Note on tomorrow’s quiz:
There will be 10 multiple-choice questions.

Review: Definition of Potential
• Potential is defined as potential energy per unit charge. Since change in potential energy is work done, this means

$$ \Delta V = -\int E_x \, dx \quad \text{and} \quad E_x = -\frac{dV}{dx} \quad \text{etc.} $$

The potential difference between any two points is the work required to carry a unit positive test charge between those two points.

Review: Coulomb’s Law
So we now have a third form of Coulomb’s Law:

1. \( F = kQq / r^2 \)
2. \( E = kQ / r^2 \)
3. \( V = kQ / r \)

Review: Basics about Potentials
• \( U = qV \)
• \( V = kQ/r \)
• \( W_{AB} = q(V_B - V_A) \)

$$ \Delta V = -\int E_x \, dx $$
$$ E_x = -\frac{dV}{dx} \quad \text{etc.} $$

Field of a Uniformly Charged Sphere
Good example for main ideas of this week.

• Given a non-conducting sphere with a uniform volume charge density.
• Use Gauss’s Law to find the electric field inside the sphere. (Ch. 23)
• Side effect: again prove the shell theorem!
• Use that solution to find potential inside and outside this sphere. (Ch. 24)

Uniformly charged sphere

\( \rho = \text{volume charge density} \)
\( Q = \text{Charge/Volume} = \text{Coulomb/Meter} \)
\( \rho = \text{volume charge density} \)

Total charge
\( Q = \frac{4\pi}{3} R^3 \rho \)

Field outside
\( E = \frac{Q}{\varepsilon_0 \rho r^2}, \quad \text{at} \ r > R \)

Field inside
\( Q \Rightarrow Q(r) = \frac{4\pi}{3} r^3 \rho, \quad E = \frac{Q(r)}{\varepsilon_0 \rho r^2} = \frac{4\pi}{3\varepsilon_0} \rho \pi r^2, \quad \text{at} \ r < R \)
Q.23-3

If the uniformly charged sphere has radius \( R = 2 \) cm and total charge \( Q = 24 \) nC, how much charge \( Q_{in} \) lies within \( r = 1 \) cm of the center?

1) 12 nC  
2) 8 nC  
3) 6 nC  
4) 4 nC  
5) 3 nC

Solution:

Volume of sphere is \( (\text{vol.}) = \frac{4}{3} \pi r^3 \)

So charge inside is \( Q = (\text{vol.}) \rho = \frac{4}{3} \pi r^3 \rho \)

So charge within a sphere is proportional to the radius cubed: \( Q \propto r^3 \)

So if sphere with \( r = 1 \) has \((1/2)^3 = 1/8 \) the volume, and \(1/8 \) the charge.

\[ Q_{in} = (1/8)Q_{tot} = \frac{24 \text{ nC}}{8} = 3 \text{ nC} \]

Field due to solid charged sphere.

So the field inside is: \( E = \frac{\rho r}{3\varepsilon_0} \)

And the field outside is: \( E = \frac{Q}{4\pi\varepsilon_0 r^2} \)

Exercise for the student:

Prove these two formulas agree when \( r = R \).

Hint: use \( Q = \frac{4}{3}\pi R^3 \rho \)

Potential Difference

Continuing with the uniformly charged sphere problem.

Suppose we have some given numbers:

\( R = 3 \) cm \quad \rho = 3 \times 10^{-6} \text{ C/m}^3

( So that \( \frac{4}{3}\pi R^3 \rho = 3.4 \times 10^{-36} \text{ C} = 0.34 \text{ nC} \) )

Then

\[ \Delta V = \frac{\rho R^2}{6\varepsilon_0} = \frac{3 \times 10^{-6} \times (3 \times 10^{-2})^2}{6 \times 9 \times 10^{-12}} = 50 V \]
Q.24-3 Using the same numbers, what is the potential at the surface of this sphere?

\[ \rho = 3 \times 10^{-6} \text{ C/m}^3 \]
\[ Q = 3.4 \times 10^{-19} \text{ C} \]

1) 250 V
2) 100 V
3) 50 V
4) 25 V
5) 0

Q.24-3 Outside, E is given by Coulomb’s Law (shell theorem), so work to bring in test charge from infinity is same as for point charge:

\[ V = kQ / r = (9 \times 10^9)(3.4 \times 10^{-19}) = 102 V \]

1) 250 V
2) 100 V
3) 50 V
4) 25 V
5) 0

A new unit of energy

• One electron-volt (eV) is the energy to move an electron between points with a potential difference of one volt.
• This is not an SI unit but is universally used in discussing processes involving electrons, atoms, nano-scale structures, etc.

\[ U = qV \]

So if \( V = 1 \text{ V} \) and \( q = e \) then:

\[ U = e \times V = (1 \text{ V})(e) = 1 \text{ eV} \]
\[ = (1 \text{ V}) \times (1.6 \times 10^{-19} \text{ C}) = 1.6 \times 10^{-16} \text{ J} \]

A new unit of energy

\[ 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \]

So in our previous example, where we have \( V = 50 \text{ V} \) and \( q = e \) then the work required to move an electron from the center to the surface is:

\[ U = qV = (50 \text{ V})(e) = 50 \text{ eV} \]
\[ = (1.6 \times 10^{-19} \text{ C}) \times (50 \text{ V}) = 8 \times 10^{-18} \text{ J} \]

Much simpler to just give 50 eV as the answer.

More about units

• The SI unit for potential is the volt.
• The SI unit for electric field is N/C.
• But \( E = dV/dx \) so another unit for E could be volts/meter.

It is correct to say that the SI unit for \( E \) is V/m.

This is more commonly used in applications.

Equipotential surfaces

Any surface which is everywhere perpendicular to the electric field is an equipotential surface. The potential has the same value everywhere on this surface. This is true because no work is required to move a test charge from place to place on the surface.
For a point charge, equipotentials are spheres:

For a dipole, the shape of the equipotentials is more complicated:

Electron gun: potential $V$ gives electron energy in eV.  
\[ K = \frac{1}{2} m v^2 = qV \]
So if $V = 500$ volts, electron energy is $K = 500$ eV

Be careful with SI units
Suppose we want to calculate the speed of this electron. We must first convert the energy from eV to SI units.
\[
K = \frac{1}{2} m v^2 = qV = 500 \text{ eV} \\
= (500 \text{ eV}) \times (1.6 \times 10^{-19} \text{ J/eV}) = 8 \times 10^{-17} \text{ J} \\
v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 8 \times 10^{-17} \text{ J}}{9.1 \times 10^{-31} \text{ kg}}} = 1.3 \times 10^7 \text{ m/s}
\]

High energies
- For X-ray machines, accelerate electrons with potentials of thousands of volts, so we speak of kinetic energies in keV: \(10^3\) eV
- In nuclear physics, accelerators produce beams of particles with energies in MeV: \(10^6\) eV
- In elementary-particle physics, high-energy particles beams have energies measured in giga-volts: GeV: \(10^9\) eV