Potentials and Fields

Note on tomorrow's quiz:

There will be 10 multiple-choice questions.

Review: Definition of Potential

- *Potential* **is defined as** *potential energy* **per unit charge. Since change in potential energy is work done, this means**

$$\Delta V = -\int E_x dx \qquad \text{and} \qquad E_x = -\frac{dV}{dx} \quad etc.$$

The potential difference between any two points is the work required to carry a unit positive test charge between those two points.

Review: Coulomb's Law

So we now have a ***third form of Coulomb's Law:***

1. $F = kQq/r^2$
2. $E = kQ/r^2$
3. $V = kQ/r$

Review: Basics about Potentials

- **U = qV**
- **V = kQ/r**
- $\mathbf{W_{AB} = q\,(V_B - V_A)}$

$$\Delta V = -\int E_x dx$$

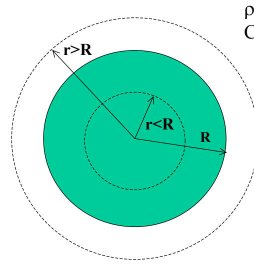
$$E_x = -\frac{dV}{dx} \quad etc.$$

Field of a Uniformly Charged Sphere

Good example for main ideas of this week.

- **Given a non-conducting sphere with a uniform volume charge density.**
- **Use *Gauss's Law* to find the electric field inside the sphere. (Ch. 23)**
- **Side effect: again prove the shell theorem!**
- **Use that solution to *find potential* inside and outside this sphere. (Ch. 24)**

Uniformly charged sphere



ρ= volume charge density= Charge/Volume= Coulomb/Meter

Total charge

$$Q = \frac{4\pi}{3} R^3 \rho$$

Field outside

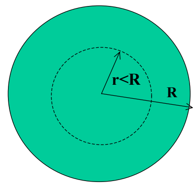
$$E = \frac{Q}{\varepsilon_0 r^2}, \quad r > R$$

Field inside

$$Q \Rightarrow Q(r) = \frac{4\pi}{3} r^3 \rho, \qquad E = \frac{Q(r)}{\varepsilon_0 r^2} = \frac{4\pi}{3\varepsilon_0} \rho r, \quad r < R$$

Q.23-3

If the uniformly charged sphere has radius R=2 cm and total charge Q = 24 nC, how much charge Q_{in} lies within r = 1 cm of the center?



1) 12 nC
2) 8 nC
3) 6 nC
4) 4 nC
5) 3 nC

Q.23-3

Solution:

Volume of sphere is $(Vol.) = \frac{4}{3}\pi r^3$

So charge inside is $Q = (Vol.)\rho = \frac{4}{3}\pi r^3 \rho$

So charge within a sphere is proportional to the radius cubed: $\boxed{Q \propto r^3}$

So if sphere with r=1 has (1/2)3 = 1/8 the volume, and 1/8 the charge.

$$Q_{in} = (1/8)Q_{tot} = \frac{24\ nC}{8} = 3\ nC$$

Field due to solid charged sphere.

So the field inside is: $E = \frac{\rho r}{3\varepsilon_0}$

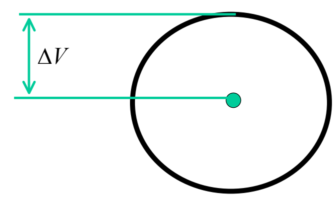
And the field outside is: $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

Exercise for the student:

Prove these two formulas agree when r=R.

Hint: use $Q = \frac{4}{3}\pi R^3 \rho$

Potential Difference



Continuing with the uniformly charged sphere problem.

- What is the potential difference between the center of the sphere and the surface?
- How much work would it require to move an electron *from* the center *to* the surface?

By definition $\Delta V = -\int_R^0 \vec{E}\cdot d\vec{r} = \int_0^R E\,dr$

But we already know $E = \frac{\rho r}{3\varepsilon_0}$

Therefore

$$\Delta V = \int_0^R \frac{\rho r}{3\varepsilon_0} dr = \frac{\rho}{3\varepsilon_0}\int_0^R r\,dr = \frac{\rho}{3\varepsilon_0}\frac{R^2}{2} = \frac{\rho R^2}{6\varepsilon_0}$$

Suppose we have some given numbers:

$R = 3\ cm \qquad \rho = 3\times 10^{-6}\ C/m^3$

(So that $Q = \frac{4}{3}\pi R^3 \rho = 3.4\times 10^{-10}\ C = 0.34\ nC$)

Then

$$\Delta V = \frac{\rho R^2}{6\varepsilon_0} = \frac{3\times 10^{-6}\times(3\times 10^{-2})^2}{6\times 9\times 10^{-12}} = \underline{50\ V}$$

Q.24-3 Using the same numbers, what is the potential at the surface of this sphere?

