Magnetic Fields

- **Ch.28: The magnetic field: Lorentz Force Law**
- **Ch.29: Electromagnetism: Ampere's Law**

HOMEWORK

- **Read Chapters 28 and 29**
- **Do Chapter 28 Questions 1, 7**
- **Do Chapter 28 Problems 3, 15, 33, 47**

Today

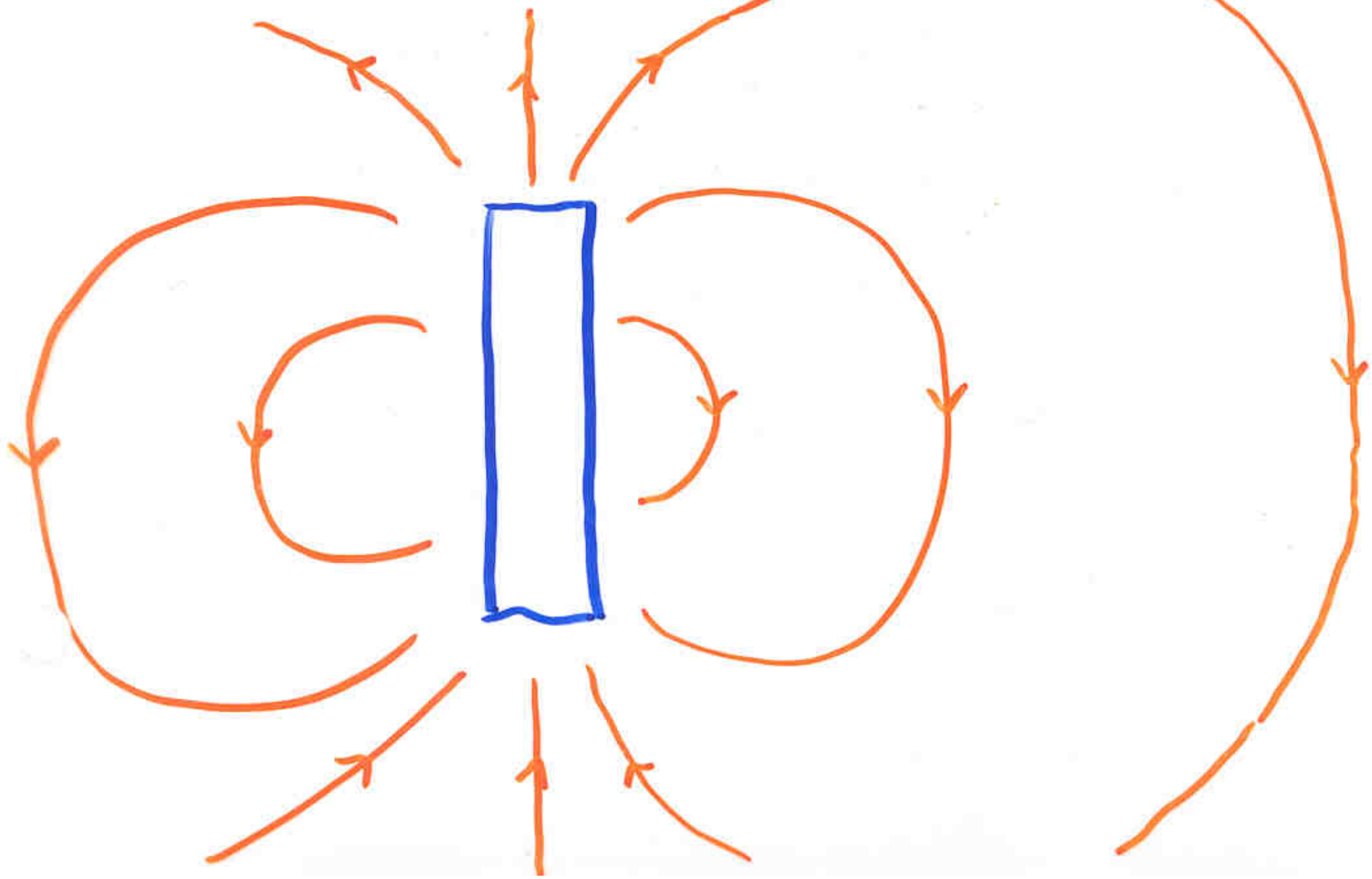
- The Magnetic Field B.
  - Field lines
  - Direction: compass needle
- Gauss's Law for B.
- The Lorentz Force.
- Force on current-carrying wire.
- Motion of charged particles in uniform B field.
- Vector cross product and right-hand rule!

The Magnetic Field

- Another <u>vector</u> <u>field</u> $\vec{B}(\vec{r})$.
- Lines of $\vec{B}$:
  - Direction indicated by compass needle.
  - Never begin or end.
  - Density indicates field strength.

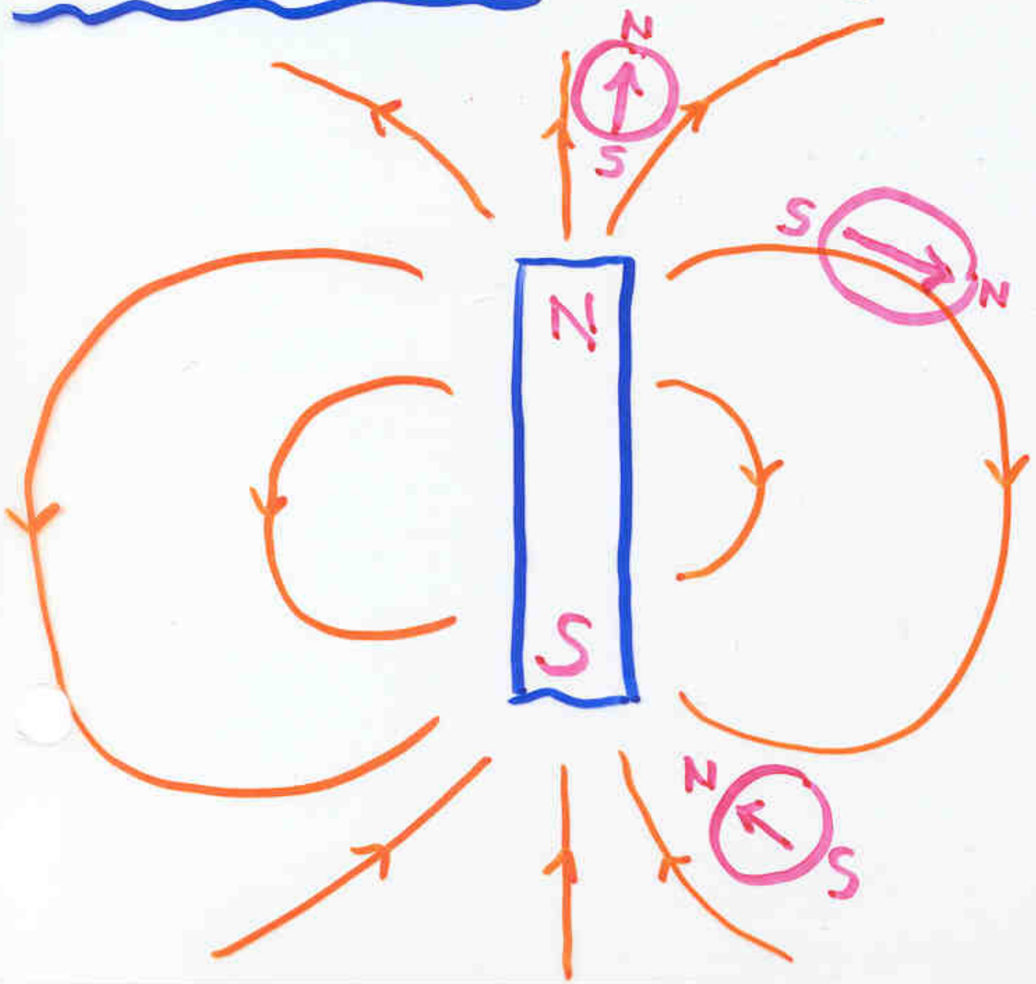
Bar magnets

[DIPOLE FIELDS]



Bar magnets

[DIPOLE FIELDS]



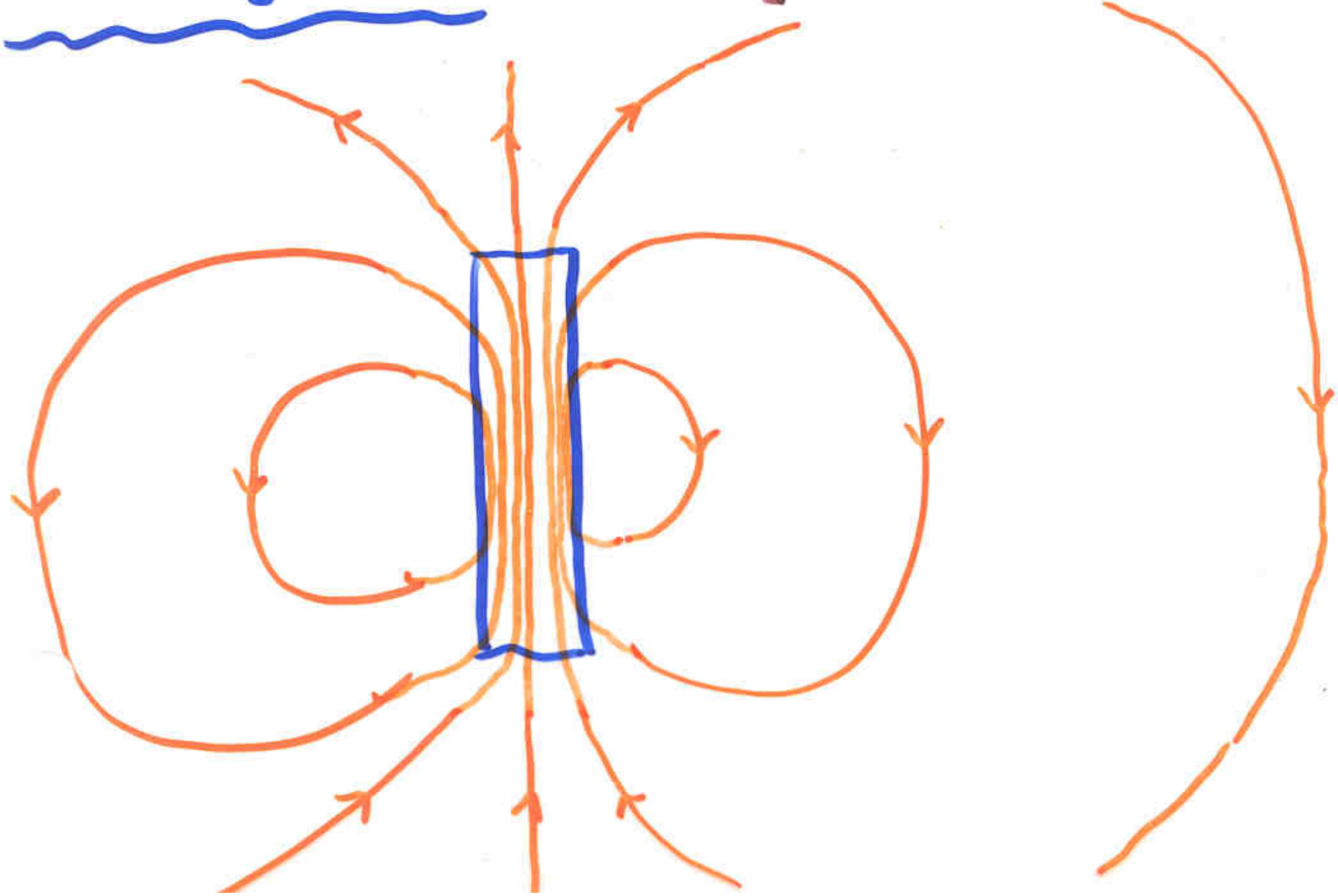
N + S Poles
Like poles repel
Unlike poles attract

HOWEVER:

Monopoles do not exist! Only DIPOLES!

