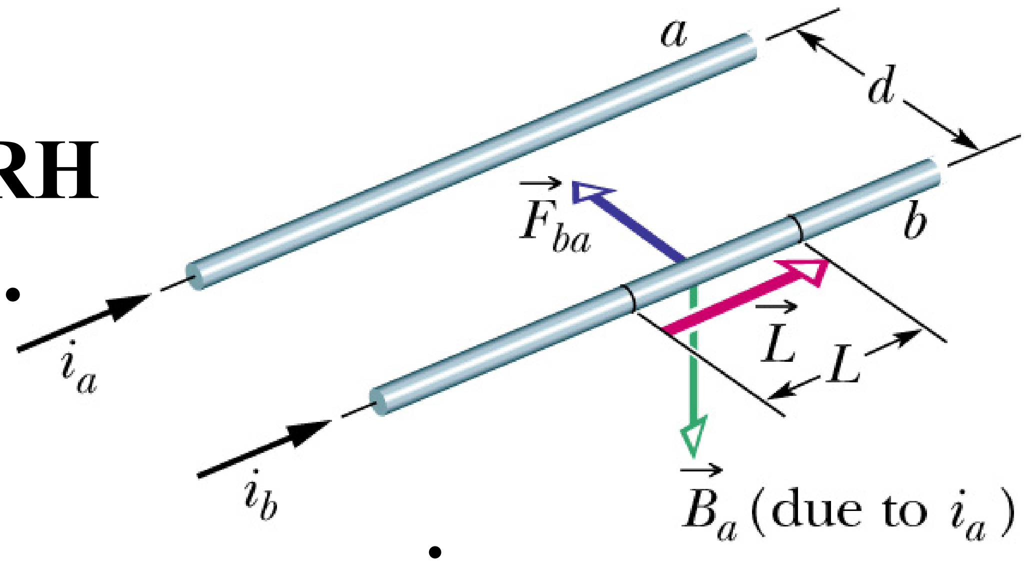
$$= (17 + 8) \times 10^{-5} / \sqrt{2} = 180\ \pi T$$

$$B = \sqrt{B_x{}^2 + B_y{}^2}$$

$$= \sqrt{64^2 + 180^2} = 190\ \mu T$$

Force between two wires

Get direction from RH rule (applied twice!).



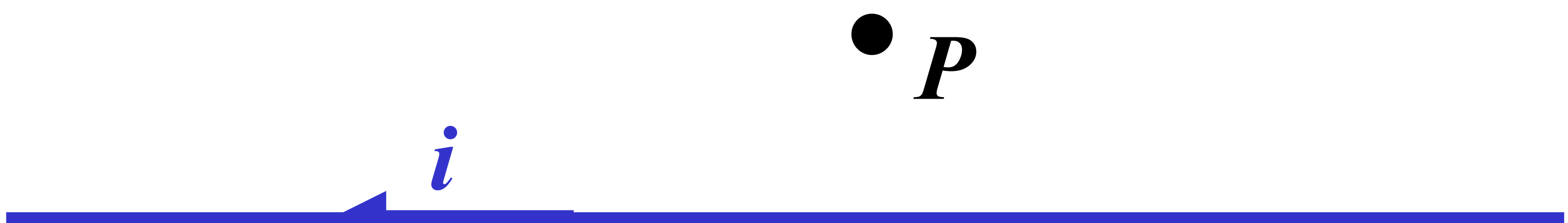
Field at ***b*** due to ***a*** is $B = \dfrac{\mu_0 i_a}{2\pi d}$ $\quad \vec{F}_{ba} = ?$

Force on ***b*** due to this ***B*** is

$$F_{ba} = i_b L B = \frac{\mu_0 L i_a i_b}{2\pi d}$$

Q.29-1

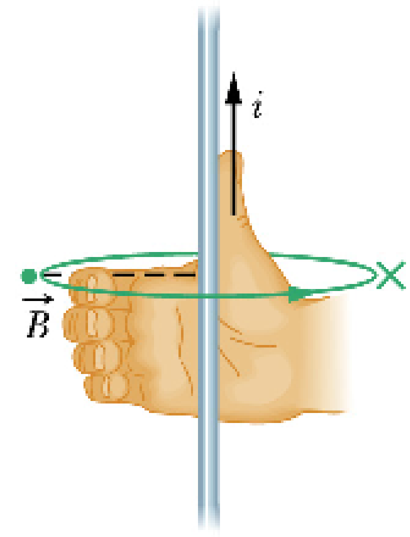
A long straight horizontal wire carries a current ***i*** in the direction shown. What is the direction of the magnetic field at point ***P***, vertically above the wire?



(1) Up (2) Down (3) Right (4) Left

(5) Into the screen (6) Out of the screen

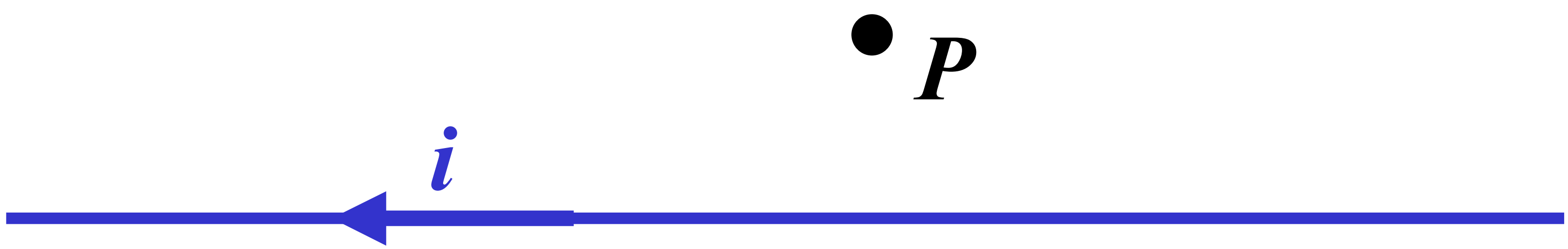
Q.29-1



(b)

What is the direction of the magnetic field at point ***P***?

Right-hand rule: thumb with current, field with fingers.

