Capacitance

Chapter 25 homework:

Questions 1, 10
Problems 1, 9, 25, 39
WileyPlus assignment

Midterm Exam Thursday

Electrostatics: Chapters 21-25

Your Clicker

❖ Click your reply (just like a T.V. remote) from anywhere in the classroom by selecting a letter or number response on your clicker.

❖ Battery: Powered by two coin cell CR2032 (3.0V) Lithium Batteries. Batteries typically will last over 1 year of clicker use.

❖ Keypad displays solid green for 3 seconds when response is received as verification for a clear, visual confirmation that your response has been received and recorded.

Channel Setting for the
ResponseCard® RF

1. Press and release the "GO" button.
2. While the light is flashing red and green, Enter 2 digit channel code.
   For Physics class,enter 15 for channel 15
3. After the second digit is entered, Press and release the "GO" button.
4. Press and release the "1/A" button. The light should flash yellow to confirm.

Tell me your clicker ID number

- **Use it as your student ID number on WileyPlus**

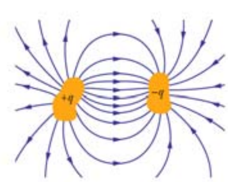
OR

- **If you can't do that for some reason send me an email containing your full name and your clicker ID number**

Capacitors

- **Capacitors store charge and energy.**
- **Definition of Capacitance: C = Q/V**
- **Calculate stored energy:** $$U = \frac{Q^2}{2C}$$

Two Conductors Form a Capacitor



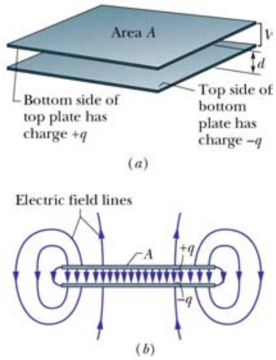
$$\Delta V = \int_{-q}^{+q} \vec{E} \cdot d\vec{s}$$

Given the amount of charge q.
Let V be the potential difference ΔV.
Define the ***capacitance*** C = q/V.
SI unit is the farad: 1 F = 1C/V

Case 1: The Parallel-plate Capacitor

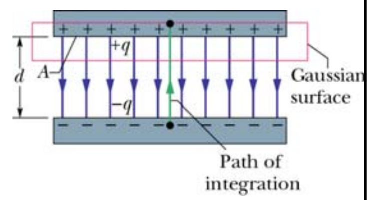
The simplest capacitor is made of two large parallel metal plates very close together.

Because of its construction, we see that all the charge is on the inner surfaces.



Parallel-Plate Capacitor

$$q = \sigma A$$
$$E = \sigma / \varepsilon_0$$
$$V = Ed$$
$$C = q / V = \frac{\sigma A}{\sigma d / \varepsilon_0} = \frac{\varepsilon_0 A}{d}$$



Potential Energy

- I must do work to charge up a capacitor.
- This energy is stored in the form of ***electric potential energy.***
- We will show that this is $U = \frac{Q^2}{2C}$
- Then we will see that this energy is stored in the electric field, with a volume ***energy density***

$$u = \tfrac{1}{2}\varepsilon_0 E^2 \qquad (J/m^3)$$

Derivation of Capacitor Energy

- If capacitor has charge q and I ***add*** charge dq, then I must do ***work*** $dW = Vdq$

*(Remember the **definition** of potential difference!)*

- For each dq I add, I must work harder because there is a stronger field against me. If I start with q=0 and end with q=Q, then my total work is

$$W = \int Vdq = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

Energy density of the electric field

- **Special case of parallel-plate capacitor:**

$$U = Q^2 / 2C = \tfrac{1}{2} CV^2 = \tfrac{1}{2} CE^2 d^2$$
$$But \quad C = \varepsilon_0 A / d \quad and \quad Vol. = Ad$$
$$\therefore \quad U = \tfrac{1}{2}(\varepsilon_0 A / d) E^2 d^2 = \tfrac{1}{2}\varepsilon_0 E^2 Ad$$

And so we get $\boldsymbol{u = (U)/(Vol.) = \tfrac{1}{2}\varepsilon_0 E^2}$

- Turns out to be true in general: an electric field ***always*** carries this energy density

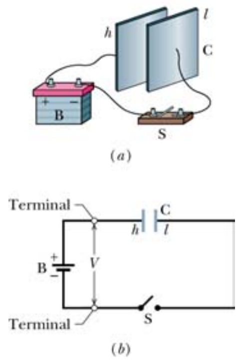
A Numerical Example

1. **Suppose we have 2 metal plates, 20cm by 30cm, set parallel to each other, separated by a gap of 3 mm. What will be the capacitance of this setup?**

$$C = \frac{\varepsilon_0 A}{d} = \frac{9 \times 10^{-12} \times 0.2 \times 0.3}{0.003}$$

$$C = 1.8 \times 10^{-10}\, F = 0.18\, nF = 180\, pF$$

2. Now suppose we connect a 12-volt battery between these two plates, so that the potential difference across this capacitor is V = 12 volts. **How much charge will be on the plates?**



2. If the potential difference is 12 volts how much charge will be on the plates?

Use the definition of capacitance:

$$C = q / V$$

$$\therefore \quad q = CV = (0.18\,nF)\times(12\,V) = 2.16\,nC$$

Thus the positive plate will have +2.16 nC and the negative plate will have –2.16 nC.