Bar magnets

[DIPOLE FIELDS]



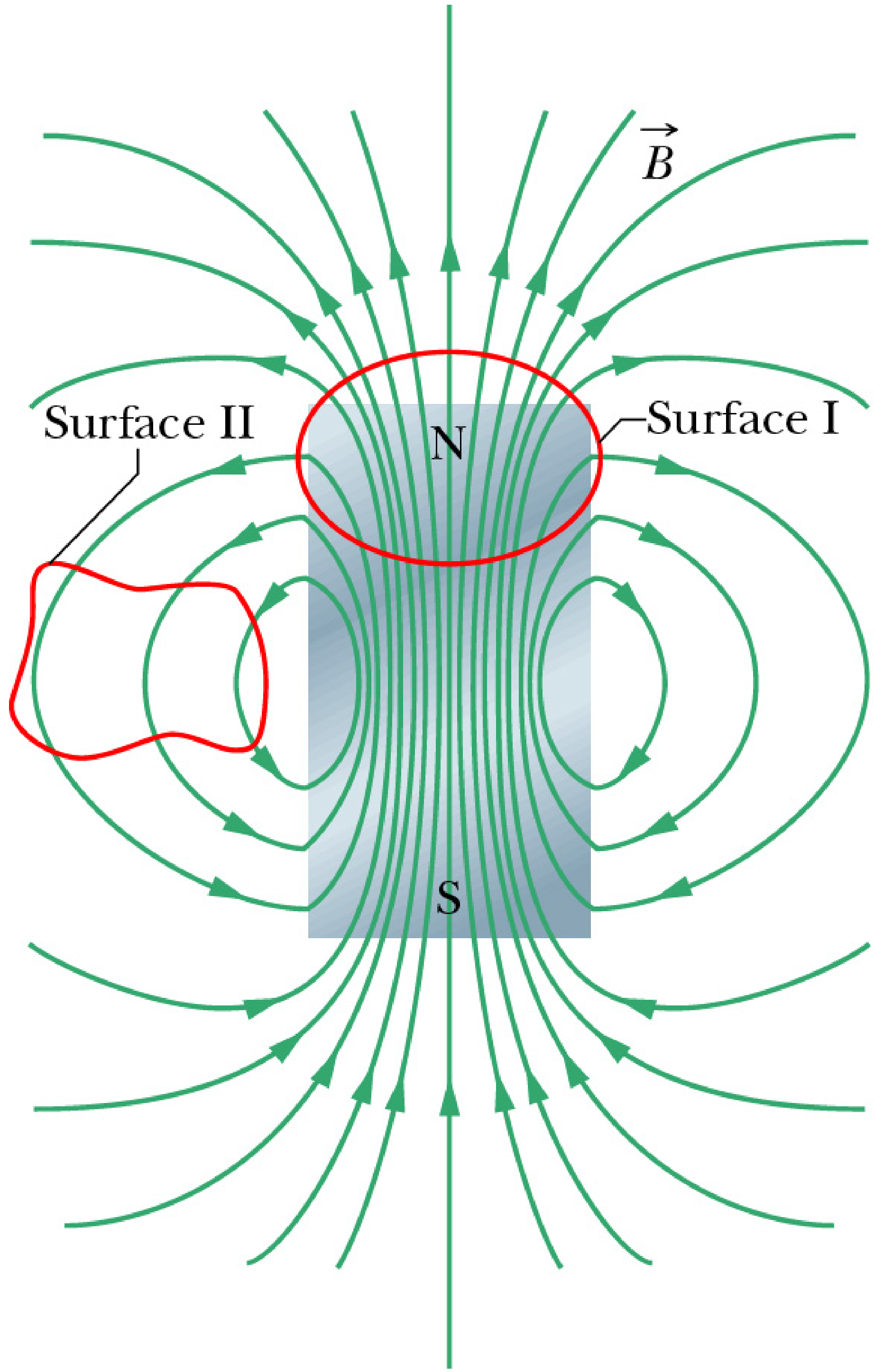
Gauss's Law for Magnetism

- **Outward *electric* flux = enclosed charge**
- **Outward *magnetic* flux = *zero.***
- **"There are no magnetic monopoles."**
- **This is Maxwell Equation #2:**

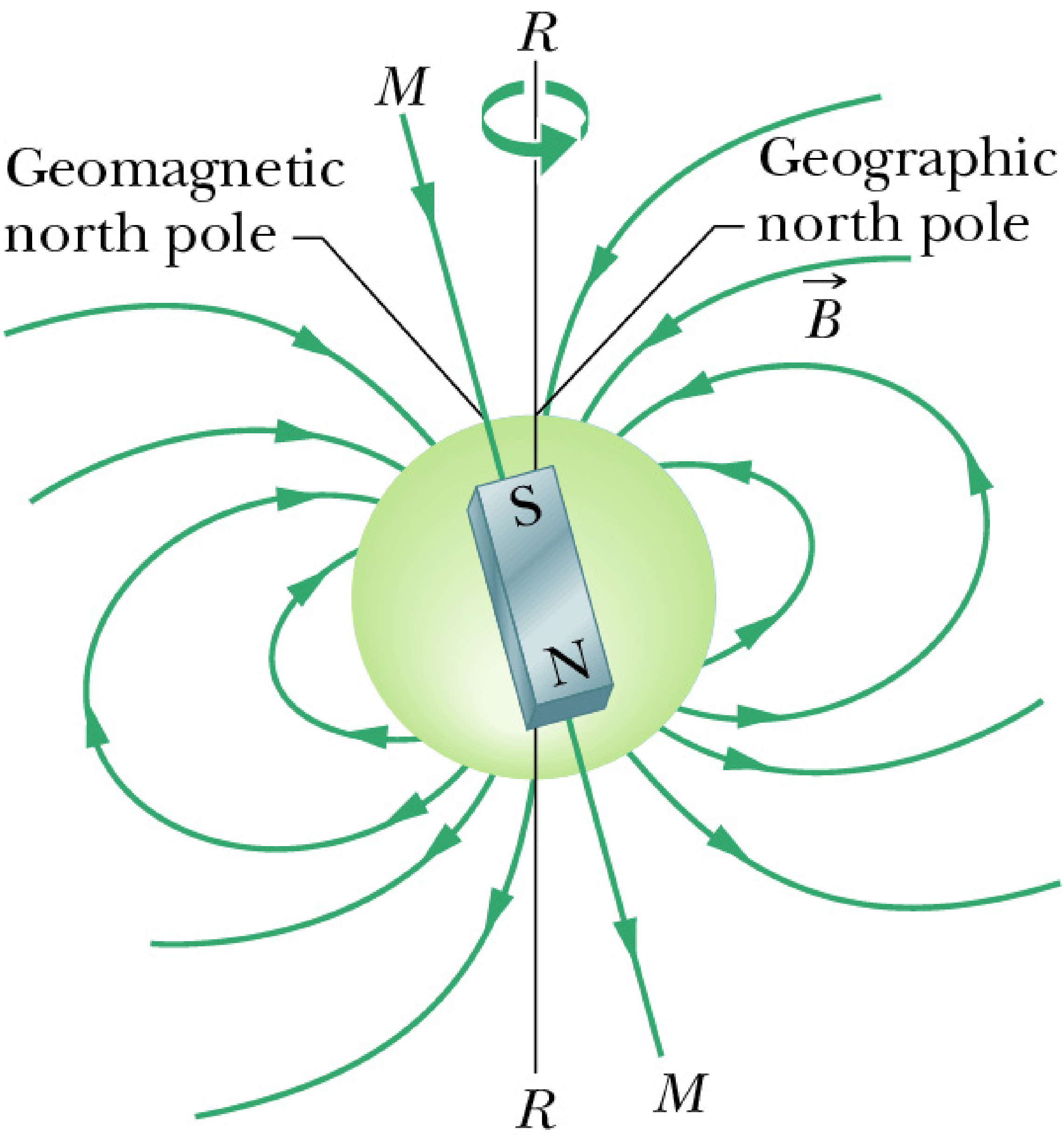
$$\oint \vec{B} \cdot d\vec{A} = 0$$

Bar magnet

Possible closed Gaussian surfaces shown in red. Zero net outward flux in both cases.



Magnetic Field of Earth



Approximately a dipole field.

The Magnetic Force

If a particle with electric charge *q* moves with velocity ***v*** through a magnetic field ***B*,** then the force by the field on the particle is

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

Unlike the electric force F = qE, the magnetic force on a charged particle is **NOT** in the direction of the magnetic field. In fact, it is ***perpendicular to it.***

Cross Product of Two Vectors

Given any two vectors ***A*** and ***B*,** and ***θ*** the angle between them, we ***define*** the ***vector product*** (cross product) $\vec{C} = \vec{A} \times \vec{B}$:

(1) $C = AB\sin\theta$

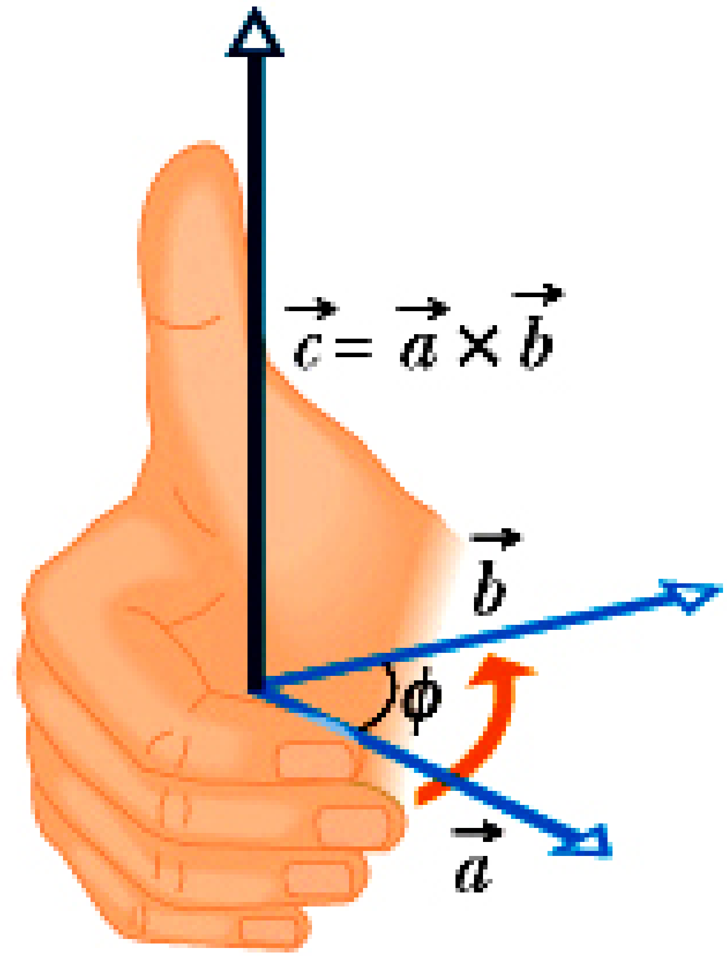
(2) $\vec{C}$ ***is perpendicular to both*** $\vec{A}$ ***and*** $\vec{B}$

(3) ***The direction of*** $\vec{C}$ ***is given by the righthand rule***

The Right-Hand Rule

If $\vec{C} = \vec{A} \times \vec{B}$

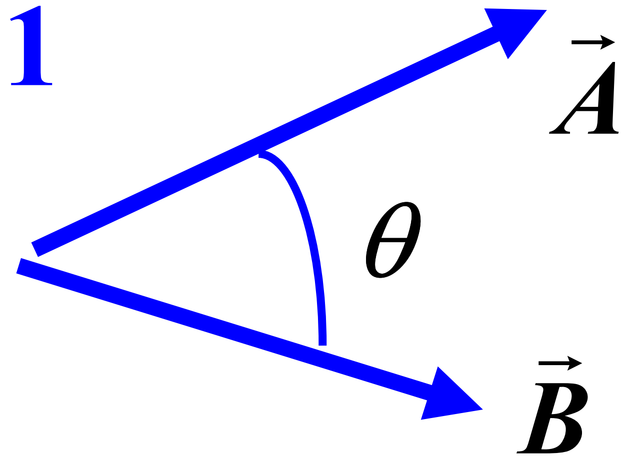
We use the right-hand rule to find the direction of the vector *C*.



Use the fingers of your right hand to rotate *a* toward *b*, then your thumb points in the direction of a×*b*.

Given vectors $\underline{A}$ and $\underline{B}$ with angle θ between them. For fixed magnitudes A, B, for what value of θ will the value of $\underline{A}\times\underline{B}$ be a maximum?

Q.28-1



1) 0°
2) 30°
3) 45°
4) 60°
5) 90°

Q.28-1

Given any two vectors ***A*** and ***B*****,** and ***θ*** the angle between them, we ***define*** the ***magnitude*** of the cross product as

$$C = AB\sin\theta$$

But $\sin\theta$ **has its maximum value when** $\theta = 90°$

So if A and B are perpendicular the magnitude of the cross product is a maximum, and is just AB.

Q.28-2

Suppose I have vector $\underline{A}$ pointing to the east and vector $\underline{B}$ pointing to the north.

What is the direction of $\underline{A} \times \underline{B}$?

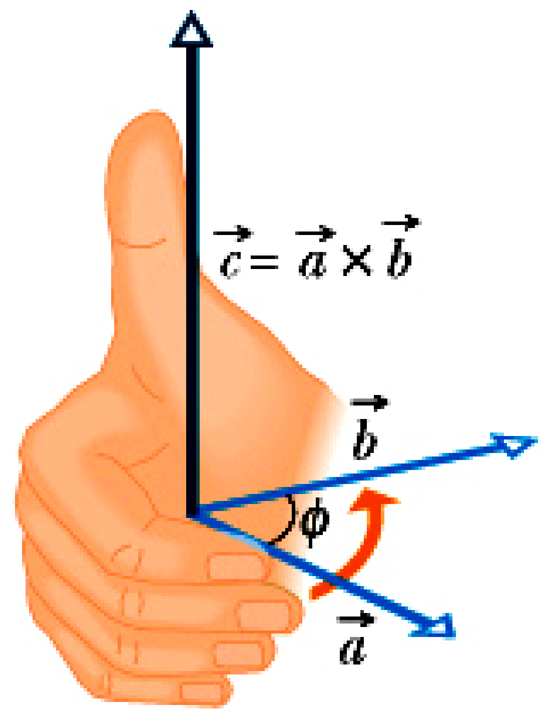
1) North
2) South
3) East
4) West
5) Up
6) Down

Q.28-2

Suppose I have vector $\underline{A}$ pointing to the east and vector $\underline{B}$ pointing to the north.