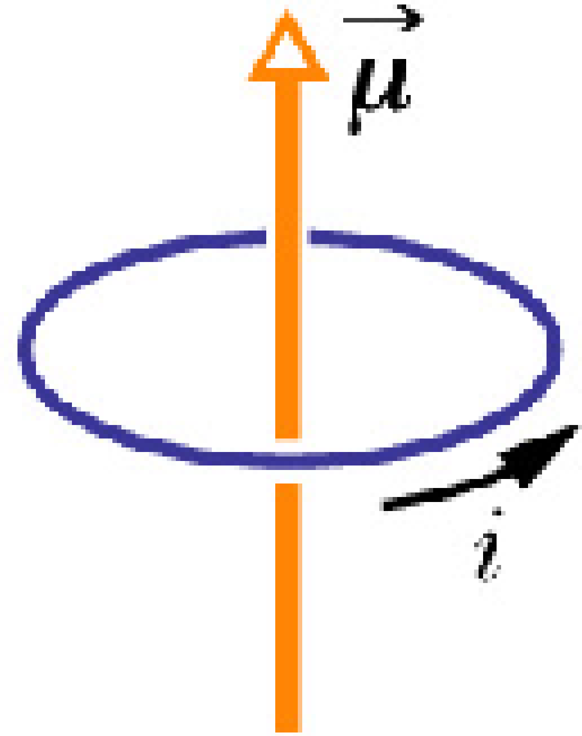


(1) Up (2) Down (3) Right (4) Left

(5) Into the screen (6) Out of the screen

Dipole Moment of a Current Loop

Definition: ***Magnetic dipole moment vector:*** $\boxed{\vec{\mu}}$

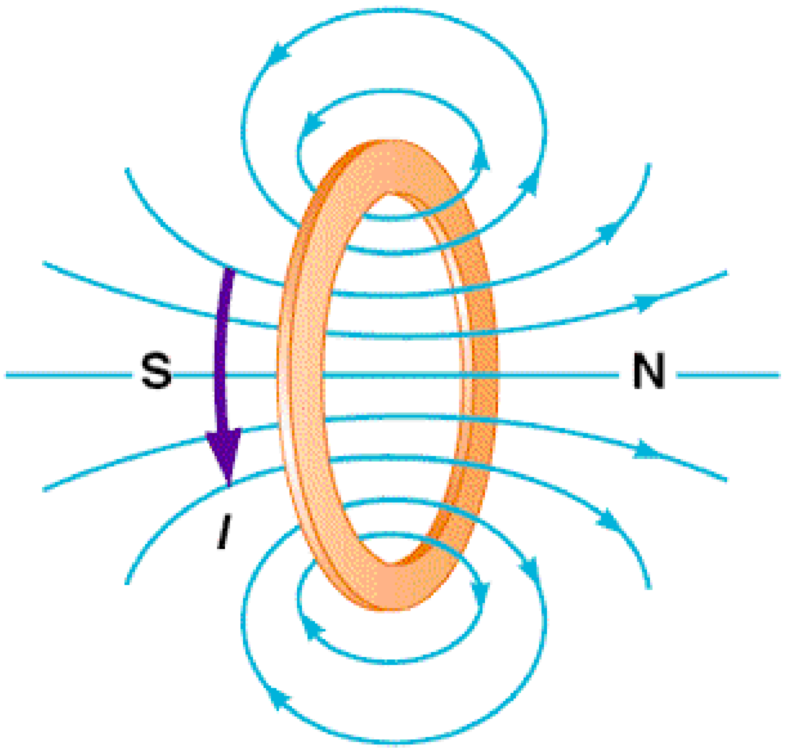


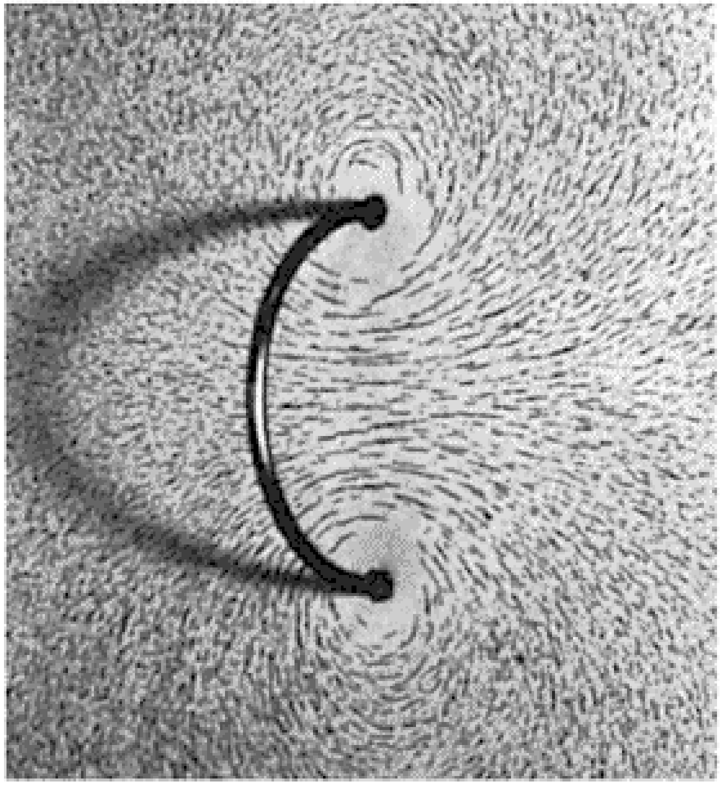
- Direction: ***RH rule***
- Magnitude: $\mu = iA$

Analogous to electric dipole moment vector $\vec{p}$

Field Due to a Current Loop

Serway, College Physics, 5/e
Text Figure 19.28a,b

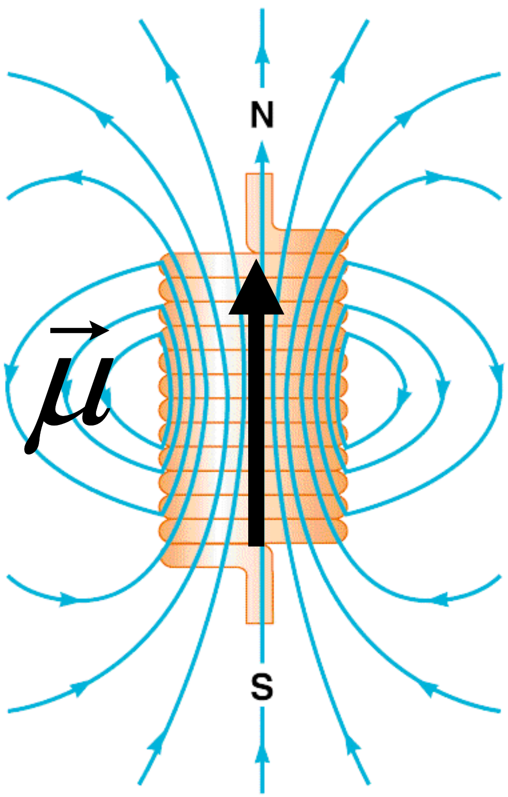




Right-hand rule: fingers with current, thumb gives direction of field on axis.

Magnetic Dipole Field

Serway, College Physics, 5/e
Text Figure 19.30a



(a)

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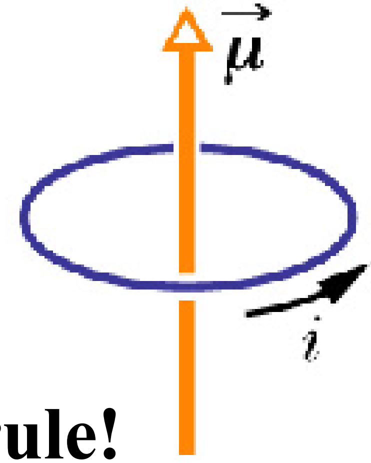
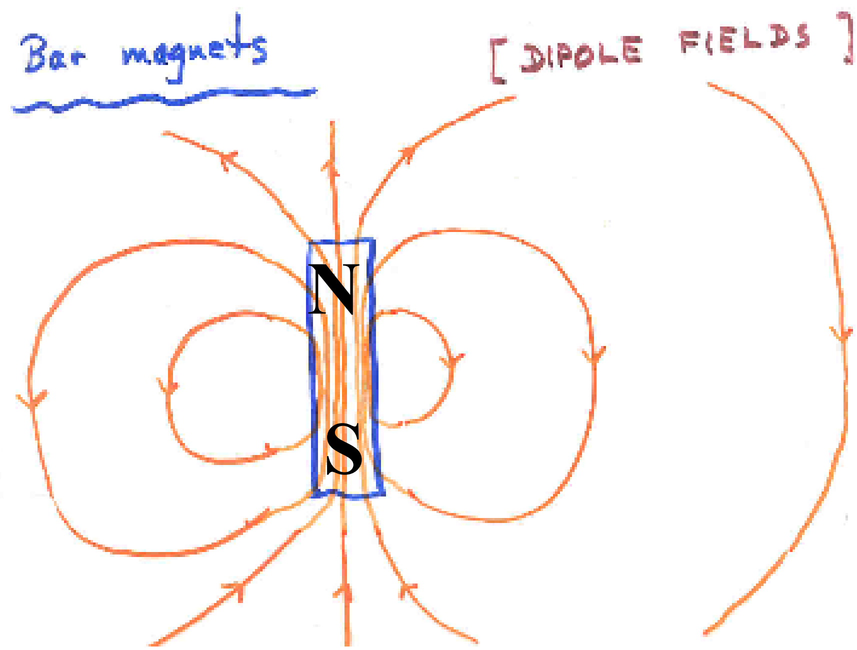
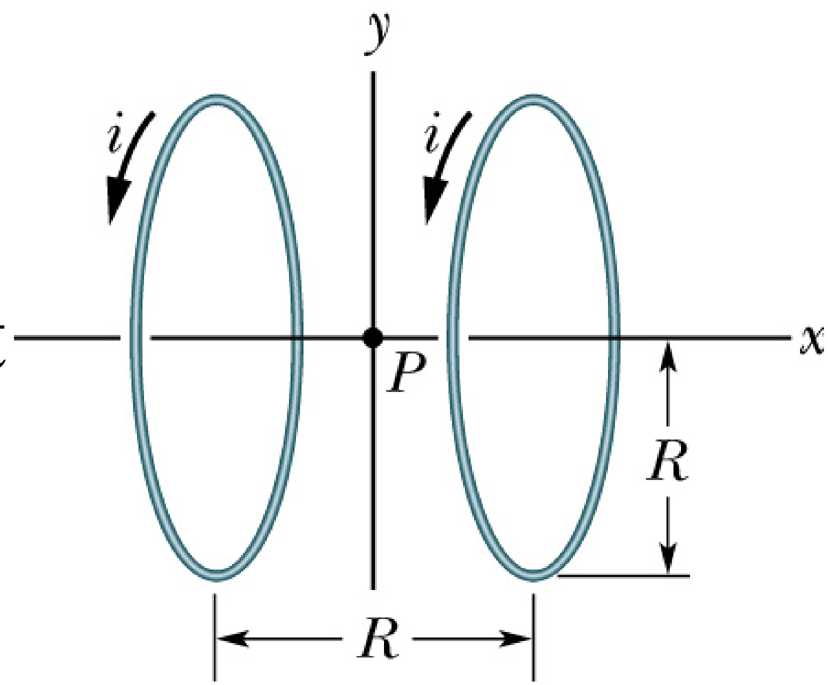


Right-hand rule!



Q.29-2

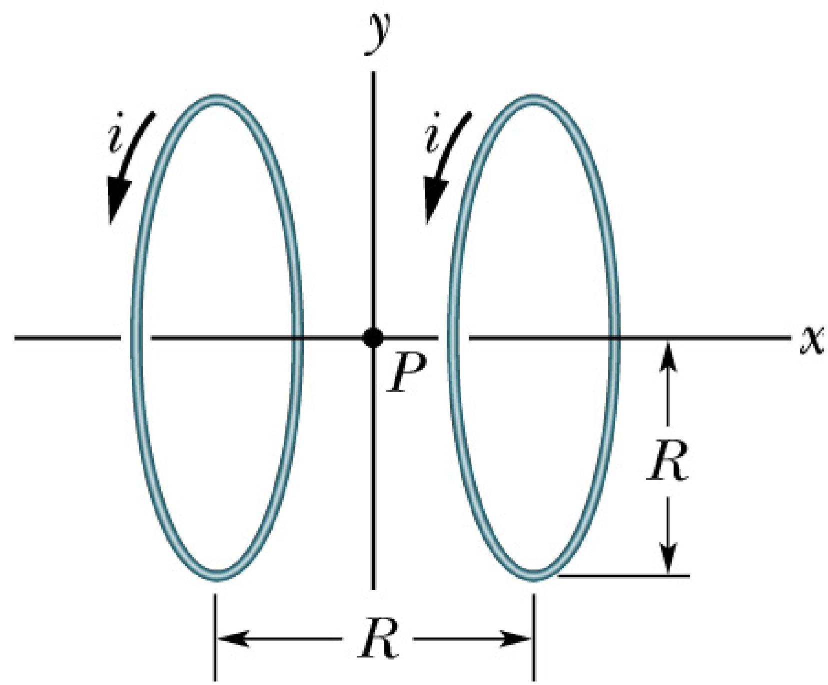
Two wire loops carry currents as shown. If I put a small compass needle at point P, in which direction will it point?