3. What is the strength of the electric field between the plates?

Method A: Remember V is work to move 1 C between plates, force times distance, so

$$V = Ed$$

$$E = V / d = 12V / .003m = 4000\,V / m$$

Method B: From Gauss's Law E=σ/ε_0, so

$$E = (Q / A) / \varepsilon_0 = \frac{2.16\times10^{-9}}{.2\times.3\times9\times10^{-12}} = 4000\,V / m$$

4. How much energy is stored?

$$U = Q^2 / 2C$$

$$U = \frac{(2.16\,nC)^2}{2\times(180\,pF)} = \frac{4.67\times10^{-18}\ SI}{360\times10^{-12}\ SI} = 13\,nJ$$

5. What voltage would be required to store 1 joule of energy?

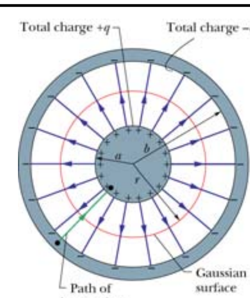
$$U = Q^2 / 2C \quad and \quad Q = CV$$

$$\therefore \quad U = \tfrac{1}{2} CV^2$$

$$So \quad V = \sqrt{2U / C} = \sqrt{2\times1 / 1.8\times10^{-10}}$$

$$= 1.1\times10^5\,V = 110\,kV$$

Case 2: The Spherical Capacitor



Use Gauss to find E(r):

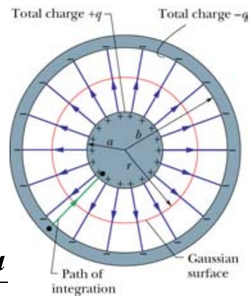
$$\Phi_{out} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(r)\cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E(r) = \frac{q}{4\pi r^2 \varepsilon_0} = \frac{kq}{r^2}$$

$$V = \int_a^b E(r)\,dr$$

Spherical Capacitor Continued



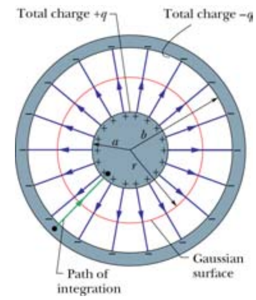
$$V = \int E dr = \int_a^b \frac{kq}{r^2} dr$$

$$= kq\left(\frac{1}{a} - \frac{1}{b}\right) = kq\frac{b-a}{ab}$$

$$\therefore \quad C = q/V = \frac{ab/k}{b-a} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

Case 3: The Cylindrical Capacitor

Application: capacitance per unit length of a coaxial cable.



Cross section of cable

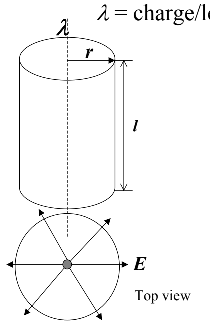
λ = charge per unit length

$$V = \int_a^b E(r) dr$$

What is E(r) in this case? Again use Gauss's Law.

Review: Long Line of Charge

λ = charge/length = linear charge density



$$\Phi = \frac{Q}{\varepsilon_0}$$ Gauss' law

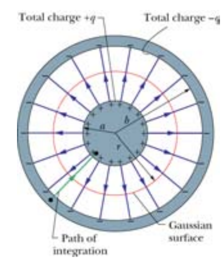
$$Q = l\lambda$$

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = EA$$

$$A = 2\pi r l$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

Back to Cylindrical Capacitor

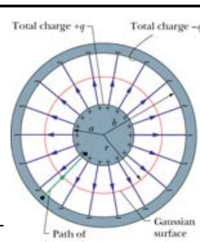


$$V = \int_a^b E(r) dr$$

$$= \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}[\ln r]_a^b = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}$$

Finish Cylindrical Capacitor



Capacitance per unit length:

$$C = q/V$$

$$\text{so} \quad \frac{C}{L} = \frac{\lambda}{V} = \frac{\lambda}{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}}$$

$$= \frac{2\pi\varepsilon_0}{\ln(b/a)}$$