$R = 3\ cm$

$\rho = 3 \times 10^{-6}\ C/m^3$

$Q = 3.4 \times 10^{-10}\ C$

1) 250 V
2) 100 V
3) 50 V
4) 25 V
5) 0

Q.24-3 Outside, E is given by Coulomb's Law (shell theorem), so work to bring in test charge from infinity is same as for point charge: $E = kQ/r^2 \qquad V = kQ/r$

$$V = kQ/R = \frac{\left(9\times10^{9}\right)\left(3.4\times10^{-10}\right)}{3\times10^{-2}} = 102\ V$$

1) 250 V
2) $\boxed{\text{100 V}}$
3) 50 V
4) 25 V
5) 0

A new unit of energy

- One *electron-volt* (eV) is the energy to move an electron between points with a potential difference of one volt.
- This is not an SI unit but is universally used in discussing processes involving electrons, atoms, nano-scale structures, etc.

$U = qV$

So if V=1 V and q=e then:

$$U = e \times V = (1\ V)(e) = \underline{1\ eV}$$
$$= (1\ V) \times (1.6 \times 10^{-19}\ C) = \underline{1.6 \times 10^{-16}\ J}$$

A new unit of energy

$$1\ eV = 1.6 \times 10^{-19}\ J$$

So in our previous example, where we have V=50 V and q=e then the work required to move an electron from the center to the surface is:

$$U = qV = (50\ V)(e) = 50\ eV$$
$$= (1.6 \times 10^{-19}\ C) \times (50\ V) = 8 \times 10^{-18}\ J$$

Much simpler to just give 50 eV as the answer.

More about units

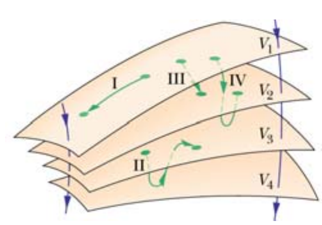
- The SI unit for potential is the volt.
- The SI unit for electric field is N/C.
- But E = dV/dx so another unit for E could be volts/meter.

It is correct to say that the SI unit for E is V/m.

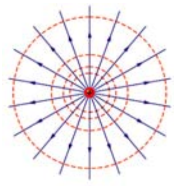
This is more commonly used in applications.

Equipotential surfaces

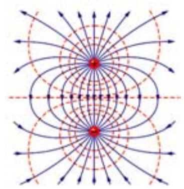
Any surface which is everywhere ***perpendicular*** to the electric field is an equipotential surface. The potential has the same value everywhere on this surface. This is true because no work is required to move a test charge from place to place on the surface.



For a point charge, equipotentials are spheres:


(b)

For a dipole, the shape of the equipotentials is more complicated: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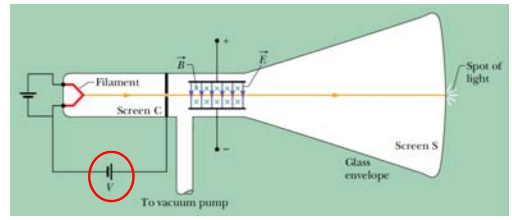
Cathode Ray Tube



Electron gun: potential V gives electron energy in eV.

$$K = \frac{1}{2}mv^2 = qV$$

So if V = 500 volts, electron energy is K = 500 eV

Be careful with SI units

Suppose we want to calculate the speed of this electron. We must first convert the energy from eV to SI units.

$$K = \frac{1}{2}mv^2 = qV = 500\ eV$$

$$= (500\ eV) \times (1.6 \times 10^{-19}\ J/eV) = 8 \times 10^{-17}\ J$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 8 \times 10^{-17}\ J}{9.1 \times 10^{-31}\ kg}} = 1.3 \times 10^{7}\ m/s$$

High energies

- **For X-ray machines, accelerate electrons with potentials of thousands of volts, so we speak of kinetic energies in *keV.*** $10^3\ eV$
- **In nuclear physics, accelerators produce beams of particles with energies in *MeV.*** $10^6\ eV$
- **In elementary-particle physics, high-energy particles beams have energies measured in giga-volts: *GeV.*** $10^9\ eV$