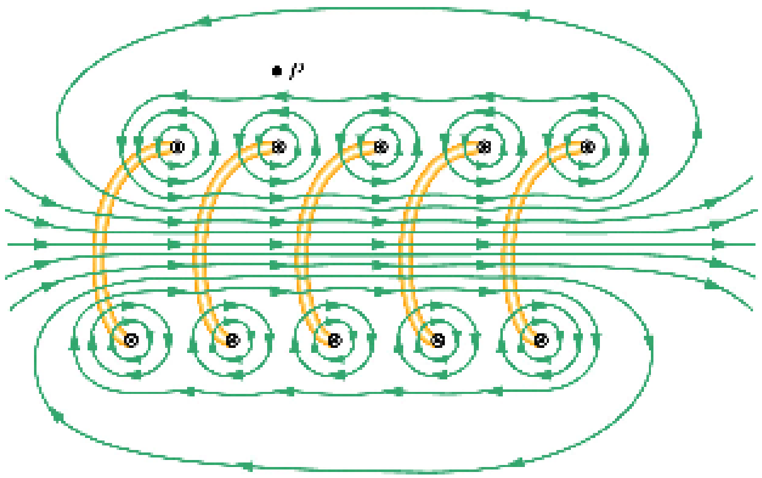


(1) +x (2) –x (3) +y (4) –y (5) +z (6) -z

Q.29-2

This is like a small coil,
producing a dipole-type field.





Right-hand rule: fingers with current,
thumb gives field inside the loop.

$\boxed{\text{(1) +x}}$ (2) –x (3) +y (4) –y (5) +z (6) -z

Field Due to a Solenoid

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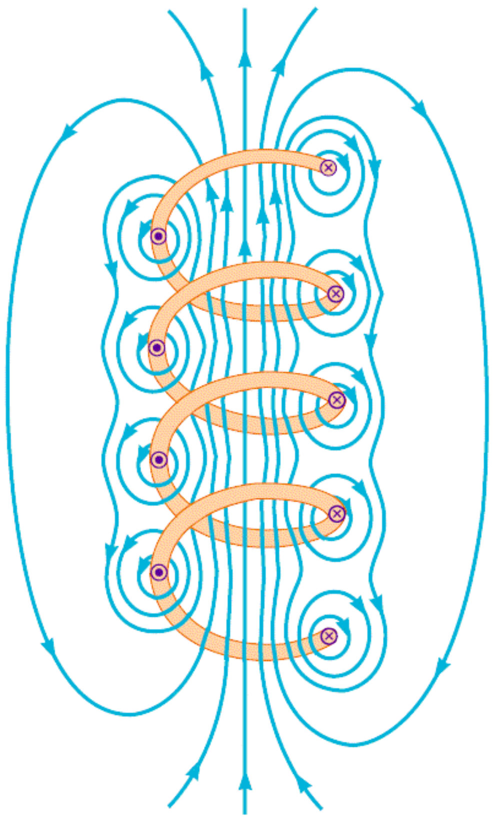
Text Figure 19.29



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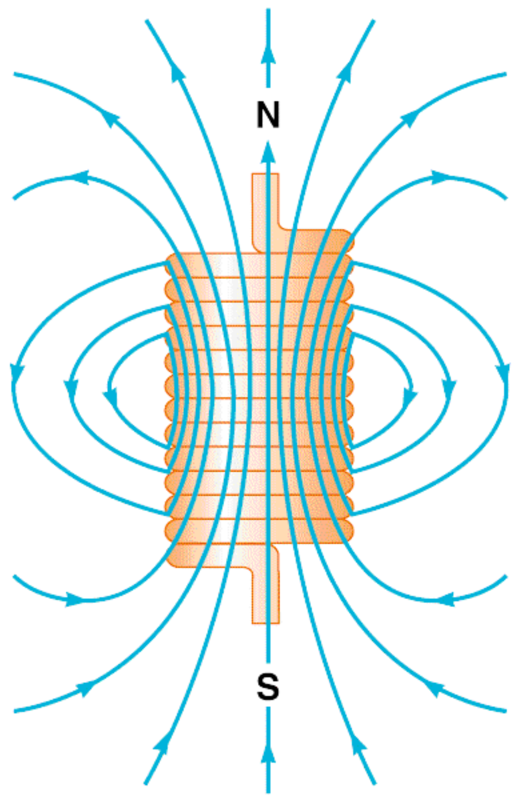
Text Figure 19.30a



(a)

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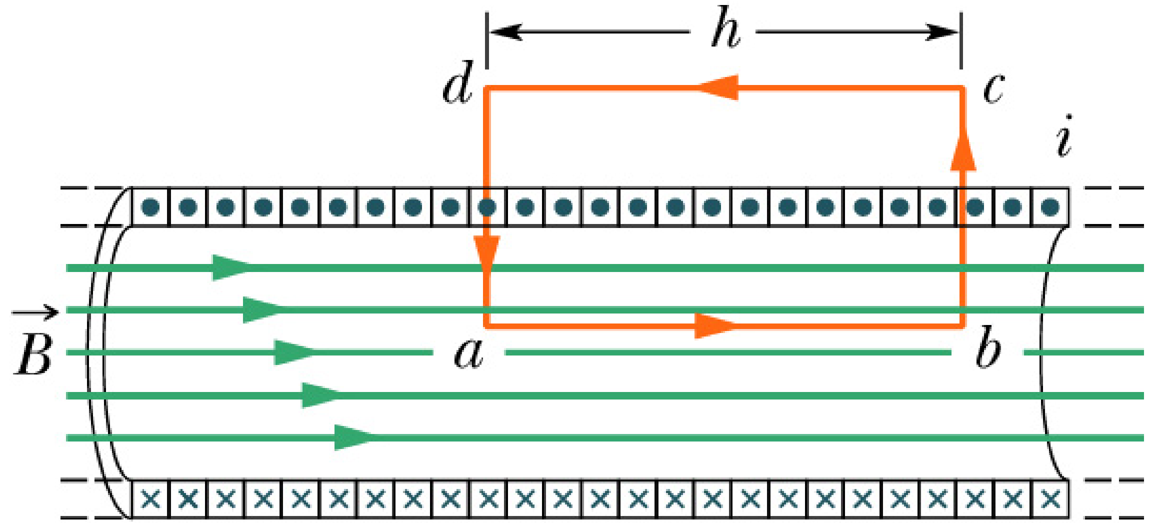
Calculating the Field in a Solenoid

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$



$$\oint_C \vec{B} \cdot d\vec{s} = Bh + 0 + 0 + 0$$

$$i_{enc} = (nh)i$$

$$B = \mu_0 n i$$

Example: Problem 29-40

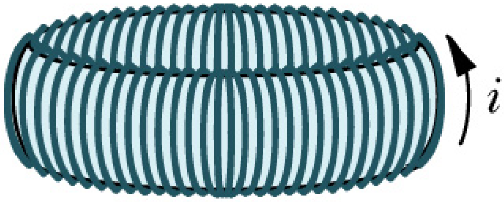
Solenoid of length 1.0 m and diameter 5 cm has 1200 turns and carries current of 4 A. Calculate the magnetic field inside.

$$B = \mu_0 n i$$

$$\boldsymbol{n = \frac{1200}{0.5} = 2400}$$ **turns per meter.**

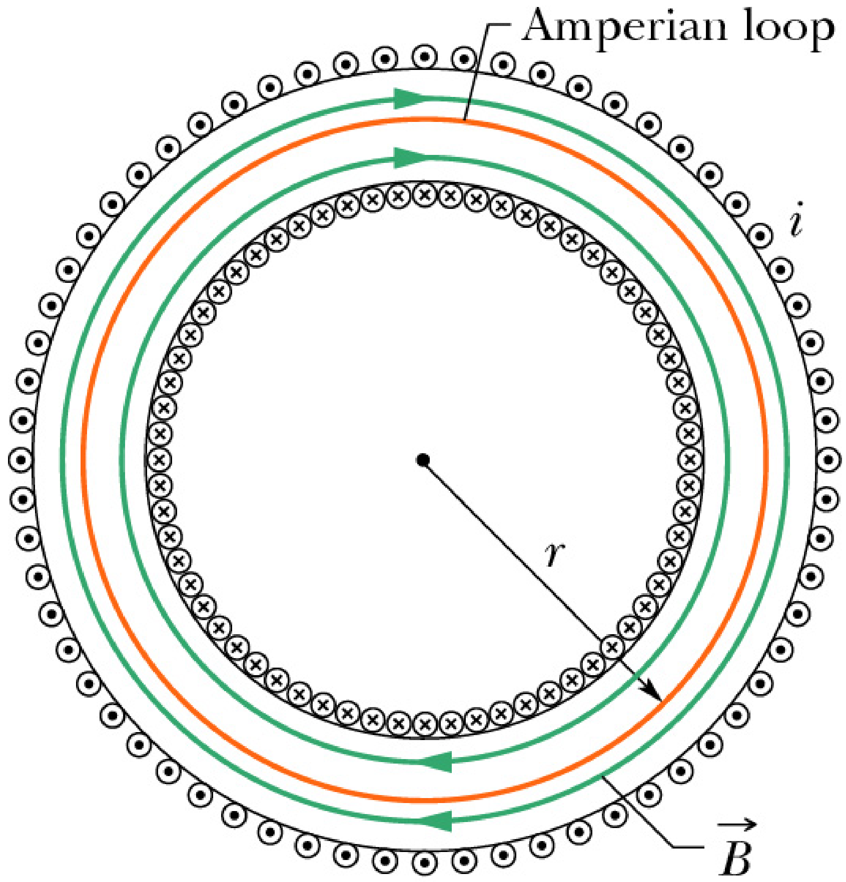
$$\boldsymbol{B = \mu_0 n i = 4\pi \times 10^{-7} \times 2400 \times 3 = 9 \times 10^{-3}\ T}$$

Field in a Toroid



(a)

The textbook derives the field in a solenoid. A toroid is just a solenoid bent into a circle.