What is the direction of $\underline{A} \times \underline{B}$?

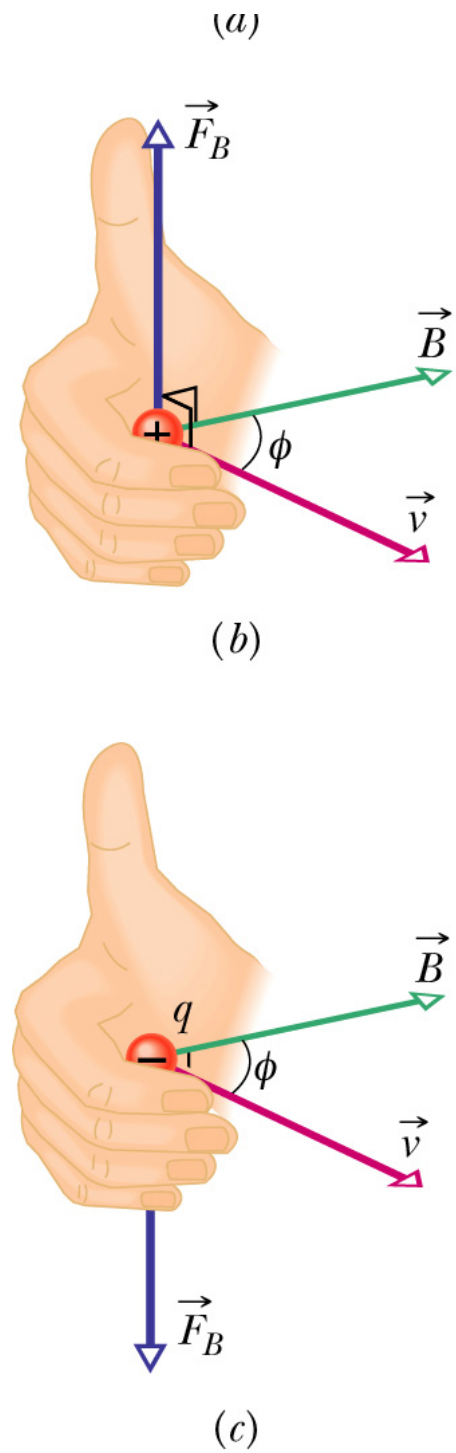
1) North
2) South
3) East
4) West
5) $\boxed{\text{Up}}$
6) Down



The Right-Hand Rule

If you use the fingers of your right hand to rotate ***v*** toward ***B***, then your thumb points in the direction of $\boldsymbol{v} \times \boldsymbol{B}$.

So in the figure the force on +q is upward, but the force on –q is downward.



Units

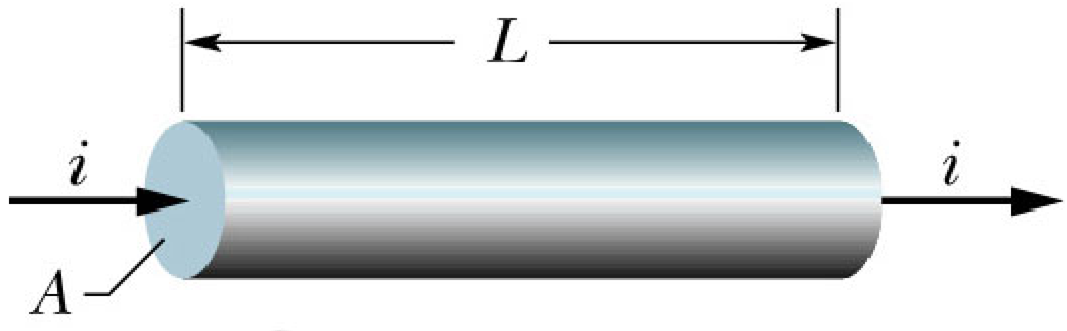
The SI unit for the magnetic field is the ***tesla*** (T).
Since F = qvB, we have B = F/qv and 1T = 1Ns/Cm.

Another unit sometimes used is the ***gauss*** (G).

$$\mathbf{1T = 10^4\ G}$$

The field of the earth is typically about 1 G.

Force on Current-carrying Wire



For wire perpendicular to B we have

$$F = q\,vB$$

But $i = \left(\dfrac{q}{L}\right)v$

So $F = qvB = \underline{BiL}$

$\boxed{\textbf{Force on length L}}$

If B is at angle θ with wire: F = BiL sin(θ)

The Magnetic Force

If a particle with electric charge q moves with velocity $\boldsymbol{v}$ through a magnetic field $\boldsymbol{B}$, then the force by the field on the particle is

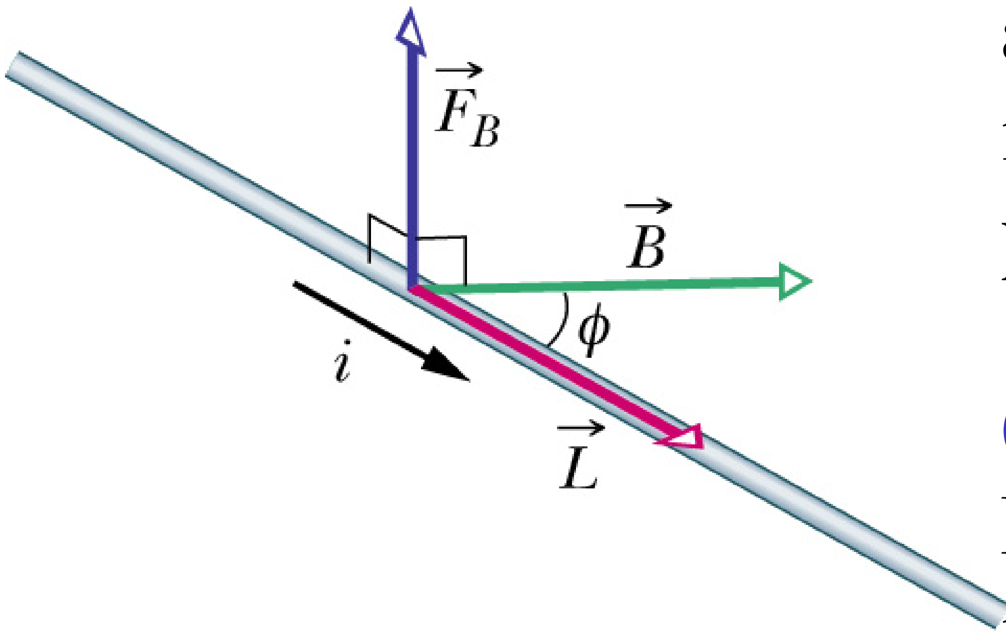
$$\vec{F} = q\vec{v} \times \vec{B}$$

If a wire of length L carries a current i through a field B, the force by the field on the wire is

$$\vec{F} = i\vec{L} \times \vec{B}$$

Example

$$\boxed{\vec{F} = i\vec{L} \times \vec{B}}$$



Given wire in field with angle φ = 37°. B = 0.3T, i = 20 mA. **Find force per unit length on wire.**

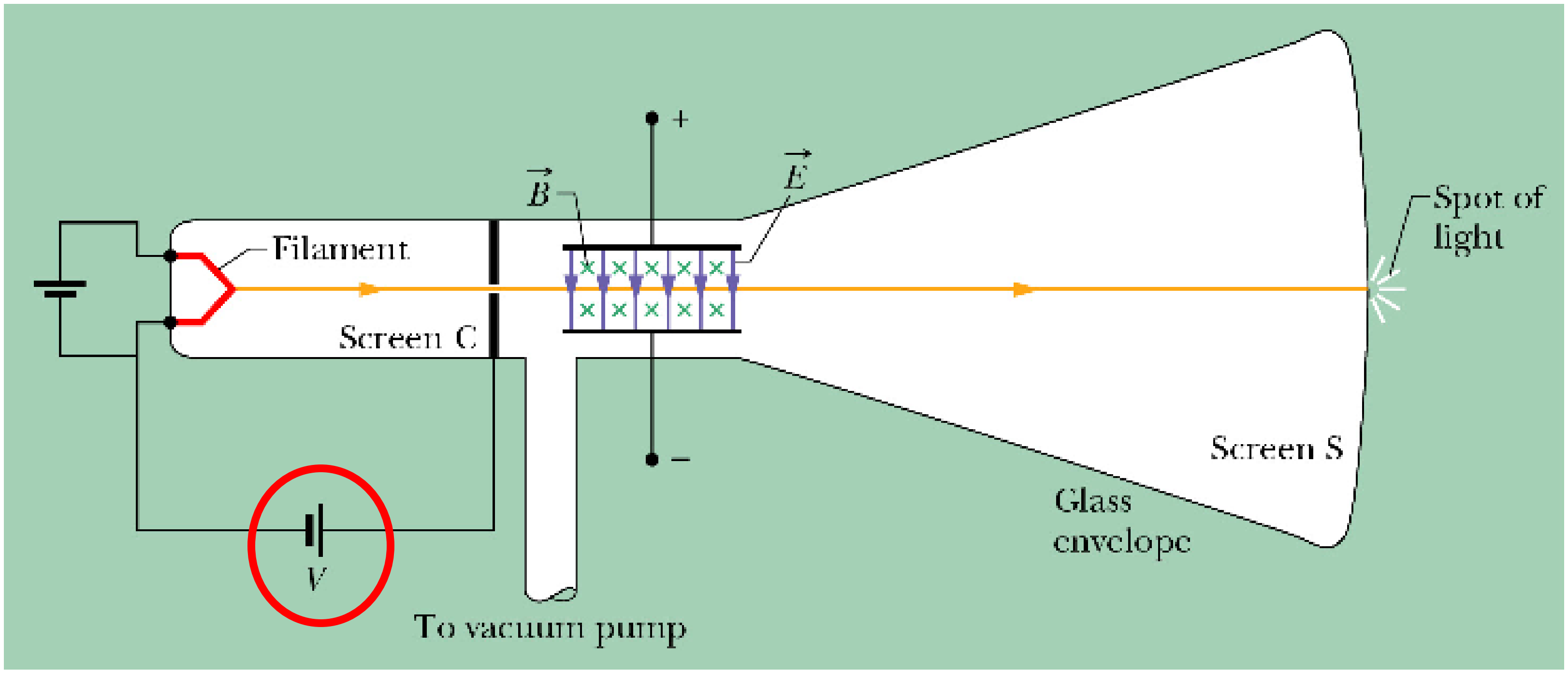
(1) Direction of force:
By right-hand rule, force is upward as shown.

(2) Magnitude of force:

$$F = i\left|\vec{L} \times \vec{B}\right| = iLB\sin\phi$$

$$F / L = iB\sin\phi = .02 \times .3 \times .6 = 3.6 \times 10^{-3}\ N / m$$

Cathode Ray Tube

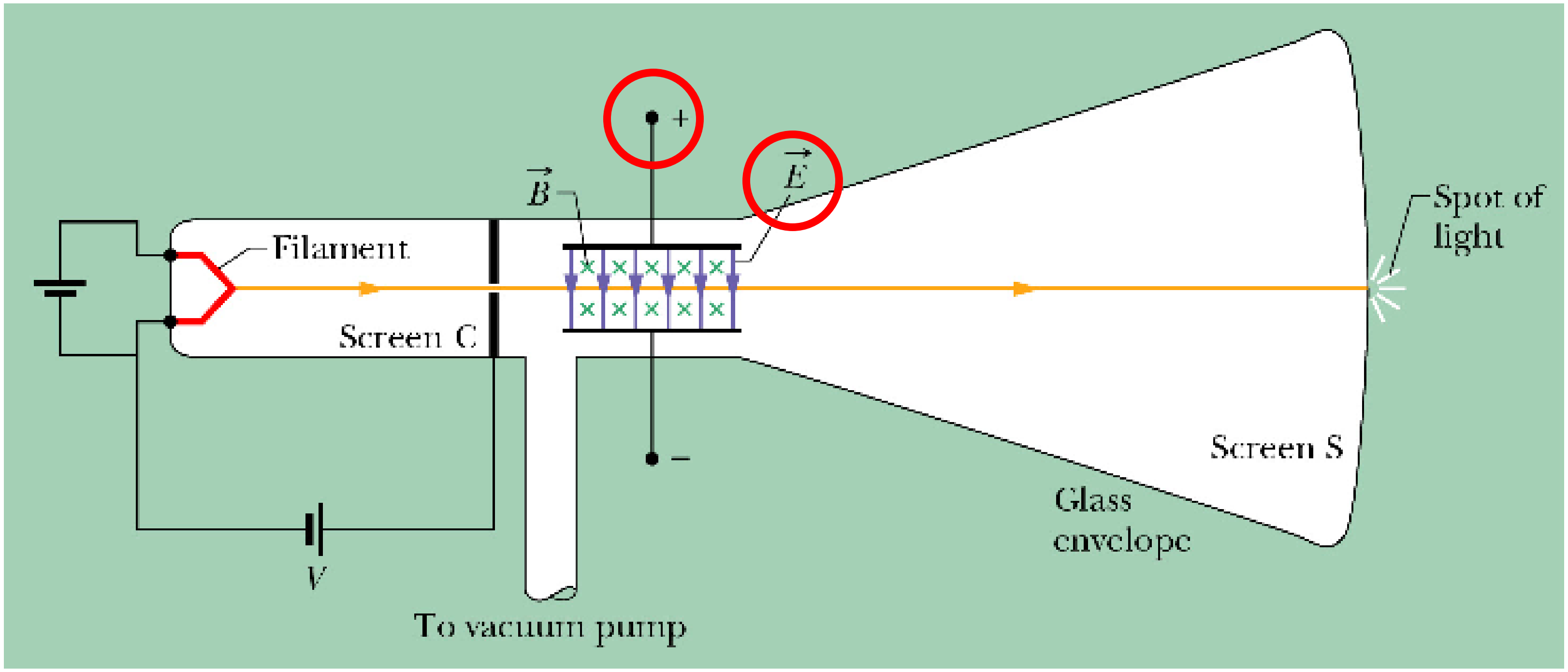


Electron gun: potential V gives electron energy in eV.

$$K = \frac{1}{2}mv^2 = qV$$

So if V = 500 volts, electron energy is K = 500 eV.

