Magnetic Fields

- Ch.28: The magnetic field: Lorentz Force Law
- Ch.29: Electromagnetism: Ampere’s Law

HOMEWORK

- Read Chapters 28 and 29
- Do Chapter 28 Questions 1, 7
- Do Chapter 28 Problems 3, 15, 33, 47
Today

- The Magnetic Field B.
  - Field lines
  - Direction: compass needle
- Gauss’s Law for B.
- The Lorentz Force.
- Force on current-carrying wire.
- Motion of charged particles in uniform B field.
- Vector cross product and right-hand rule!
The Magnetic Field

- Another vector field $\vec{B}(F)$.
- Lines of $\vec{B}$:
  - Direction indicated by compass needle.
  - Never begin or end.
  - Density indicates field strength.
Bar magnets

[Dipole Fields]
Bar magnets

[DIPOLE FIELDS]

N + S Poles
Like poles repel
Unlike poles attract

However:
Monopoles do not exist! Only dipoles!
Bar magnets

[DIPOLAR FIELDS]
Gauss’s Law for Magnetism

• Outward *electric* flux = enclosed charge
• Outward *magnetic* flux = zero.
• “There are no magnetic monopoles.”
• This is Maxwell Equation #2:

\[ \oint \vec{B} \cdot d\vec{A} = 0 \]
Bar magnet

Possible closed Gaussian surfaces shown in red. Zero net outward flux in both cases.
Magnetic Field of Earth

Approximately a dipole field.
The Magnetic Force

If a particle with electric charge $q$ moves with velocity $\mathbf{v}$ through a magnetic field $\mathbf{B}$, then the force by the field on the particle is

$$\mathbf{F} = q \mathbf{v} \times \mathbf{B}$$

Unlike the electric force $F = qE$, the magnetic force on a charged particle is **NOT** in the direction of the magnetic field. In fact, it is *perpendicular to it*. 
Cross Product of Two Vectors

Given any two vectors \( \mathbf{A} \) and \( \mathbf{B} \), and \( \theta \) the angle between them, we define the vector product (cross product)

\[
\mathbf{C} = \mathbf{A} \times \mathbf{B} : \\
\]

(1) \( C = AB \sin \theta \)

(2) \( \mathbf{C} \) is perpendicular to both \( \mathbf{A} \) and \( \mathbf{B} \)

(3) The direction of \( \mathbf{C} \) is given by the righthand rule
The Right-Hand Rule

\[
\vec{C} = \vec{A} \times \vec{B}
\]

If \( \vec{C} = \vec{A} \times \vec{B} \)

We use the right-hand rule to find the direction of the vector \( \vec{C} \).

Use the fingers of your right hand to rotate \( \vec{a} \) toward \( \vec{b} \), then your thumb points in the direction of \( \vec{a} \times \vec{b} \).
Given vectors $\vec{A}$ and $\vec{B}$ with angle $\theta$ between them. For fixed magnitudes $A$, $B$, for what value of $\theta$ will the value of $\vec{A} \times \vec{B}$ be a maximum?

1) 0°
2) 30°
3) 45°
4) 60°
5) 90°
Q.28-1

Given any two vectors $A$ and $B$, and $\theta$ the angle between them, we define the magnitude of the cross product as

$$ C = AB \sin \theta $$

But $\sin \theta$ has its maximum value when $\theta = 90^\circ$

So if $A$ and $B$ are perpendicular the magnitude of the cross product is a maximum, and is just $AB$. 
Suppose I have vector \( \mathbf{A} \) pointing to the east and vector \( \mathbf{B} \) pointing to the north.

What is the direction of \( \mathbf{A} \times \mathbf{B} \)?

1) North
2) South
3) East
4) West
5) Up
6) Down
Suppose I have vector $\vec{A}$ pointing to the east and vector $\vec{B}$ pointing to the north.

What is the direction of $\vec{A} \times \vec{B}$?

1) North
2) South
3) East
4) West
5) Up
6) Down
The Right-Hand Rule

If you use the fingers of your right hand to rotate $\mathbf{v}$ toward $\mathbf{B}$, then your thumb points in the direction of $\mathbf{v} \times \mathbf{B}$.

So in the figure the force on $+q$ is upward, but the force on $-q$ is downward.
Units

The SI unit for the magnetic field is the *tesla* (T). Since $F = qvB$, we have $B = F/qv$ and $1\text{T} = 1\text{Ns}/\text{Cm}$.