(b)

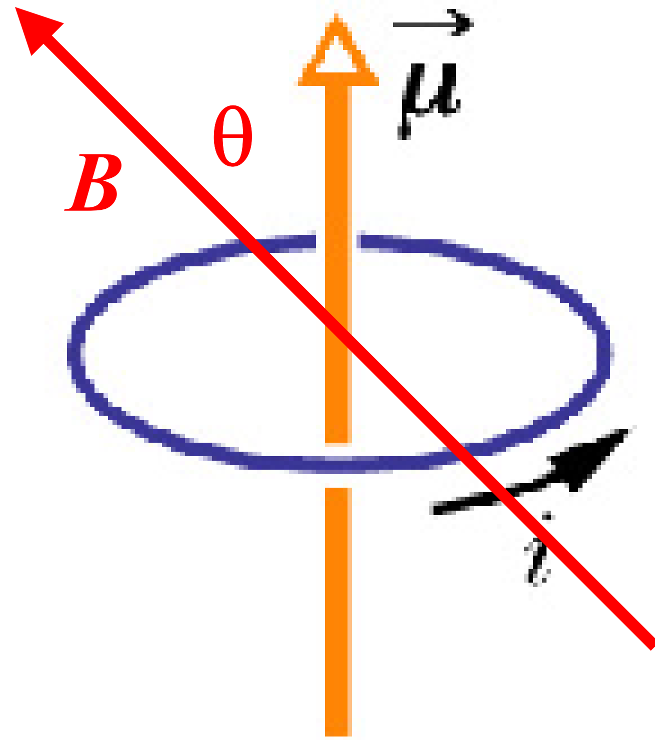
N = total number of turns

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} = \mu_0 iN$$

So $$B = \frac{\mu_0 iN}{2\pi r}$$

Torque on a Current Loop

Given dipole $\vec{\mu}$
placed in magnetic field $\vec{B}$



Torque on loop due to field: $\vec{\tau}$

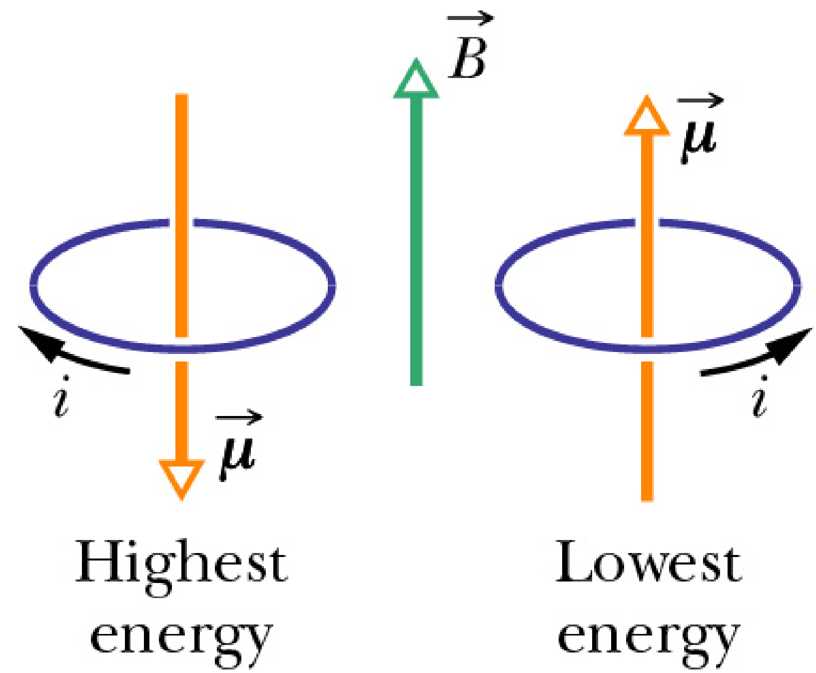
- Direction: ***turns*** μ ***toward B.***
- Magnitude: $\tau = \mu B \sin\theta$

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

Analogous to electric case $\vec{\tau} = \vec{p} \times \vec{E}$

Potential Energy of Current Loop

Work required to turn dipole moment ***against*** the field.



$$U = -\vec{\mu} \cdot \vec{B}$$

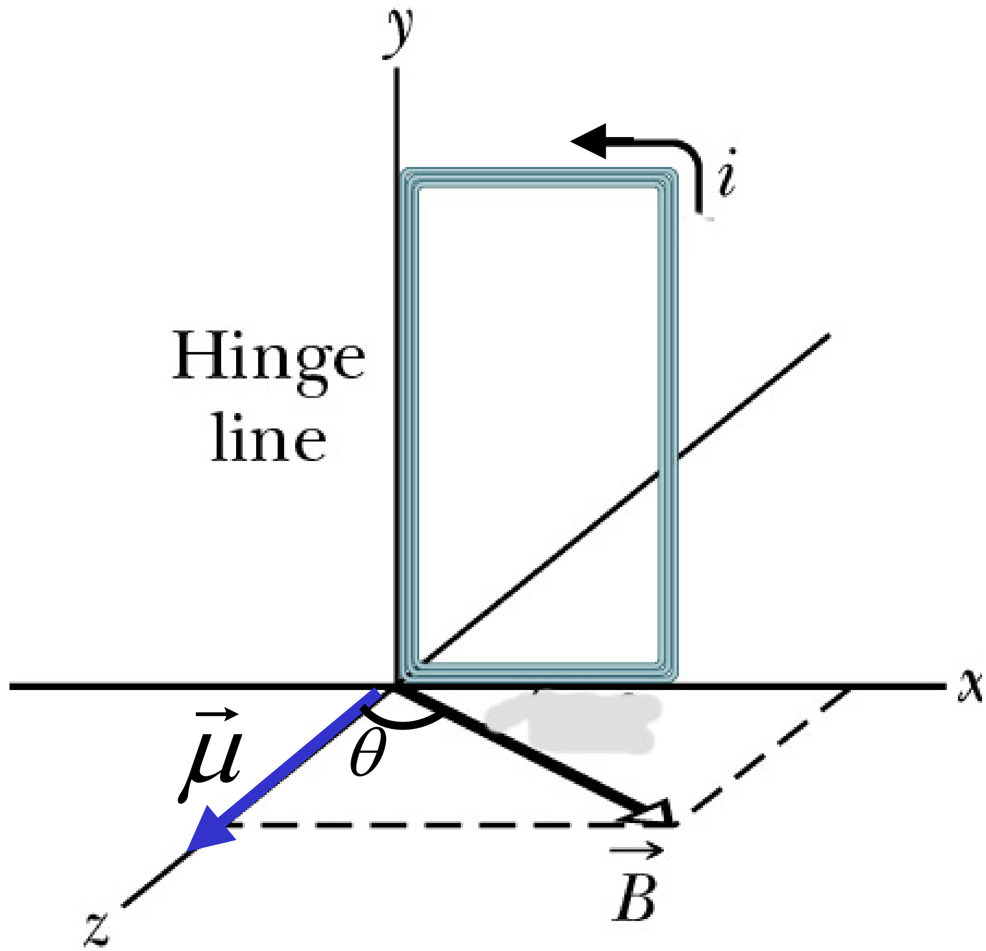
Example (28-39)

Hinged coil in B field.