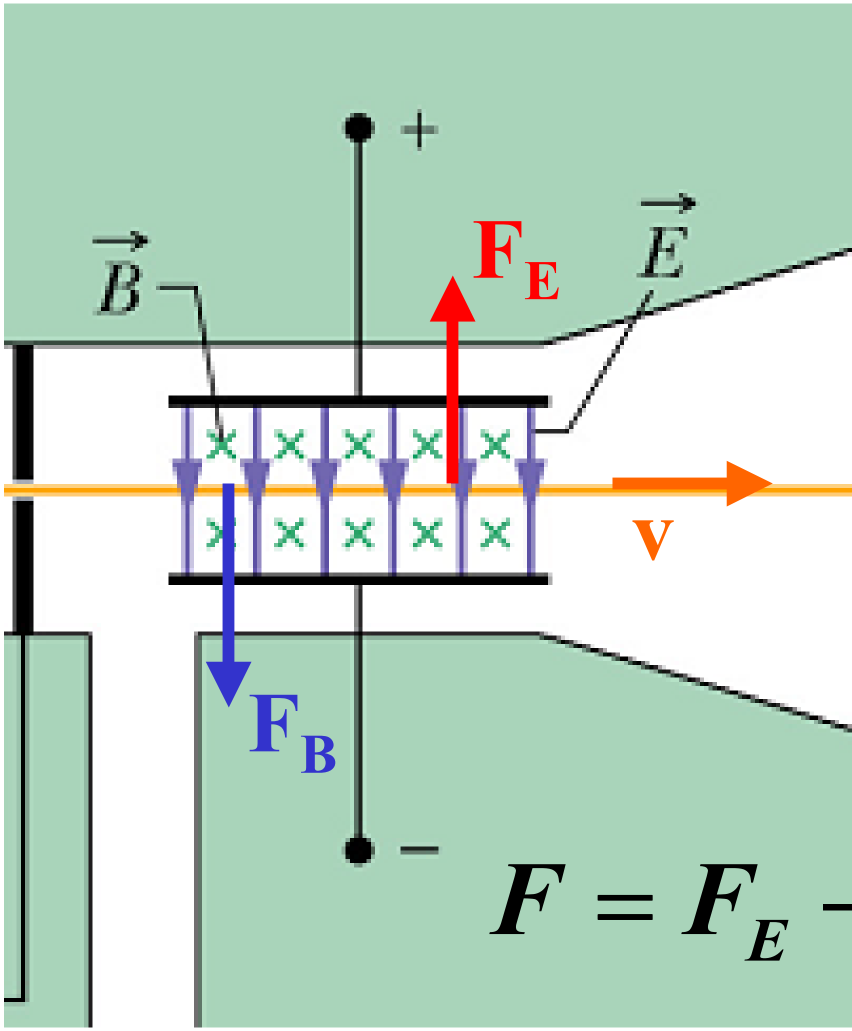
Crossed Fields



Crossed *E* and *B* fields:

$$\boldsymbol{If \quad qE = qvB \quad then \quad F = 0}$$

To deflect beam upward, increase E.



Forces on electron.

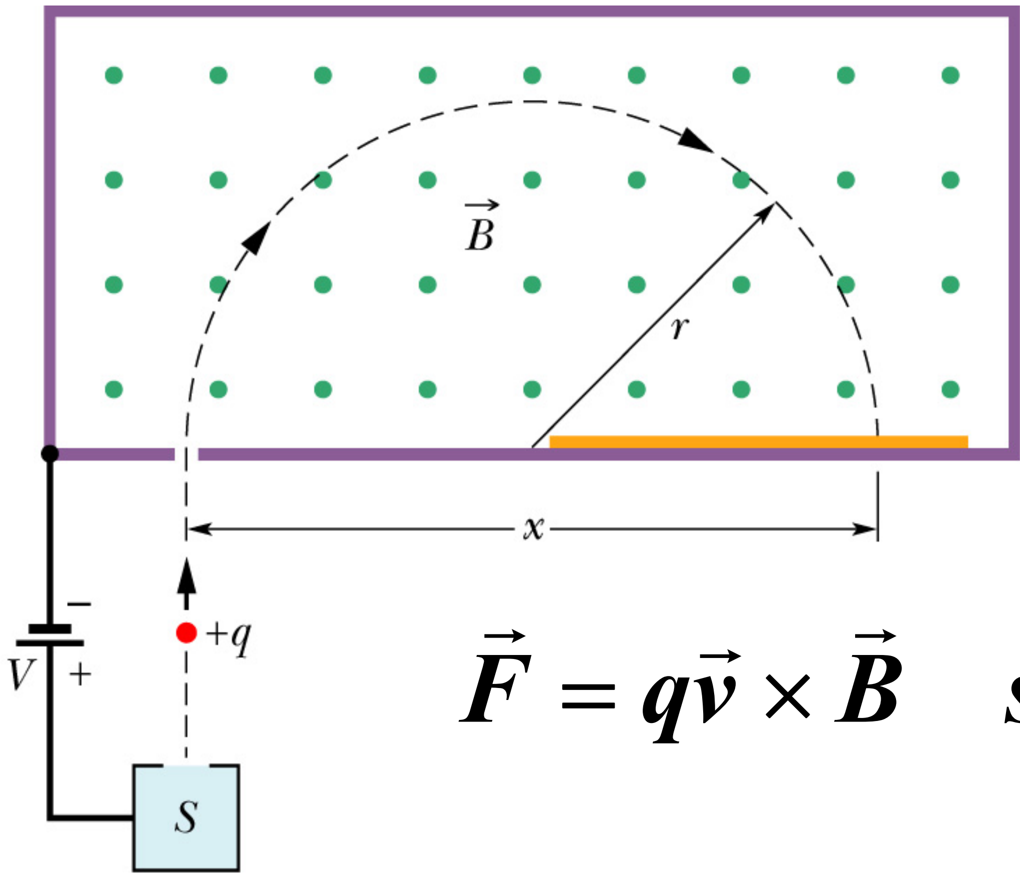
$$F_B = evB$$

$$F_E = eE$$

$$F = F_E - F_B = e(E - vB)$$

$$If \quad v = \frac{E}{B} \quad then \quad F = 0$$

Charge in Uniform B Field



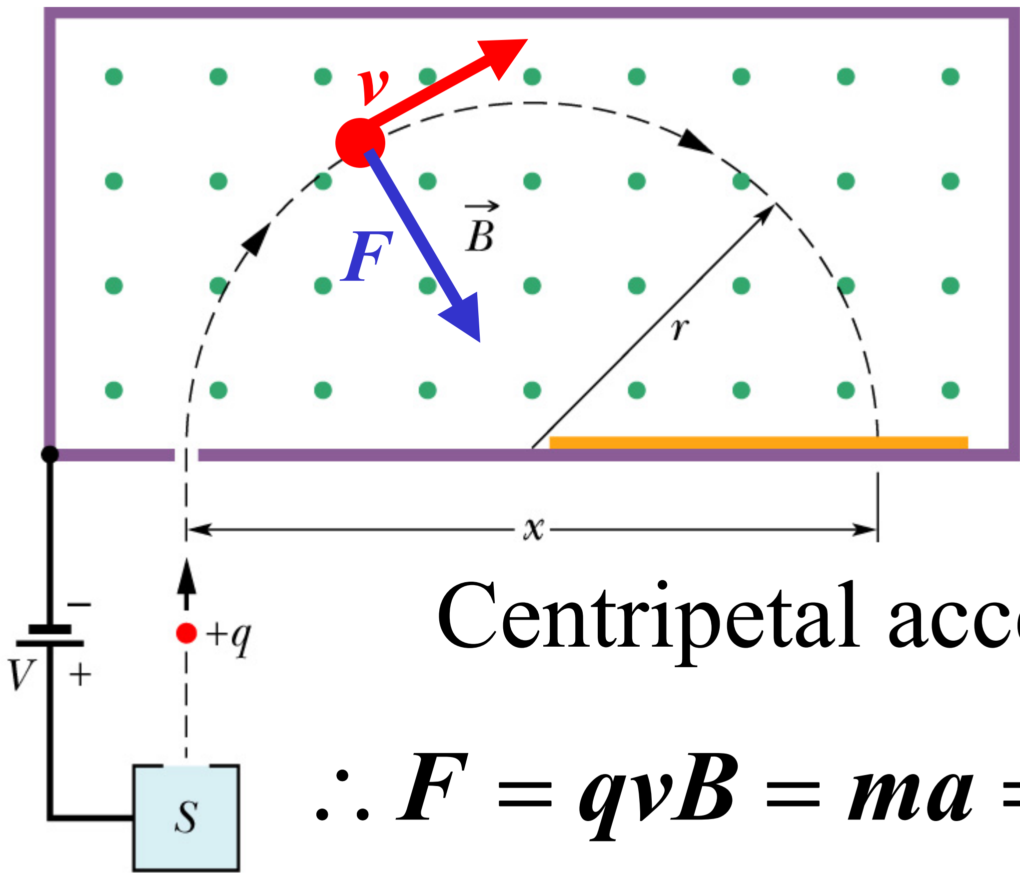
Positive ion enters field with energy

$$K = \frac{1}{2}mv^2 = qV$$

$$\vec{F} = q\vec{v} \times \vec{B} \quad so \quad \vec{F} \cdot \vec{v} = 0$$

So F does no work so K remains constant and so as the ion moves through field its ***speed*** remains constant .

Charge in Uniform B Field



B is ***out of*** screen so F is ***toward center*** so ion moves in circle with constant speed.

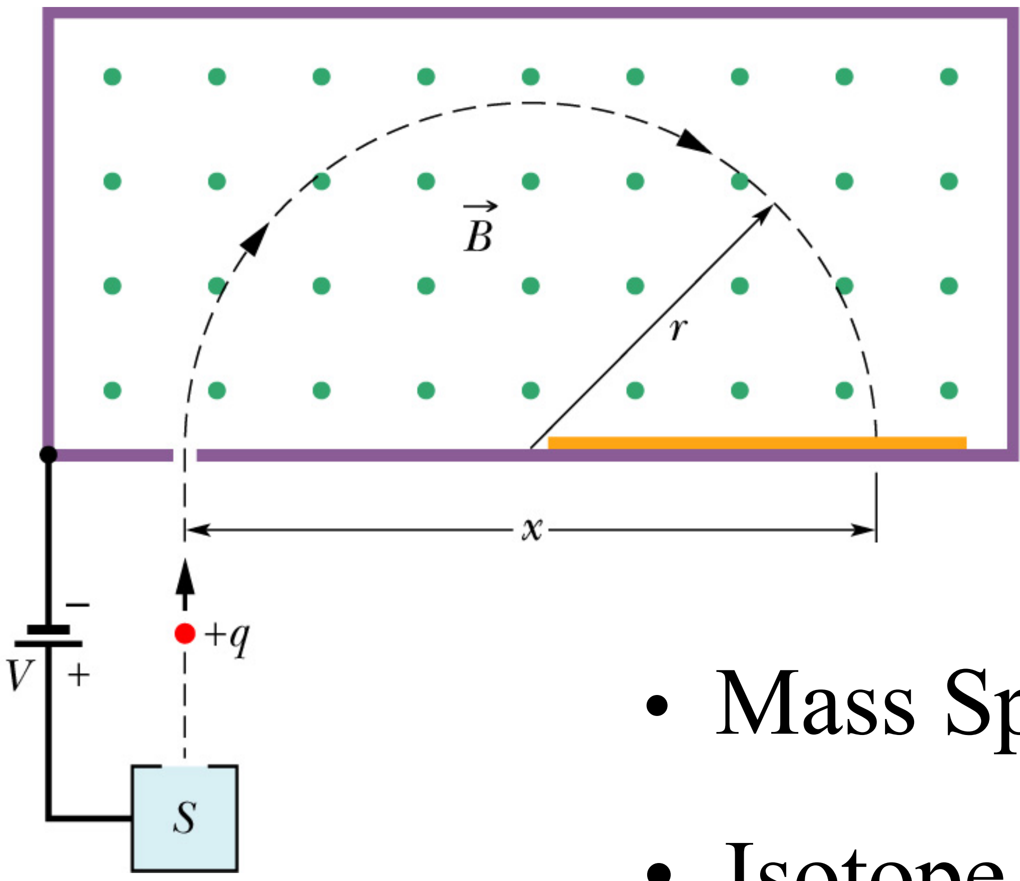
Centripetal acceleration is $\boldsymbol{a = v^2 / r}$

$$\therefore \boldsymbol{F = qvB = ma = mv^2 / r}$$

So solving for ***r*** gives the radius of curvature of the path:

$$\boldsymbol{r = \frac{mv}{qB}}$$

Charge in Uniform B Field



Applications

- Mass Spectrometer
- Isotope Separator
- Particle Accelerator

Electromagnetic Fields

- Ch.28: The magnetic field: Lorentz Force Law
- Ch.29: Electromagnetism: Ampere’s Law