Another unit sometimes used is the *gauss* (G).

$$1\text{T} = 10^4 \text{G}$$

The field of the earth is typically about 1 G.
Force on Current-carrying Wire

For wire perpendicular to B we have

\[ F = qvB \]

But \[ i = \left( \frac{q}{L} \right) v \]

So \[ F = qvB = BiL \]

Force on length L

If B is at angle \( \theta \) with wire: \[ F = BiL \sin(\theta) \]
The Magnetic Force

If a particle with electric charge $q$ moves with velocity $\vec{v}$ through a magnetic field $\vec{B}$, then the force by the field on the particle is

$$\vec{F} = q \vec{v} \times \vec{B}$$

If a wire of length $L$ carries a current $i$ through a field $B$, the force by the field on the wire is

$$\vec{F} = i \vec{L} \times \vec{B}$$
Example

Given wire in field with angle $\phi = 37^\circ$. $B = 0.3T$, $i = 20$ mA. Find force per unit length on wire.

(1) Direction of force: By right-hand rule, force is upward as shown.

(2) Magnitude of force:

$$F = i |\vec{L} \times \vec{B}| = iLB \sin \phi$$

$$F / L = iB \sin \phi = .02 \times .3 \times .6 = 3.6 \times 10^{-3} \text{ N} / \text{m}$$
Electron gun: potential $V$ gives electron energy in eV.

$$K = \frac{1}{2}mv^2 = qV$$

So if $V = 500$ volts, electron energy is $K = 500$ eV.
Crossed Fields

Crossed $E$ and $B$ fields:

If $qE = qvB$ then $F = 0$

To deflect beam upward, increase E.
Forces on electron.

\[ F_B = e\nu B \]

\[ F_E = eE \]

\[ F = F_E - F_B = e(E - \nu B) \]

If \( \nu = \frac{E}{B} \), then \( F = 0 \)
Positive ion enters field with energy

\[ K = \frac{1}{2}mv^2 = qV \]

So \( F \) does no work so \( K \) remains constant and so as the ion moves through field its \textit{speed} remains constant.
Charge in Uniform B Field

B is *out of* screen so F is *toward center* so ion moves in circle with constant speed.

Centripetal acceleration is $a = v^2 / r$

\[ \therefore F = qvB = ma = mv^2 / r \]

So solving for $r$ gives the radius of curvature of the path:

\[ r = \frac{mv}{qB} \]
Charge in Uniform B Field

Applications

- Mass Spectrometer
- Isotope Separator
- Particle Accelerator
Electromagnetic Fields

• Ch.28: The magnetic field: Lorentz Force Law
• Ch.29: Electromagnetism: Ampere’s Law

• Chapter 28 Questions 1, 7
• Chapter 28 Problems 3, 15, 33, 47
TODAY: Electromagnetism

Production of magnetic field by a current

- $B$ field due to a current in a long straight wire
- $B$ field due to a current in a short bit of wire
- **Ampere’s Law**: the third of Maxwell’s Equations
Field Due to a *Long Straight Wire*

Lines of $B$ make circles around wire!
BUT FIRST REVIEW:
The Lorentz Force

If a particle with electric charge $q$ moves with velocity $v$ through a magnetic field $B$, then the force by the field on the particle is

$$\vec{F} = q \vec{v} \times \vec{B}$$

If a wire of length $L$ carries a current $i$ through a field $B$, the force by the field on the wire is

$$\vec{F} = iL \times \vec{B}$$
REVIEW: The cross product

Given vectors \( \mathbf{v} \) and \( \mathbf{B} \), and \( \theta \) the angle between them, we define the vector product (cross product) \( \mathbf{v} \times \mathbf{B} \):

1. Magnitude is \( vB\sin\theta \).
2. Right-hand rule gives direction, perpendicular to both \( \mathbf{v} \), \( \mathbf{B} \).
Example

Given wire in field with angle $\phi = 37^\circ$. $B = 0.3\, \text{T}$, $i = 20\, \text{mA}$. Find force per unit length on wire.

(1) Direction of force:
By right-hand rule, force is upward as shown.

(2) Magnitude of force:
\[ F = i|\vec{L} \times \vec{B}| = iLB \sin \phi \]

\[ F / L = iB \sin \phi = 0.02 \times 0.3 \times 0.6 = 3.6 \times 10^{-3}\, \text{N} / \text{m} \]
An electron with speed $v$ enters a magnetic field $B$ as shown. What is the direction of the force on the electron?

1) Out of the screen
2) Into the screen
3) In the direction of $B$
4) In the direction of $v$
What is the direction of the force on the electron?

Solution: \( \vec{F} = q \, \vec{v} \times \vec{B} \)

\( \vec{v} \times \vec{B} \) is out of screen by RH rule. But \( q \) is negative, so \( F \) is into the screen.

(1) Out of the screen. \hspace{1cm} (2) Into the screen.