Use: $\vec{\tau} = \vec{\mu} \times \vec{B}$

$$\mu = iA$$

$$\tau = \mu B \sin\theta$$

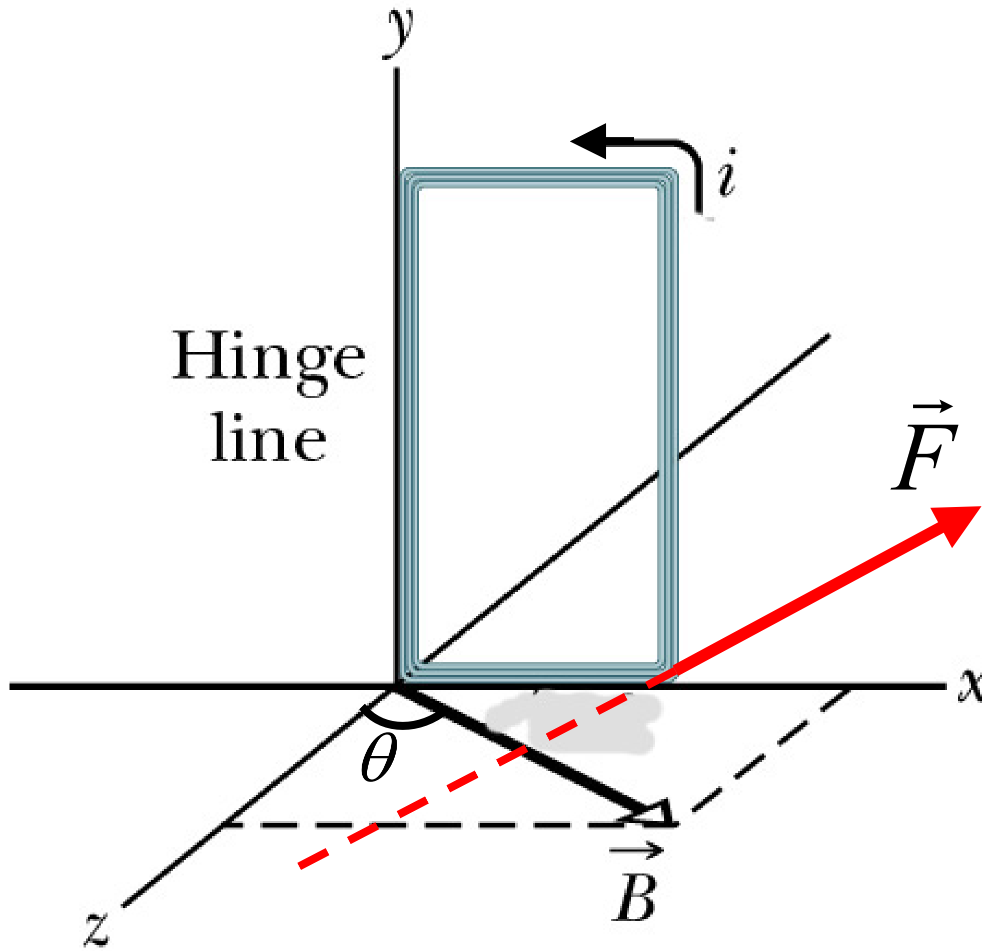


So:

$$\tau = iAB\sin\theta$$

Example (28-39)

Hinged coil in B field.



Check: $\vec{F} = i\vec{L} \times \vec{B}$

$$F = iLB$$

$$\tau = F(w \sin\theta)$$

So:

$$\tau = iLBw\sin\theta = iAB\sin\theta$$

✓

H2

Induction and Oscillations

Ch. 30: Faraday's Law

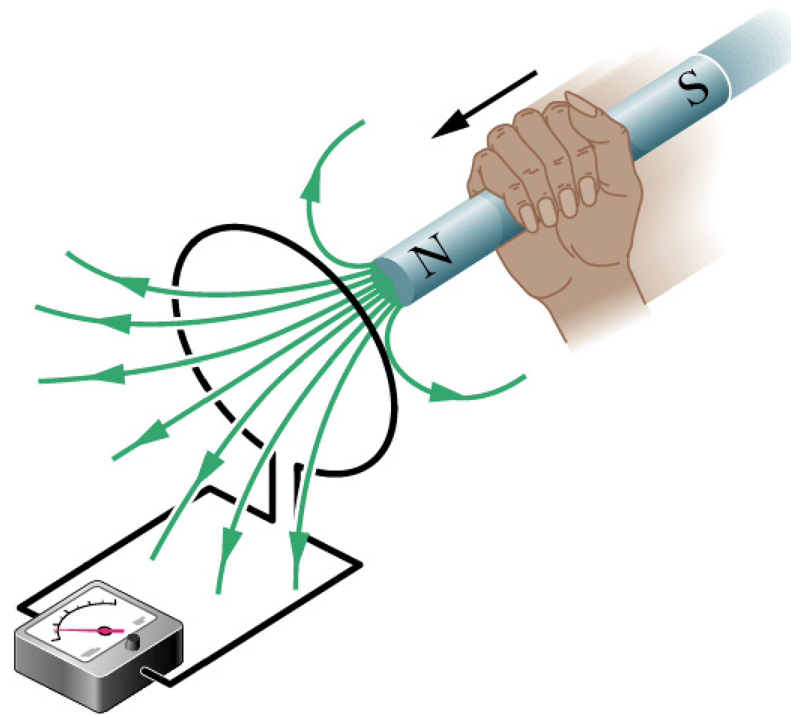
Ch. 31: AC Circuits

Induced EMF: Faraday's Law

"Time-dependent *B* creates induced *E*"

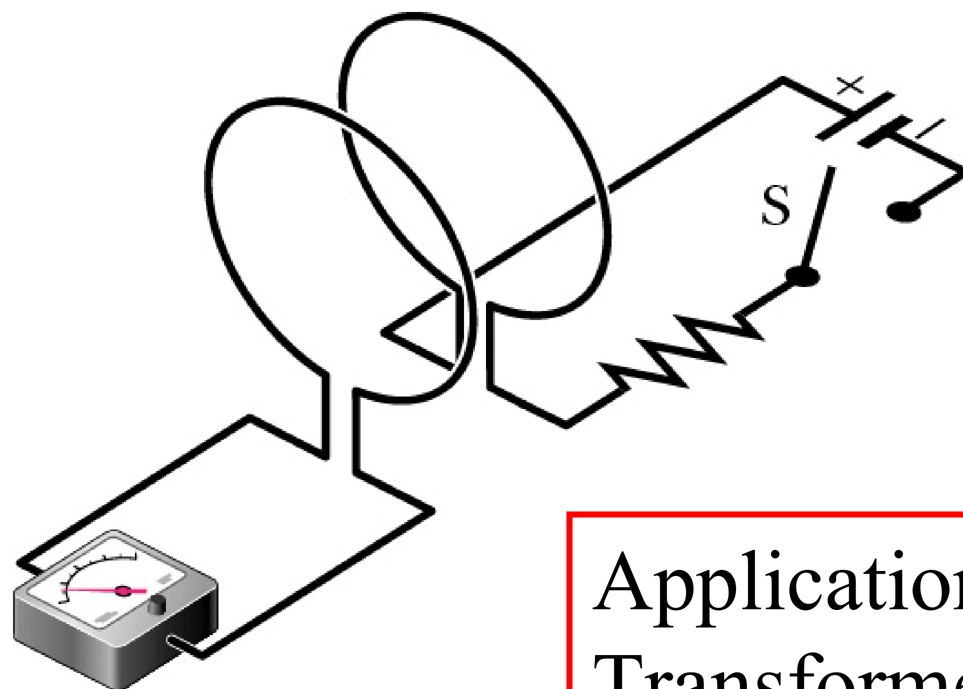
In particular: A *changing magnetic flux* creates an *emf in a circuit*:

Ammeter or voltmeter.



Electromagnetic Induction

Current in secondary circuit can be produced by a *changing* current in primary circuit.



Ammeter or voltmeter.

Application: Transformer

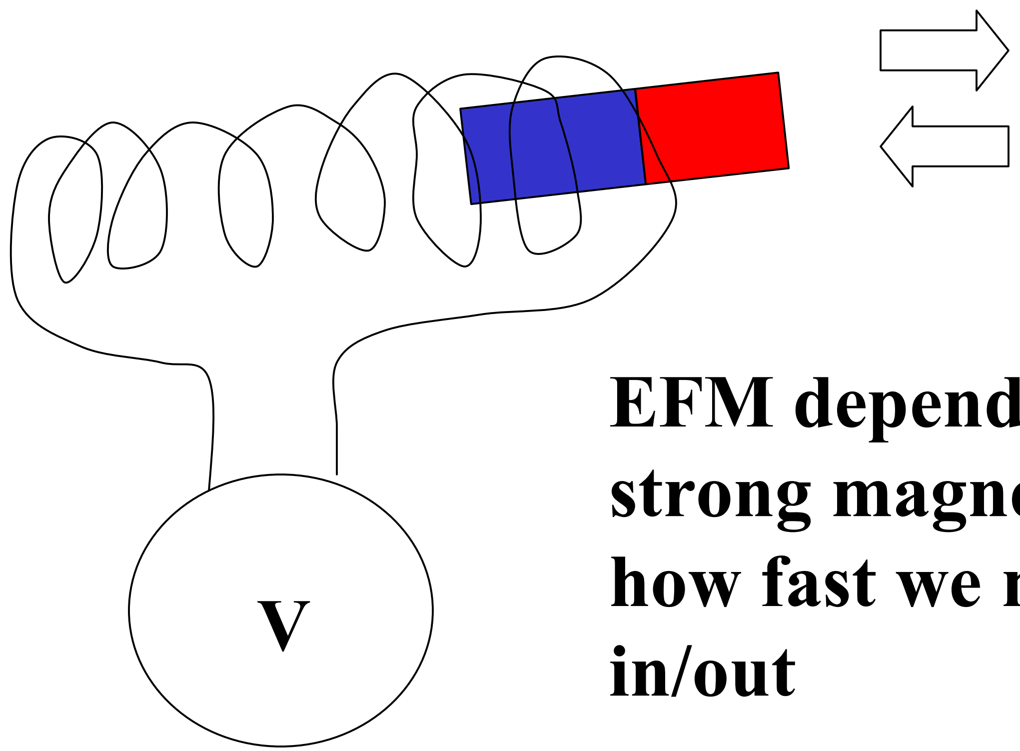
Demonstrations

- **EMF induced in a coil by moving a bar magnet**

- **EMF induced in a secondary coil by changing current in primary coil**

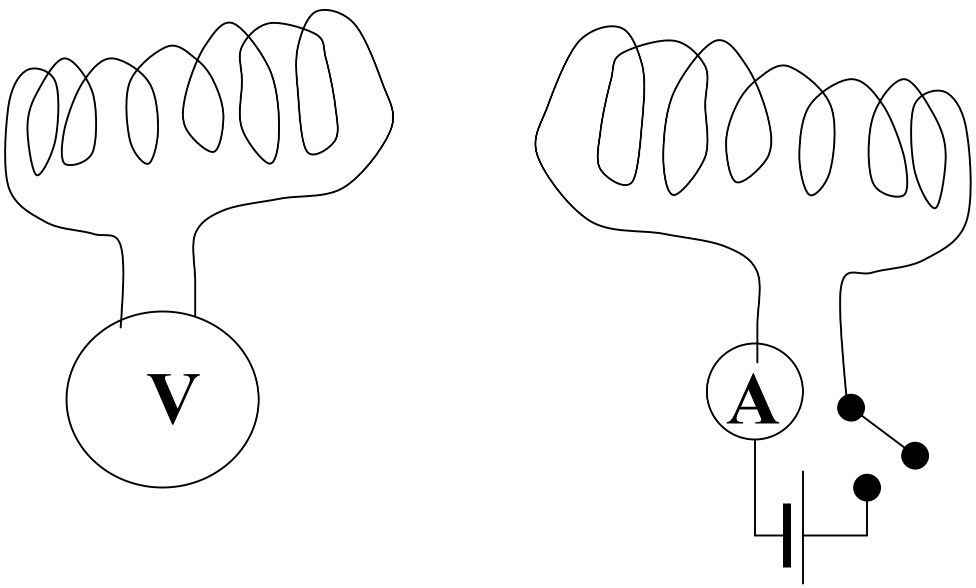
Sorry, we can't do it in this packed room … but here is the essence of it