- Chapter 28 Questions 1, 7
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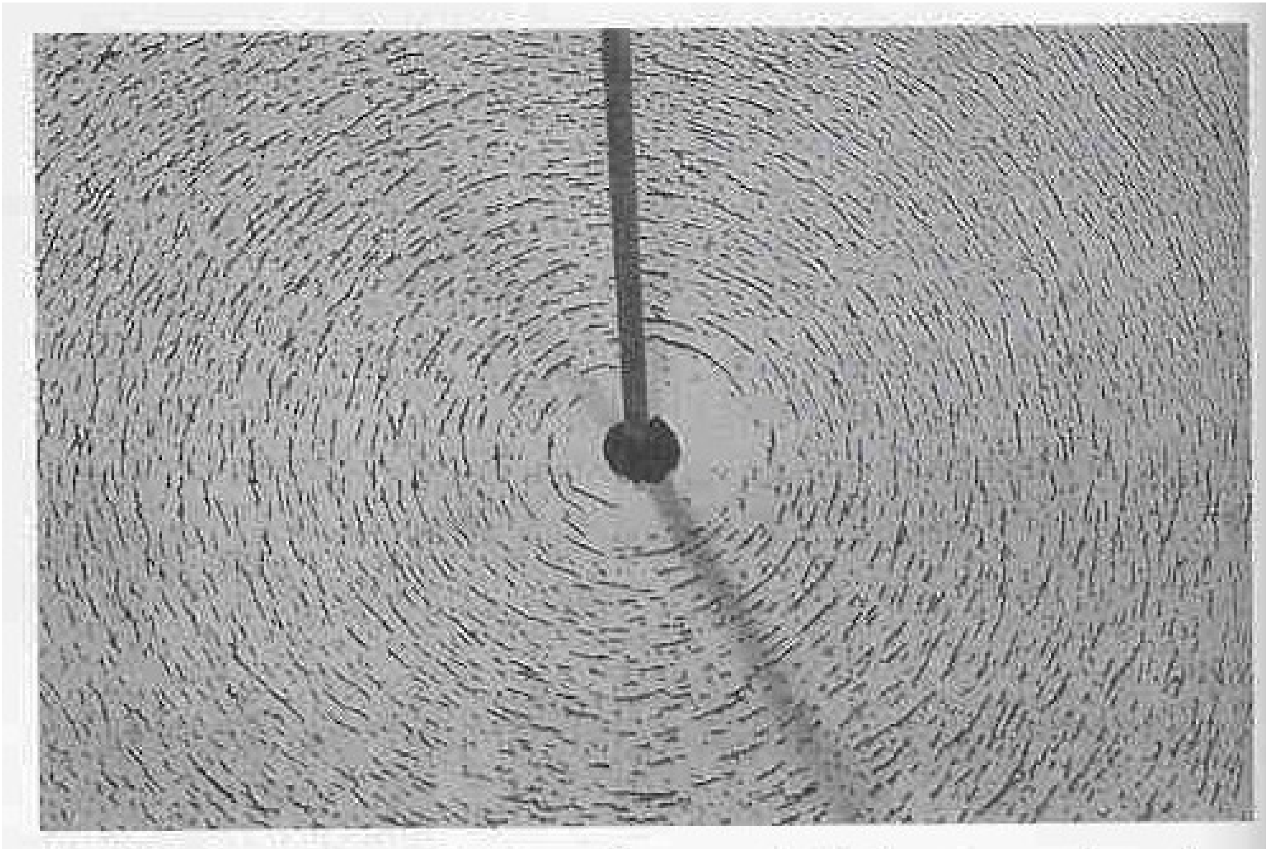
TODAY: Electromagnetism

Production of magnetic field by a current

- ***B*** **field due to a current in a long straight wire**
- ***B*** **field due to a current in a short bit of wire**
- **Ampere's Law**: the third of Maxwell's Equations

Field Due to a *Long Straight* Wire



Lines of B make circles around wire!

BUT FIRST REVIEW:
The Lorentz Force

If a particle with electric charge ***q*** moves with velocity *v* through a magnetic field ***B***, then the force by the field on the particle is

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

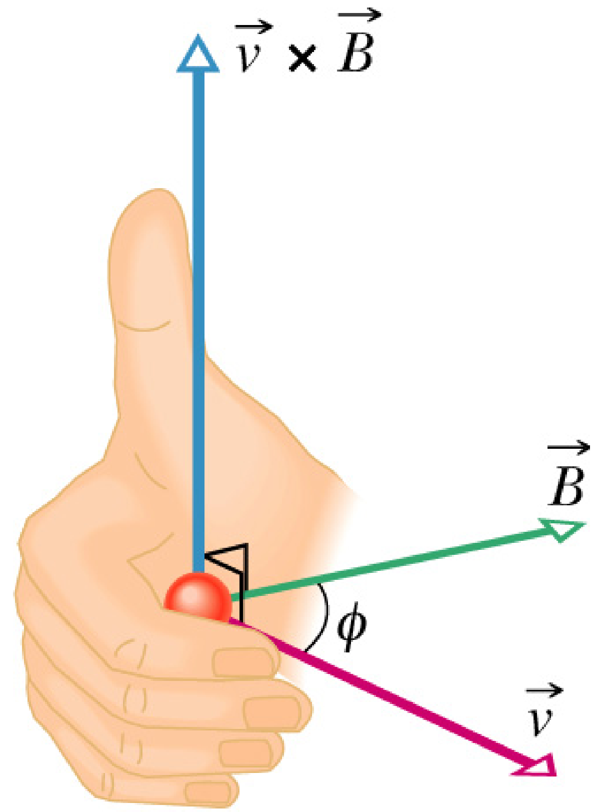
If a wire of length ***L*** carries a current ***i*** through a field ***B***, the force by the field on the wire is

$$\vec{F} = i\vec{L} \times \vec{B}$$

REVIEW: The cross product

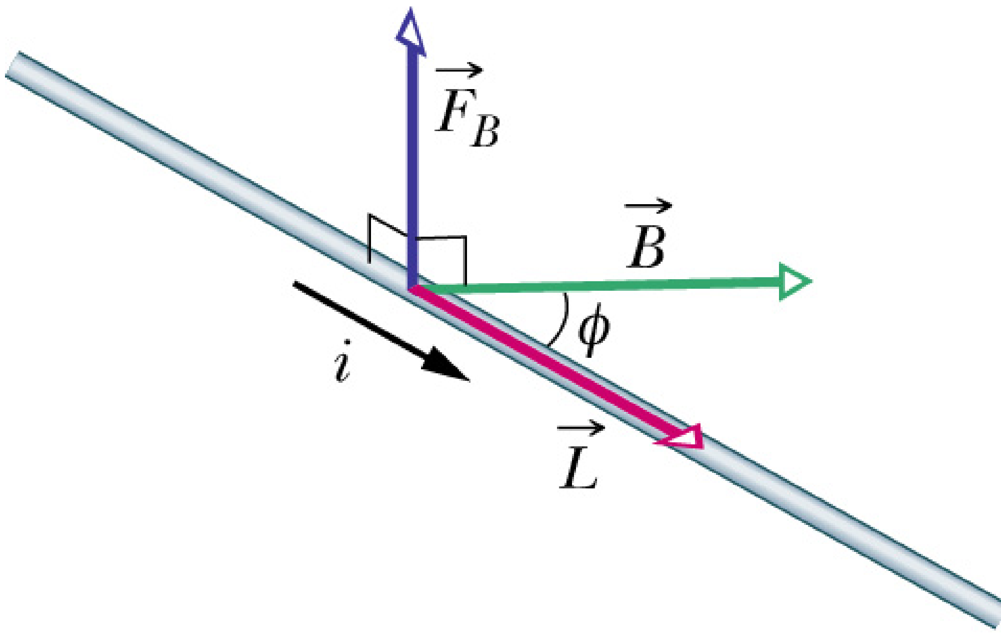
Given vectors *v* and *B*, and *θ* the angle between them, we define the vector product (cross product) $\vec{v} \times \vec{B}$

1. Magnitude is vBsinθ.
2. Right-hand rule gives direction, perpendicular to both v, B.



Example

$$\vec{F} = i\vec{L} \times \vec{B}$$



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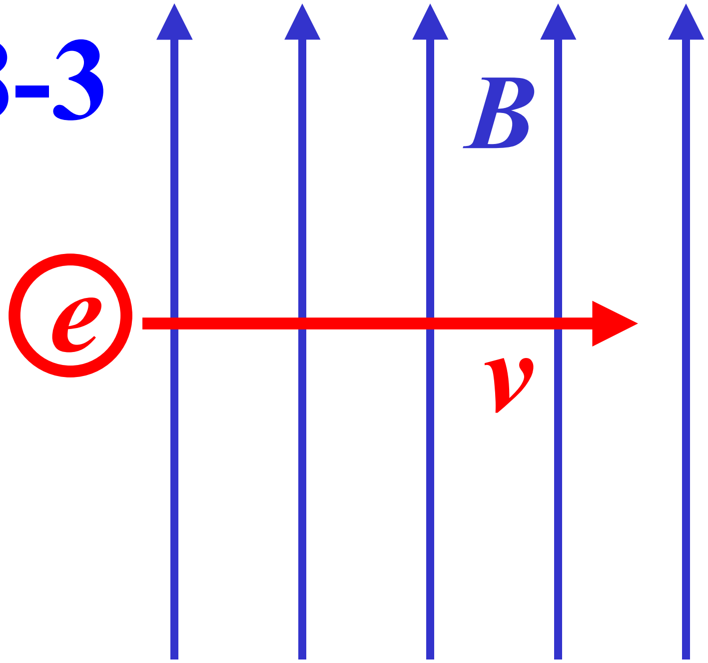
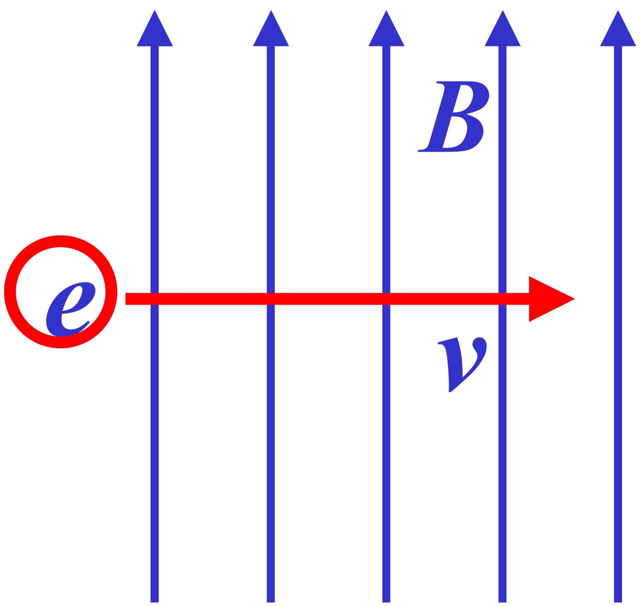
Q.28-3



An electron with speed *v* enters a magnetic field *B* as shown. What is the direction of the *force* on the electron?

1) **Out of the screen**
2) **Into the screen**
3) **In the direction of B**
4) **In the direction of v**

Q.28-3



What is the direction of the *force* on the electron?

Solution: $$\vec{F} = q\,\vec{v} \times \vec{B}$$

$\vec{v} \times \vec{B}$ **is out of screen by RH rule.**

But *q* is negative, so *F* is into the screen.