(3) In the direction of \( B \). \hspace{1cm} (4) In the direction of \( v \).
REVIEW: Charge in Uniform Field

B is out of screen so F is toward center so ion moves in circle with constant speed.

Centripetal acceleration is $a = \frac{v^2}{r}$

$\therefore F = qvB = ma = \frac{mv^2}{r}$

So solving for $r$ gives the radius of curvature of the path:

$r = \frac{mv}{qB}$
Q.28-4

An ion with charge $+e$ and mass $M_1$ follows the dashed path in given $B$ field.

Another ion with charge $+e$, but a different mass $M_2$ enters the field with the same velocity as the first, and follows the blue path. How do the masses compare?

(1) $M_1 > M_2$  (2) $M_2 > M_1$  (3) Can’t say.
Two ions, same q, v.

M1 follows dashed path.

M2 follows blue path.

How do their masses compare?
Q.28-4

Two ions with equal charges and velocities follow the two curves shown.

\[ r = \frac{mv}{qB} \]

\( M_1 \) follows the dashed, \( M_2 \) the blue path. How do the masses compare?

Blue radius is smaller so blue mass is smaller.

(1) \( M_1 > M_2 \)  (2) \( M_2 > M_1 \)  (3) Can’t say.
Field Due to a *Long Straight Wire*

*Lines of B make circles around wire!*
Field of a long straight wire

1. Direction is given by the right-hand rule!

2. Magnitude is \( B = \frac{\mu_0 i}{2\pi r} \)

3. New universal constant:
\[
\mu_0 = 4\pi \times 10^{-7} \text{ Tm} / \text{A}
\]
Example

What is the magnetic field at point \( P \)?

**Direction:** Out of the screen by right-hand rule.

**Magnitude:**

\[
B = \frac{\mu_0 i}{2\pi r} = \frac{4\pi \times 10^{-7} \times 3.0}{2\pi \times 0.20} = 3.0 \times 10^{-6} \ T
\]
Field Due to a Short Bit of Wire

Recall Coulomb: \( E \) is parallel to \( r \).

But as usual for magnetism, we find \( B \) is perpendicular to \( r \)!

\[
d\mathbf{B} \propto i \, d\mathbf{s} \times \mathbf{r}
\]

Another right-hand rule!
The Biot-Savart Law

\[ d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \vec{r}}{r^3} \]

- Field \( d\vec{B} \) is perpendicular to both \( ds \) and \( r \).
- Inverse square law like Coulomb’s Law.
- Universal constant:
  \[ \mu_0 = 4\pi \times 10^{-7} \ Tm / A \]
Example

Field at the center of a circular loop of wire.

Direction: Into the screen, by the right-hand rule.

Magnitude: Must add up (integrate) all the little $dB$ from all the little $ds$.

\[
B = \int dB = \frac{\mu_0}{4\pi} \frac{i}{r^2} \int ds = \frac{\mu_0}{4\pi} \frac{i}{r^2} 2\pi r = \frac{\mu_0 i}{2r}
\]
Ampere’s Law
\[ \oint_C \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \]

- $C =$ Any closed path
- $i_{\text{enc}} =$ Net current linking $C$ (*Right-hand rule*)
- $B =$ The total magnetic field
- $ds =$ A short step along the path

This is the third of Maxwell’s equations.
Field Due to a *Long Straight Wire*

1. Direction is given by the right-hand rule!

2. Magnitude is found by applying Ampere’s Law to a circular path of radius $r$:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i$$

$$B \cdot \oint ds = B \cdot 2\pi r = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r}$$
Example: Field in Coaxial Cable

Cable perpendicular to screen
Central wire: $i$ inward
Outer cylinder: $i$ outward

(b) Field outside the cable?

\[
\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}
\]

\[
B \cdot \oint ds = B \cdot 2\pi r = \mu_0 (i - i)
\]

$B = 0$

Outer conductor prevents field from escaping!
Example: Field in Coaxial Cable

Cable perpendicular to screen
Central wire: $i$ inward
Outer cylinder: $i$ outward

(a) Field between conductors?

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{enc}$$

$$B \cdot \oint ds = B \cdot 2\pi r = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r}$$

Just as if outer conductor did not exist!
Field Due to a Current Loop

Serway, College Physics, 5/e
Text Figure 19.28a,b

(a)

(b)
Magnetic Dipole Field

Serway, College Physics, 5/e
Text Figure 19.30a

Harcourt Brace & Company
Dipole Moment of a Current Loop

Definition: Magnetic dipole moment vector: $\vec{\mu}$

- Direction: RH rule
- Magnitude: $\mu = iA$

Analogous to electric dipole moment vector $\vec{p}$