EMF induced in a coil by moving a bar magnet



EFM depends on how strong magnet and how fast we move in/out

EMF induced in a secondary coil by changing current in primary coil



Magnetic Flux

We define ***magnetic flux*** Φ exactly as we defined the flux of the electric field. The idea is the number of lines of ***B*** that pass through an area.

$$\Phi = \int \vec{B} \cdot d\vec{A}$$

Simple case #1: uniform ***B***, $\perp$ surface: $\Phi = BA$

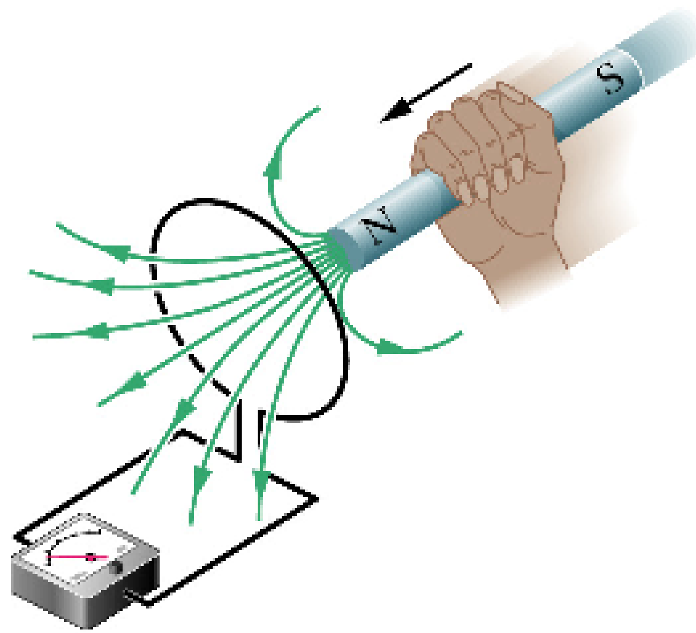
Simple case #2: surface is ***closed:*** $\Phi = 0$

Faraday's Law

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$

The emf induced in any loop or circuit is equal to the negative rate of change of the magnetic flux through that loop.

Voltmeter reading gives ***rate of change*** of the number of lines linking the loop.



Changing Magnetic Flux

$$\Phi = \int \vec{B} \cdot d\vec{A}$$

How can we get a time-changing flux, so that $\mathcal{E} = -\frac{d\Phi}{dt} \neq 0$?

- **Change the field: $\Phi = B(t)\ A$**
- **Change the area: $\Phi = B\ A(t)$**
- **Change the angle: $\Phi = B\ A \cos\theta(t)$**

Example 1

A circle of radius 20 cm in the *xy* plane is formed by a wire and a 3-ohm resistor. A uniform magnetic field is in the *z* direction; its magnitude decreases steadily from .08 tesla to 0 in a time of 4 seconds.

What emf is generated?

$$A = \pi r^2 = 0.13\, m^2 \qquad \frac{dB}{dt} = -\frac{.08\,T}{4\,s} = -.02\,T/s$$

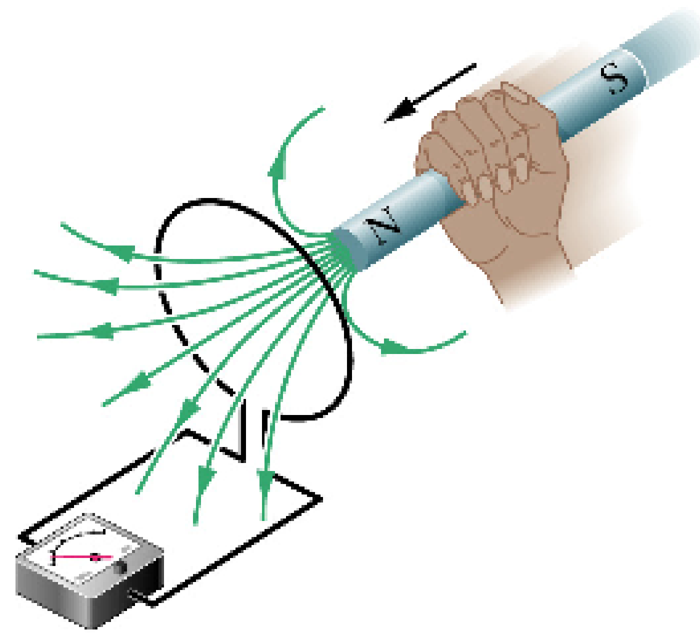
$$\mathcal{E} = -\frac{d\Phi}{dt} = -A\frac{dB}{dt} = -(.13)(-.02) = 2.6\times 10^{-3}\,V$$

Lenz's Law

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

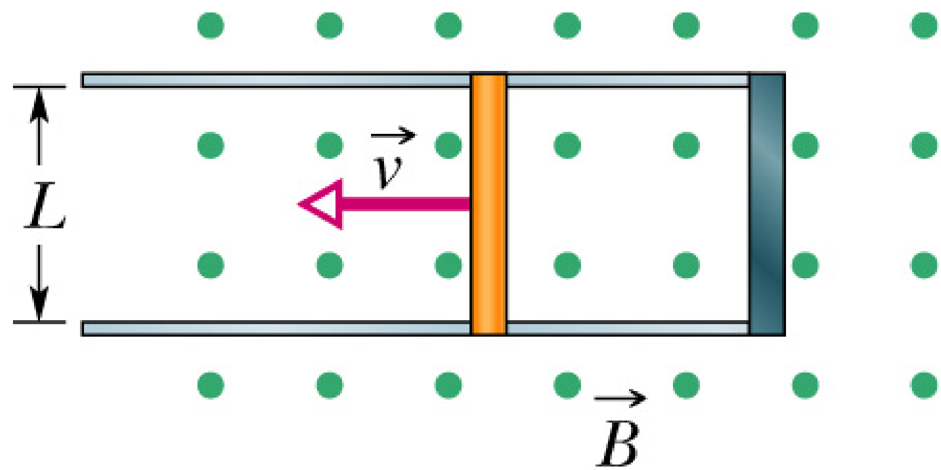
The ***direction*** of the induced emf is such as to create a current which will ***oppose the change*** in the flux.

Motion as shown produces clockwise current which makes B field opposing the increase.



Example 2

I push a rod along metal rails through a uniform magnetic field.

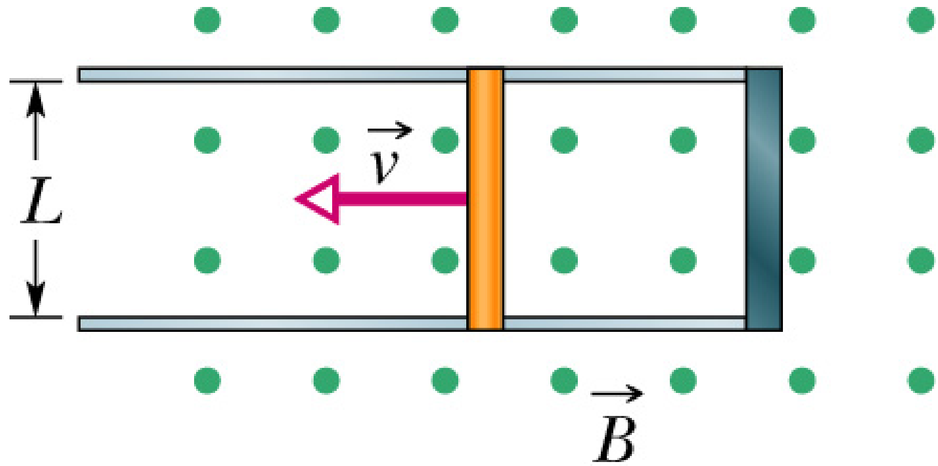


(a) What emf is generated?

(b) What current will flow?

(c) What power must I supply?