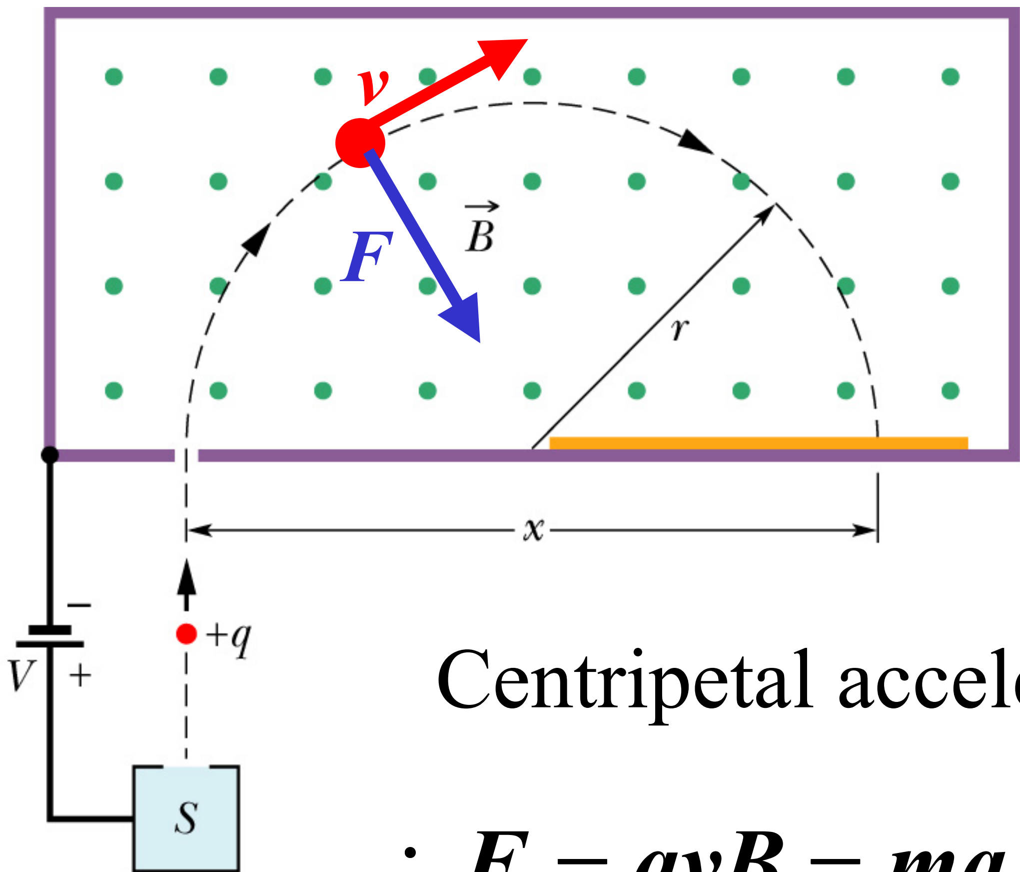
(1) Out of the screen.

$\boxed{\textbf{(2) Into the screen.}}$

(3) In the direction of B.

(4) In the direction of v.

REVIEW: Charge in Uniform Field



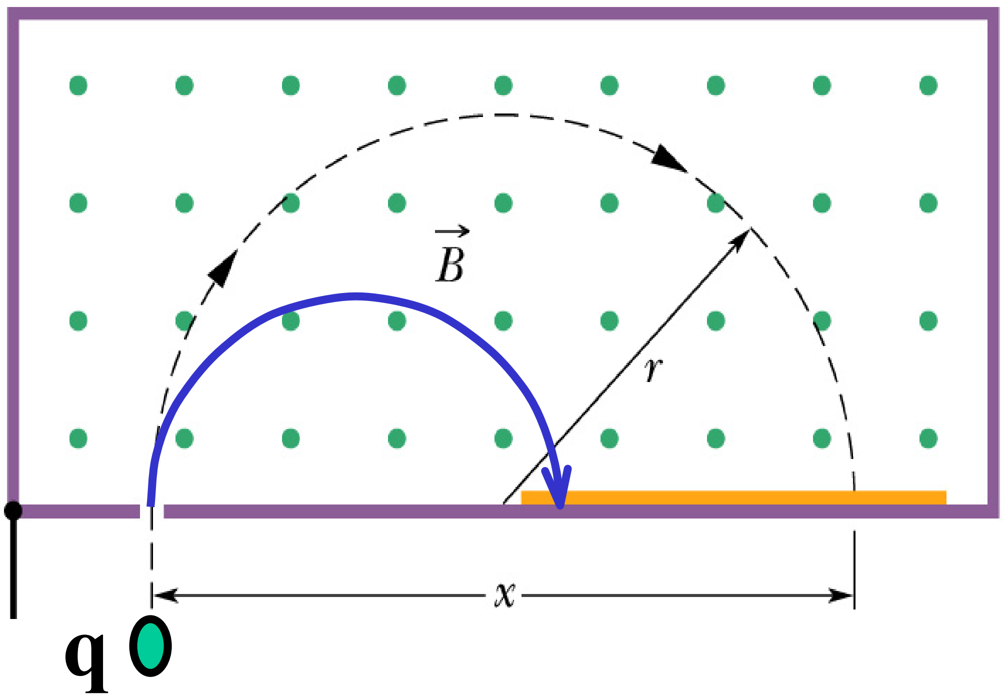
B is *out of* screen so F is *toward center* so ion moves in circle with constant speed.

Centripetal acceleration is $a = v^2 / r$

$$\therefore F = qvB = ma = mv^2 / r$$

So solving for *r* gives the radius of curvature of the path:

$$\boxed{r = \frac{mv}{qB}}$$

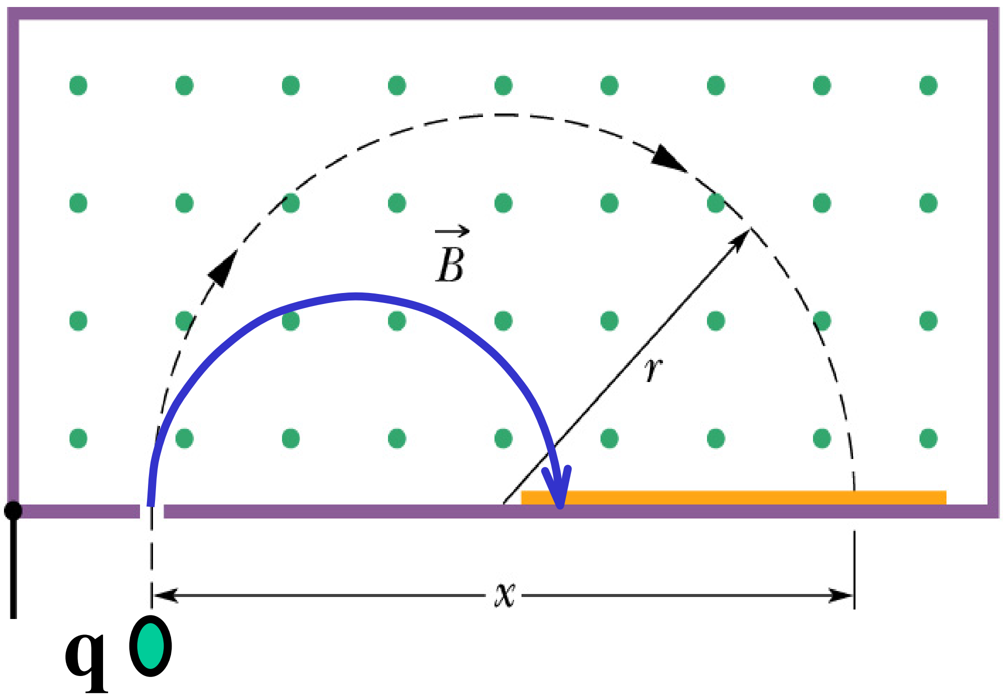


Q.28-4

An ion with charge +e and mass M_1 follows the dashed path in given B field.

Another ion with charge +e, but a different mass M_2 enters the field with the same velocity as the first, and follows the blue path. How do the masses compare?

(1) $M_1 > M_2$ (2) $M_2 > M_1$ (3) Can't say.



Q.28-4

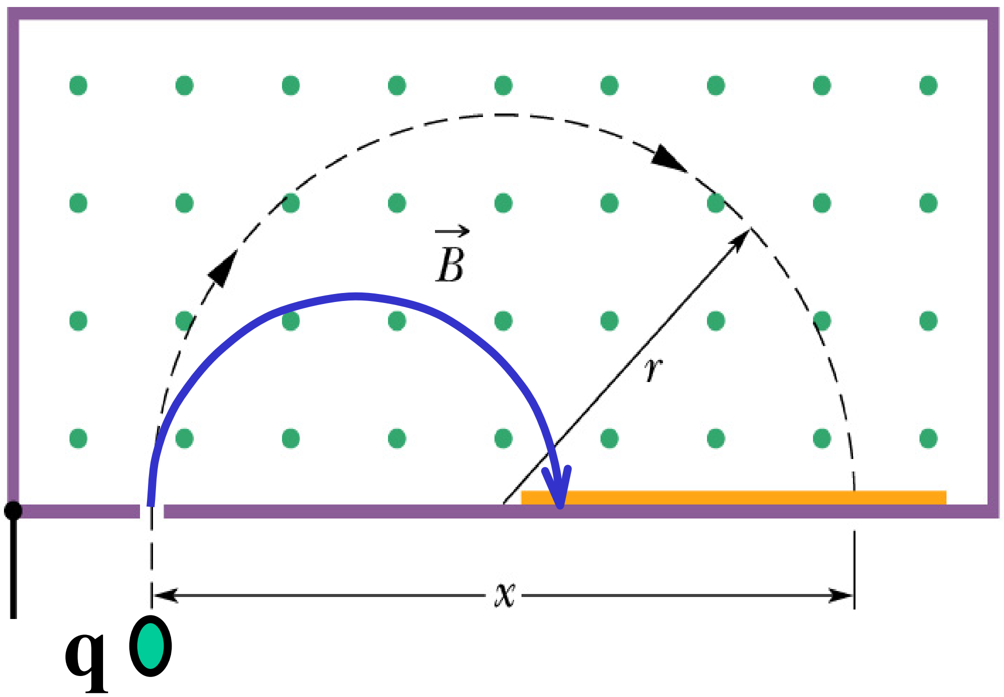
1) M1 > M2
2) M2 > M1
3) Not enough information

Two ions, same q, v.

M1 follows dashed path.

M2 follows blue path.

How do their masses compare?



Q.28-4

Two ions with equal charges and velocities follow the two curves shown.

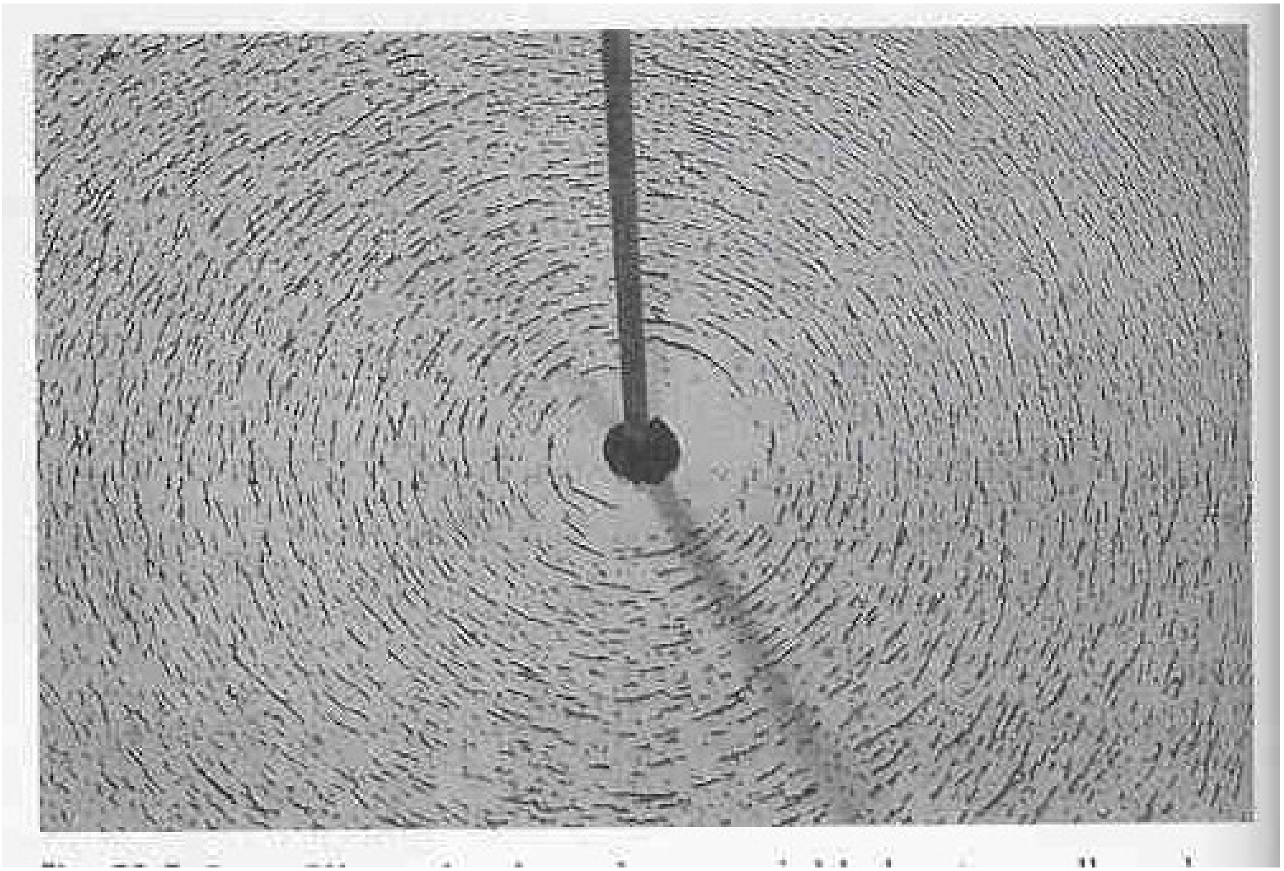
$$r = \frac{mv}{qB}$$

M_1 follows the dashed, M_2 the blue path. How do the masses compare?

Blue radius is smaller so blue mass is smaller.

$\boxed{\text{(1) } M_1 > M_2}$ (2) $M_2 > M_1$ (3) Can’t say.

Field Due to a *Long Straight* Wire



Lines of B make circles around wire!

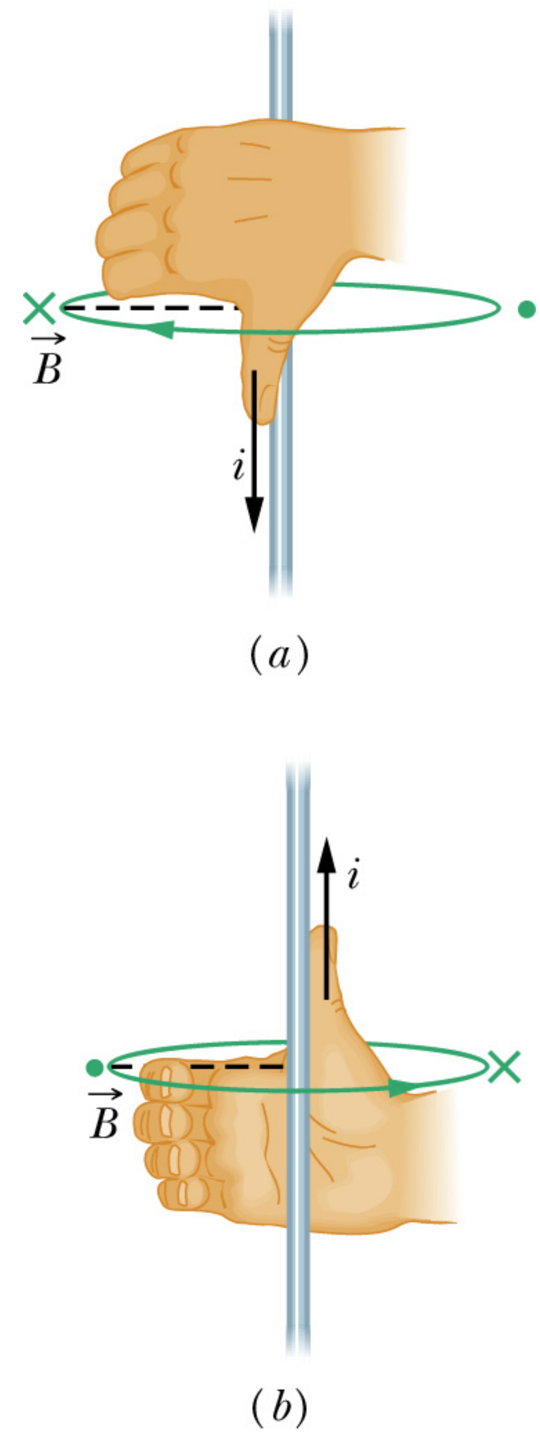
Field of a long straight wire

1. Direction is given by the right-hand rule!

2. Magnitude is $B = \dfrac{\mu_0 i}{2\pi r}$

3. New universal constant:

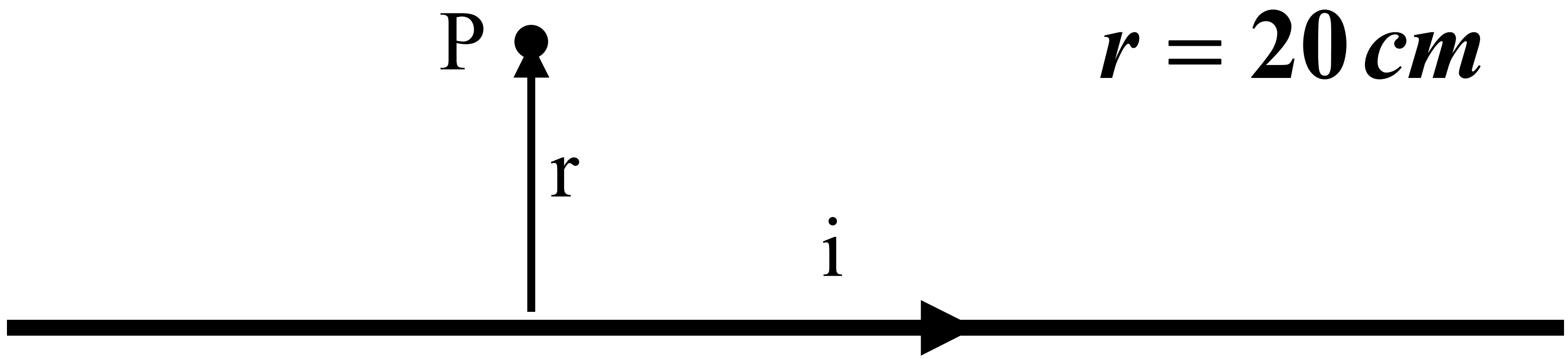
$$\mu_0 = 4\pi \times 10^{-7}\ Tm/A$$



Example

$i = 3.0\,A$

$r = 20\,cm$



What is the magnetic field at point P?

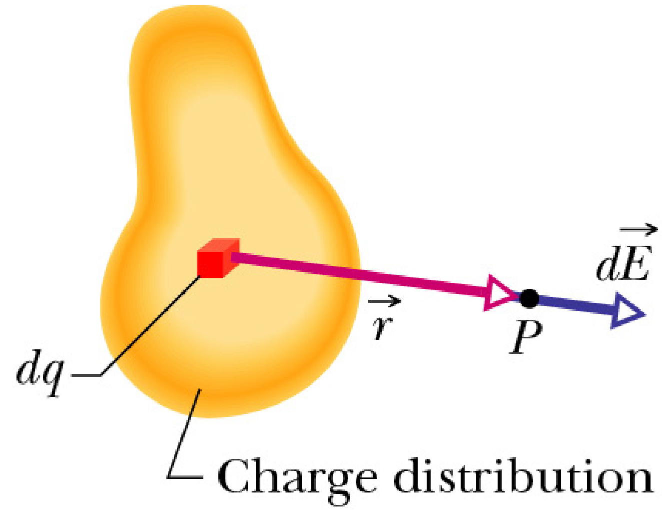
Direction: **<u>Out of the screen by right-hand rule.</u>**

Magnitude:

$$B = \frac{\mu_0 i}{2\pi r} = \frac{4\pi \times 10^{-7} \times 3.0}{2\pi \times 0.20} = 3.0 \times 10^{-6}\,T$$

Field Due to a *Short Bit* of Wire

Recall Coulomb:
E is **parallel** to ***r*.**

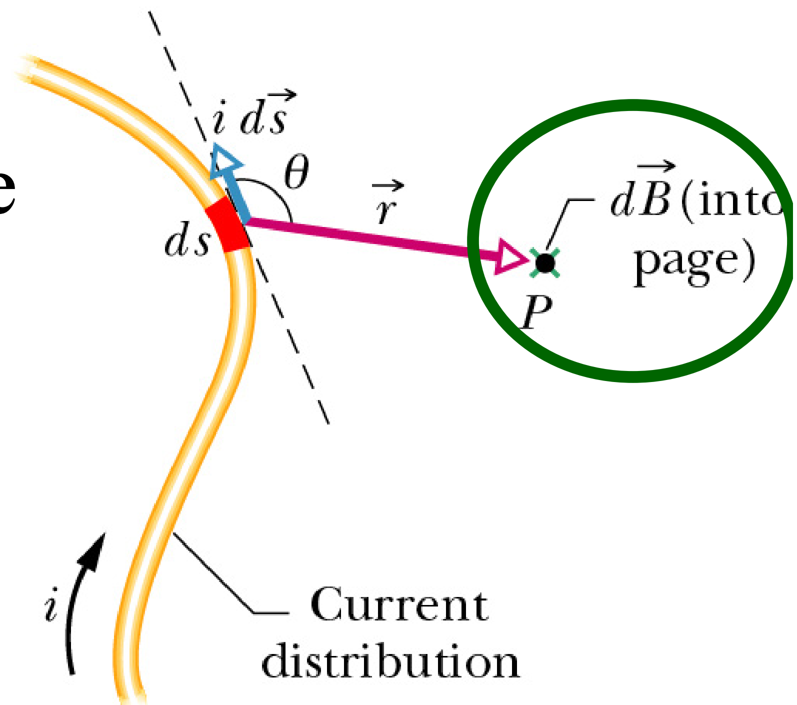


(a)

But as usual for magnetism, we find ***B*** is **perpendicular** to ***r***!

$$d\vec{B} \propto i\, d\vec{s} \times \vec{r}$$

Another right-hand rule!



(b)

The Biot-Savart Law

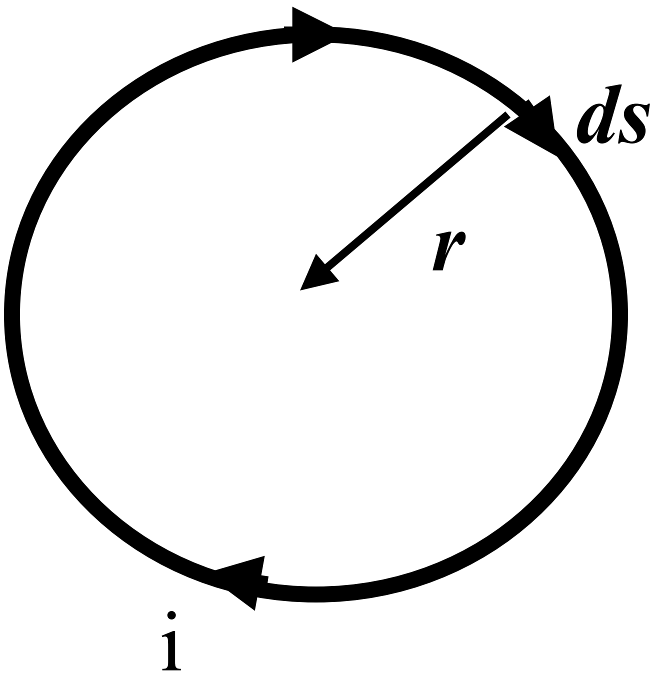
$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}}$$

- Field ***dB*** is perpendicular to both ***ds*** and ***r***.
- Inverse square law like Coulomb's Law.
- Universal constant:

$$\mu_0 = 4\pi \times 10^{-7}\ Tm/A$$

Example

Field at the center of a circular loop of wire.



Direction: **Into** the screen, by the right-hand rule.

Magnitude: Must add up (integrate) all the little ***dB*** from all the little ***ds.***

$$B = \int dB = \frac{\mu_0}{4\pi}\frac{i}{r^2}\int ds = \frac{\mu_0}{4\pi}\frac{i}{r^2}2\pi r = \frac{\mu_0 i}{2r}$$

Ampere's Law

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

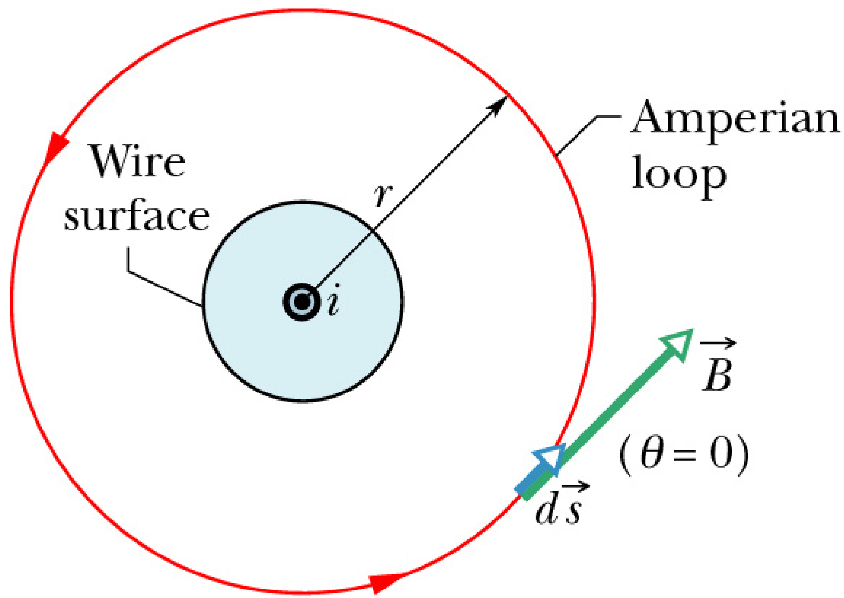
- **C = Any closed path**
- **i_{enc} = Net current linking C (*Right-hand rule)***
- ***B* = The total magnetic field**
- ***ds* = A short step along the path**

This is the third of Maxwell's equations.