Example 2a



L = 20 cm

V = 3.0 m/s

B = .05 T

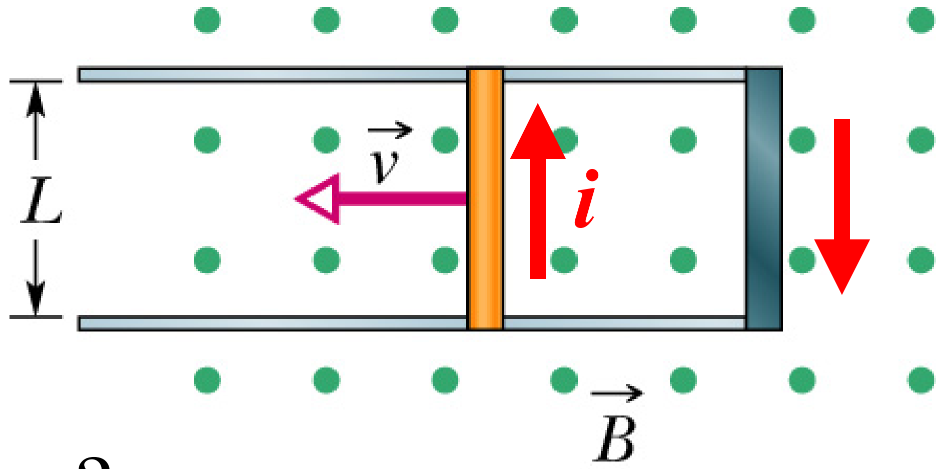
(a) What emf is generated?

$$\frac{dA}{dt} = L\frac{dx}{dt} = Lv = 0.6\,m^2/s$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -B\frac{dA}{dt} = -.05 \times 0.6 = -30\,mV$$

Example 2b

Resistance of bar: R = 15 Ω



(b) What current will flow?

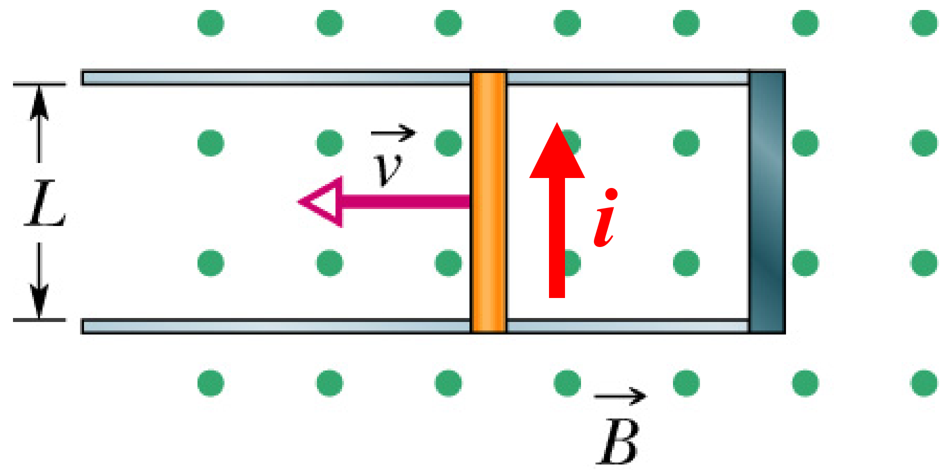
$$i = \frac{\mathcal{E}}{R} = \frac{-30\times 10^{-3}\,V}{15\,\Omega} = -2\,mA$$

Which direction does current flow?

Forget the minus sign. **Use Lenz's Law!**

Flux is ***increasing outward.*** Therefore current will ***resist that change*** by flowing ***clockwise.***

Example 2c



(c) What power must I supply?

Magnetic force: $\vec{F} = i\vec{L} \times \vec{B}$

$$F = .002 \times .2 \times .05 = 2 \times 10^{-5}\ N$$

Power: $P = Fv = (2 \times 10^{-5}\ N)(3\, m/s) = 6 \times 10^{-5}\ W$

Check Joule heating: $P = i^2 R = 6 \times 10^{-5}\ W$

Faraday's Law: General Form

For any closed curve C
and the surface S bounded by C:

$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

$\boxed{\mathcal{E}}$ $\boxed{\Phi}$

Inductance

- For any coil of wire, there is a flux Φ through the coil, which is proportional to the current.

- If that changes, Faraday's Law requires an emf ***induced*** in the coil, proportional to the ***rate of change*** of the flux.

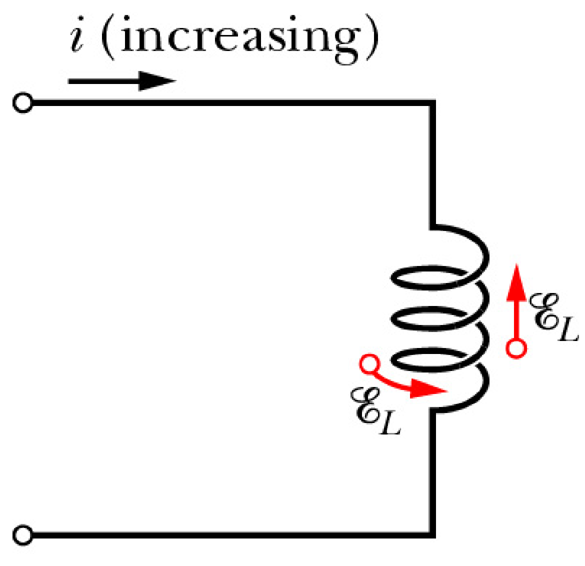
- Clearly $\Phi \propto$ i and so $\mathcal{E} = -\frac{d\Phi}{dt} \propto -\frac{di}{dt}$

- Define the proportionality constant to be the ***inductance L:*** $\boxed{\mathcal{E} = -L\frac{di}{dt}}$

- SI unit of inductance is the **henry** (**H**).

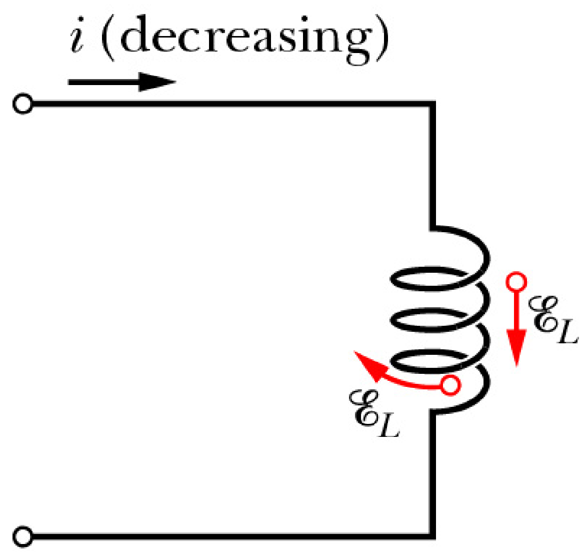
Inductors

If current is ***increasing,*** the induced emf acts against the increase, giving a voltage ***drop.***



(a)

If current is ***decreasing,*** the induced emf acts against the decrease, giving a voltage ***rise.***



(b)

Energy in an Inductor

The energy stored in an inductor equals the work required to set up the current.

$$dW = Vdq = V\frac{dq}{dt}dt = (L\frac{di}{dt})idt = Lidi$$

$$W = \int dW = L\int_0^I idi = \tfrac{1}{2}LI^2$$

So energy stored in an inductor is $\boxed{U = \tfrac{1}{2}Li^2}$

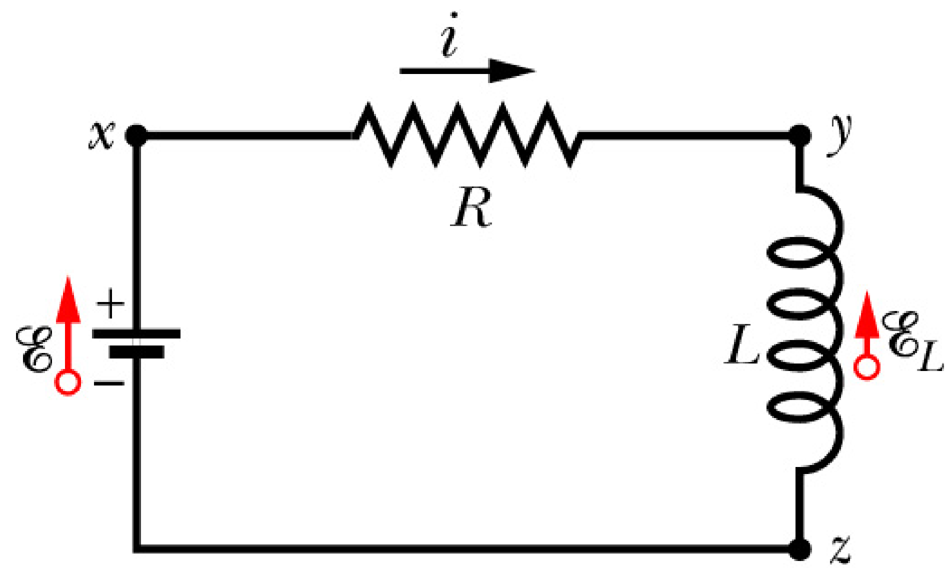
Magnetic Field Energy

The energy stored in an inductor is contained in the ***magnetic field.*** The general formula for the ***energy density*** in any magnetic field is

$$u = \frac{B^2}{2\mu_0}$$

Inductors and Resistors

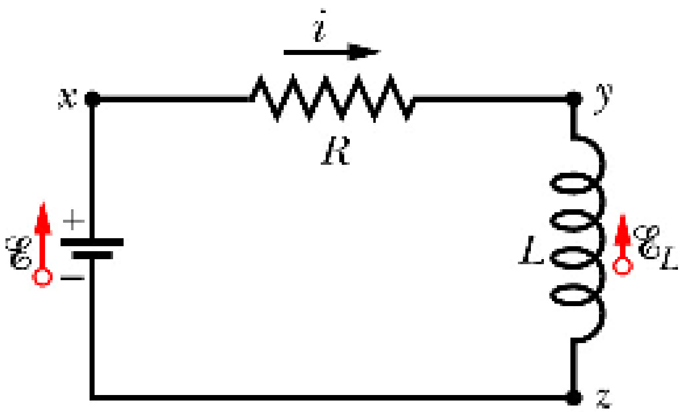
Voltage changes going clockwise around this loop:



$$+\mathcal{E} - iR - L\frac{di}{dt} = 0$$

Inductor gives voltage ***drop*** if current is ***increasing***.