Field Due to a *Long Straight* Wire

1. **Direction is given by the right-hand rule!**

2. **Magnitude is found by applying Ampere's Law to a circular path of radius *r*:**



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i$$

$$B \cdot \oint ds = B \cdot 2\pi r = \mu_0 i$$

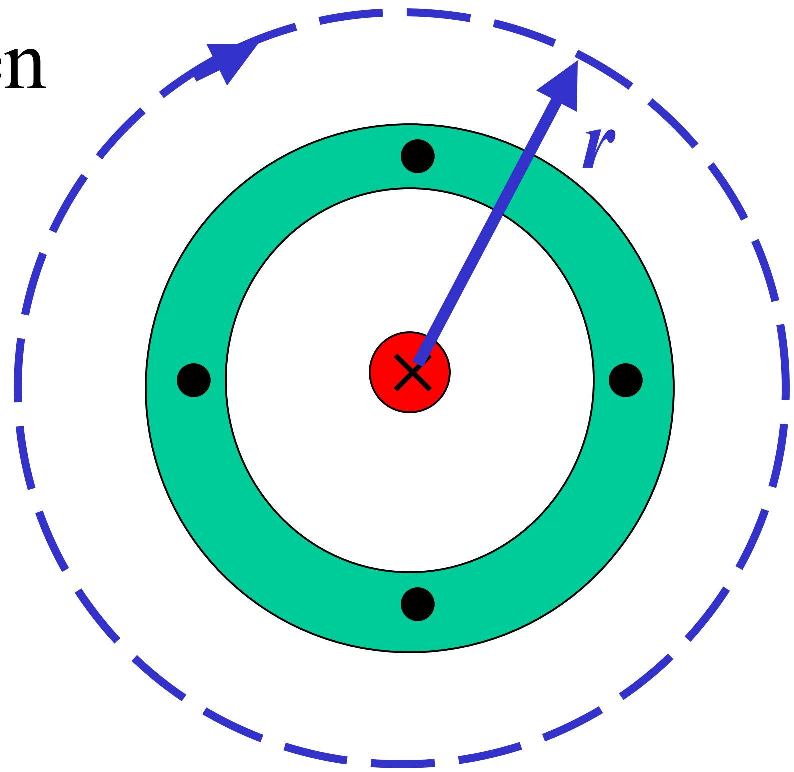
$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

Example: Field in Coaxial Cable

Cable perpendicular to screen

Central wire: ***i*** inward

Outer cylinder: ***i*** outward



(b) Field outside the cable?

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

$$B \cdot \oint ds = B \cdot 2\pi r = \mu_0 (i - i)$$

$$B = 0$$

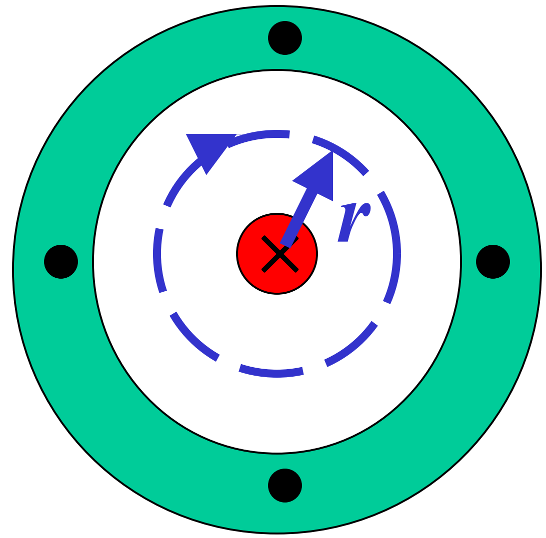
Outer conductor prevents field from escaping!

Example: Field in Coaxial Cable

Cable perpendicular to screen

Central wire: ***i*** inward

Outer cylinder: ***i*** outward



(a) Field between conductors?

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

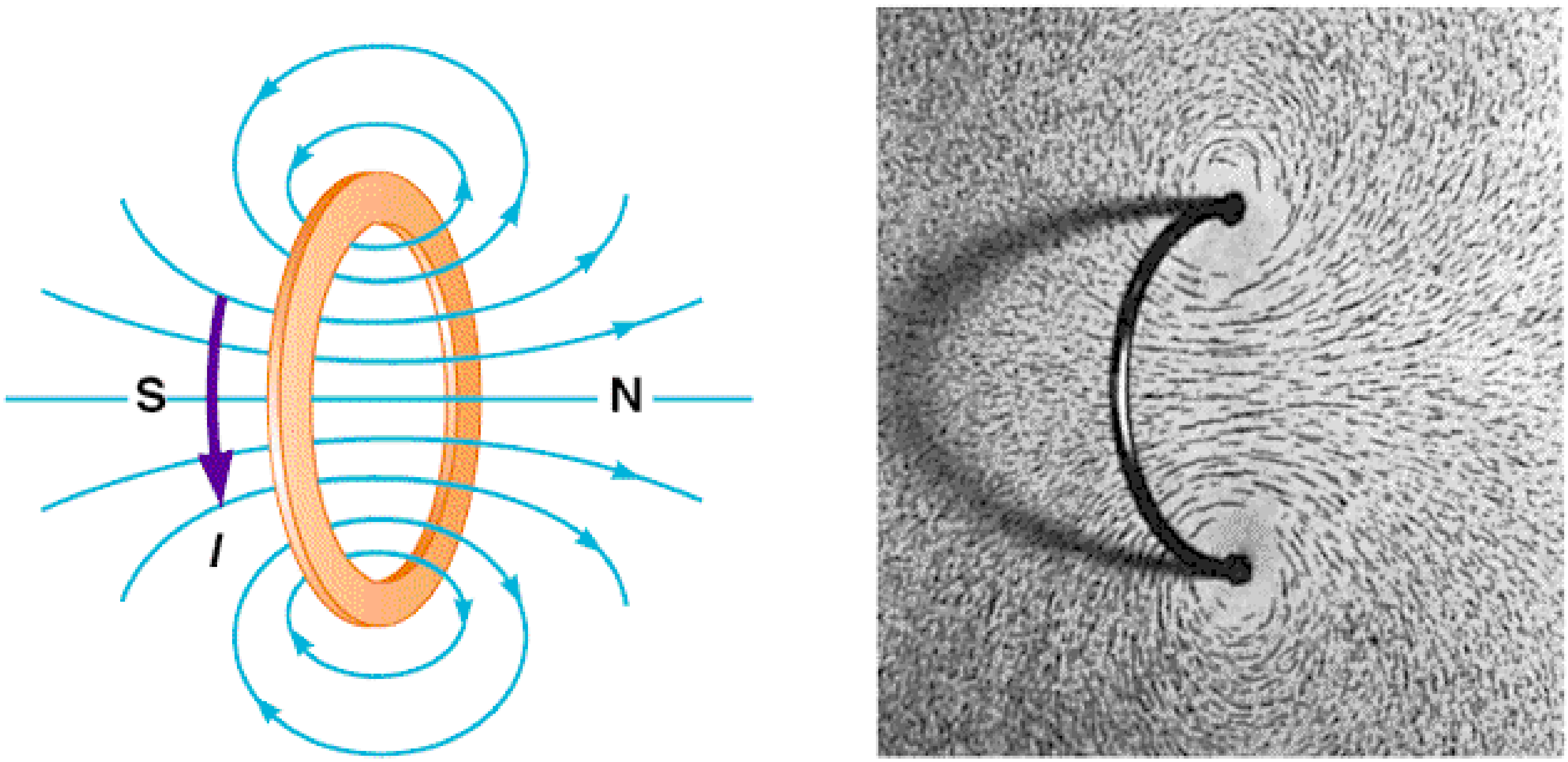
$$B \cdot \oint ds = B \cdot 2\pi r = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r}$$

Just as if outer conductor did not exist!

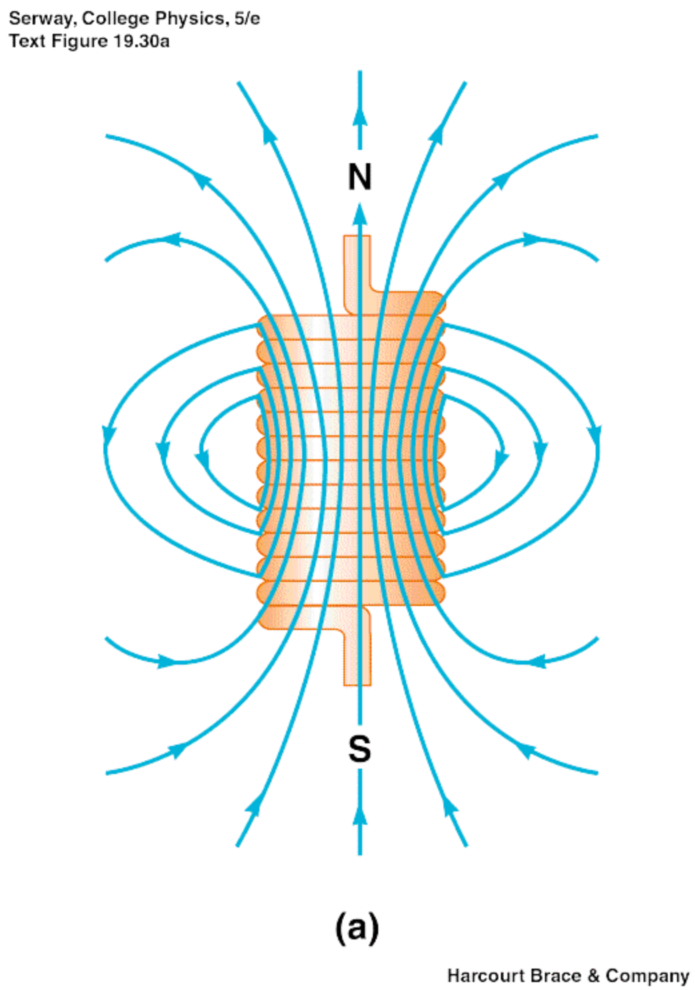
Field Due to a Current Loop

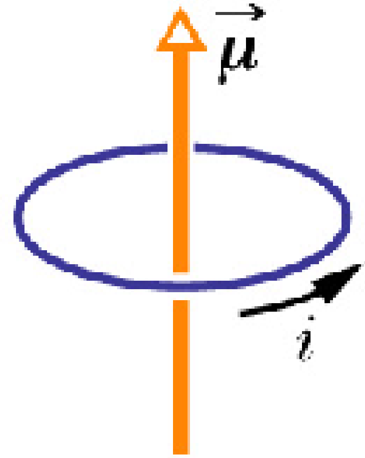
Serway, College Physics, 5/e
Text Figure 19.28a,b

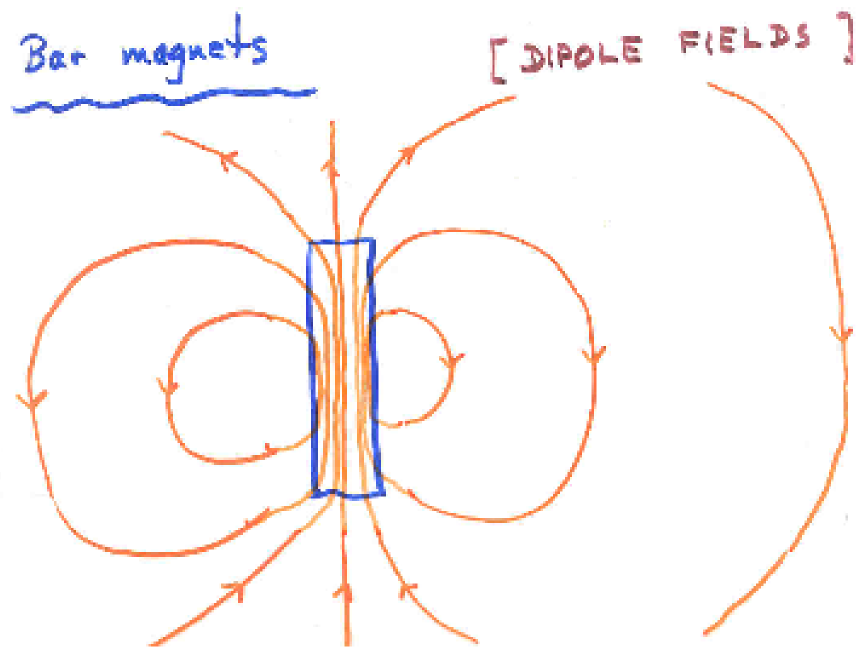


(a) (b)

Magnetic Dipole Field

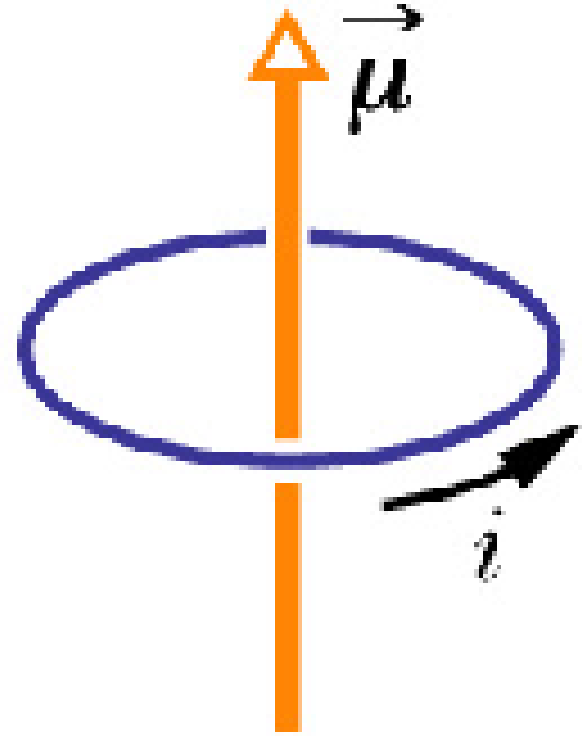






Dipole Moment of a Current Loop

Definition: ***Magnetic dipole moment vector:*** $\boxed{\vec{\mu}}$



- Direction: ***RH rule***
- Magnitude: $\mu = iA$

Analogous to electric dipole moment vector $\vec{p}$