RL Circuits



$$+\mathcal{E} - iR - L\frac{di}{dt} = 0$$

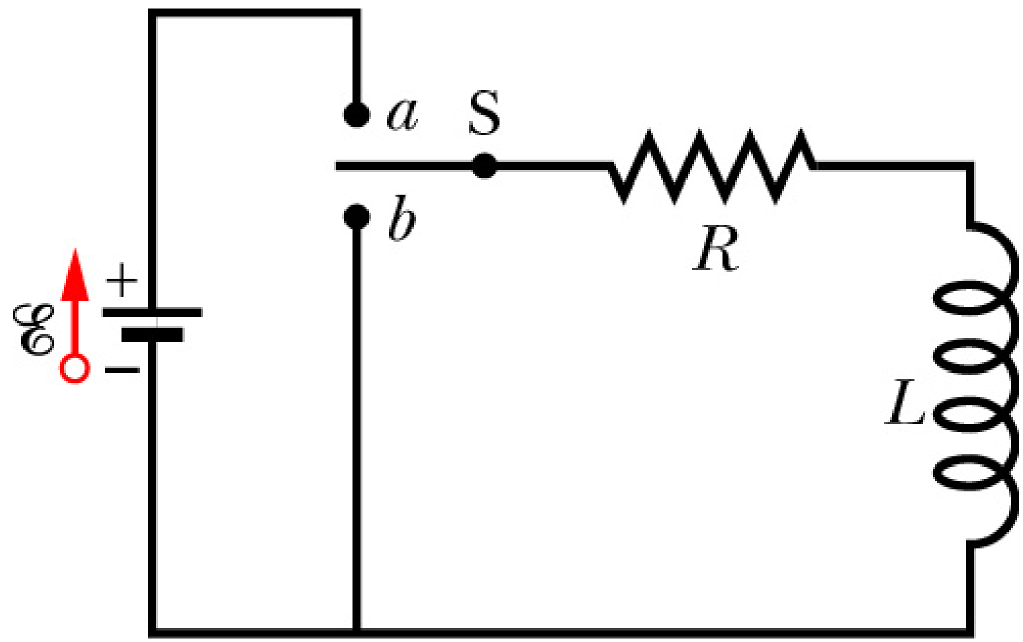
$$L\frac{di}{dt} + Ri = \mathcal{E}$$

Same equation as for charging a capacitor!

Try same kind of solution: $$i = \frac{\mathcal{E}}{R}\left\{1 - e^{-t/\tau}\right\}$$

This works, provided $$\boxed{\tau = L/R}$$

RL Summary



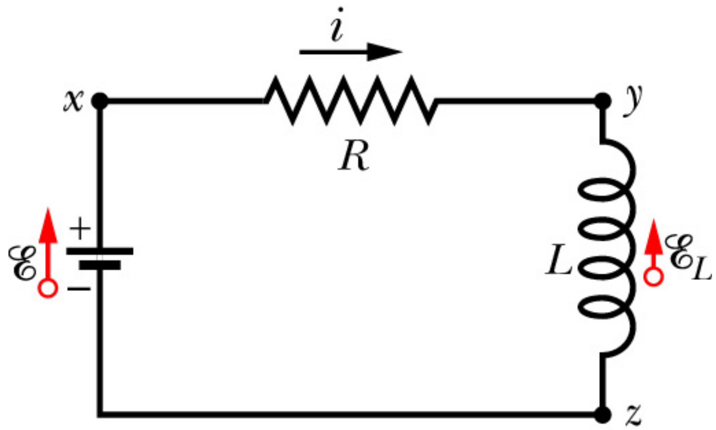
Set switch to position a:

$$i = \frac{\mathcal{E}}{R}\left\{1 - e^{-t/\tau}\right\}$$

Set switch to position b: $i = \frac{\mathcal{E}}{R} e^{-t/\tau}$

In either case time constant is: $\boxed{\tau = L / R}$

Example



$$\mathcal{E} = 30\,V$$

$$R = 5000\,\Omega$$

$$L = 15\,mH$$

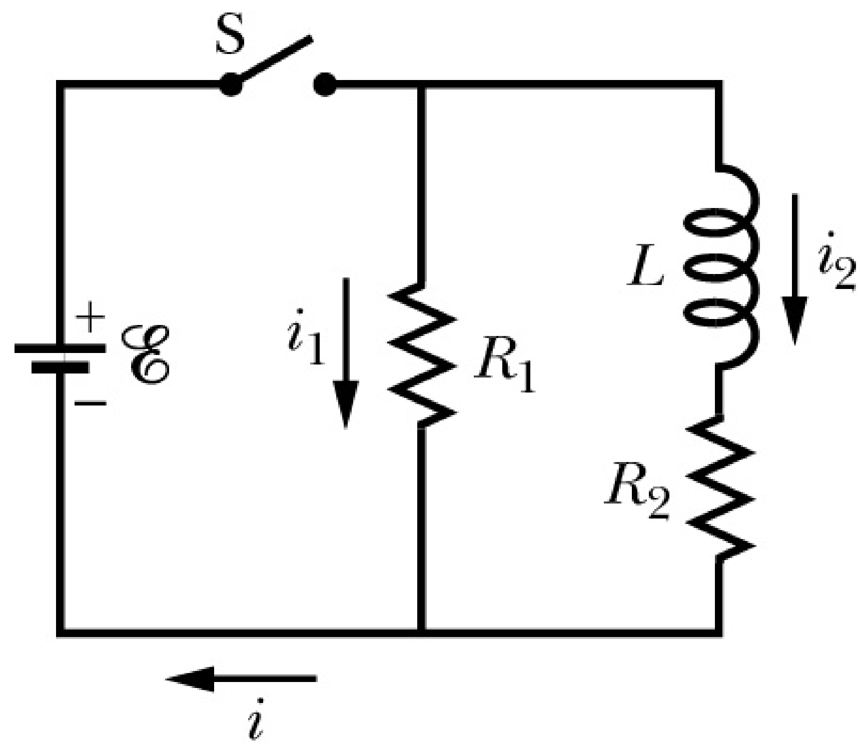
(a) What is the time constant?

$$\tau = L/R = \frac{15\times 10^{-3}}{5\times 10^{3}} = 3\times 10^{-6} = 3\,\mu s$$

(b) What is current after 1 second?

$$i = \frac{\mathcal{E}}{R}\left\{1 - e^{-t/\tau}\right\} = \frac{30}{5000}\left(1-0\right) = 6\,mA$$

Example 2:
Problem 30-89



(a) What happens ***immediately*** after switch is closed?

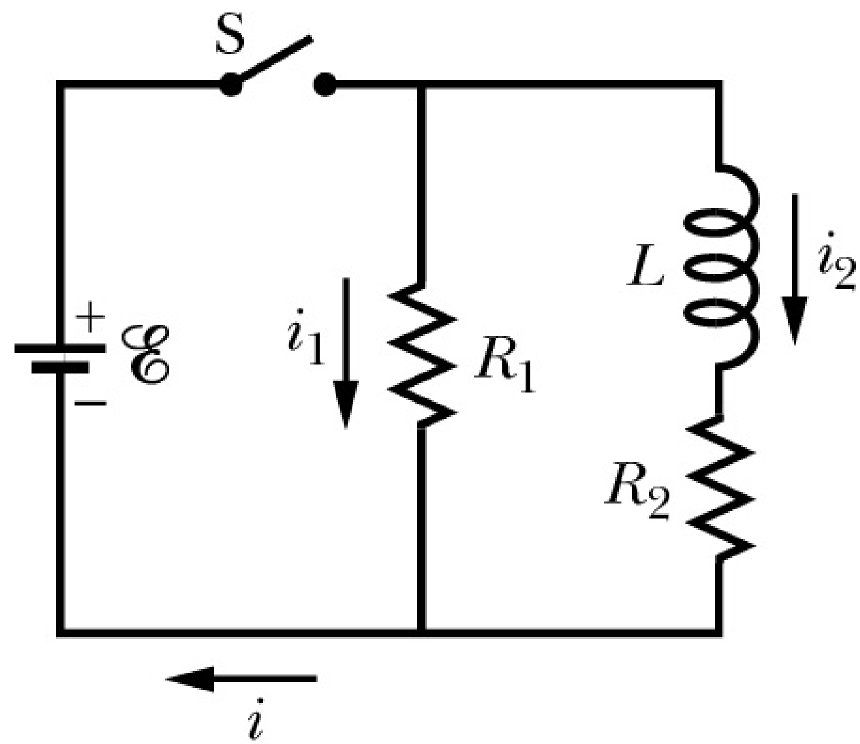
L prevents sudden change so:

$$i_2 = 0 \qquad \therefore \quad i = i_1 = \mathcal{E} / R_1$$

So: $V_{R2} = 0 \qquad \therefore \quad V_L = \mathcal{E}$ and $\dfrac{di_2}{dt} = \mathcal{E} / L$

Example 2
continued

(b) What happens a ***long time*** after switch is closed?



We have reached a steady state so:

$$\frac{di_2}{dt} = 0 \qquad \therefore \quad V_L = 0 \quad \text{and} \quad V_{R2} = \mathcal{E}$$

So: $\;i_1 = \mathcal{E} / R_1 , \quad i_2 = \mathcal{E} / R_2 , \quad i = i_1 + i_2$