DC Circuits

- Resistance Review
- Following the potential around a circuit
- Multiloop Circuits
- RC Circuits

Homework for today:

Read Chapters 26, 27

Chapter 26 Questions 1, 3, 10

Chapter 26 Problems 1, 17, 35, 77

Homework for tomorrow:

Chapter 27 Questions 1, 3, 5

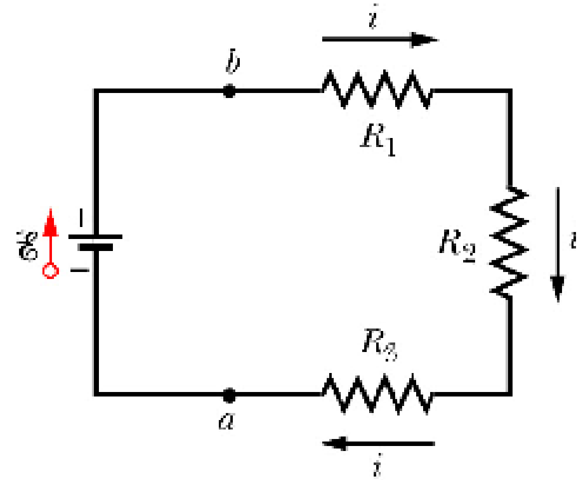
Chapter 27 Problems 7, 19, 49

WileyPlus assignment: Chapters 26, 27

Review: Series and Parallel Resistors

Series:

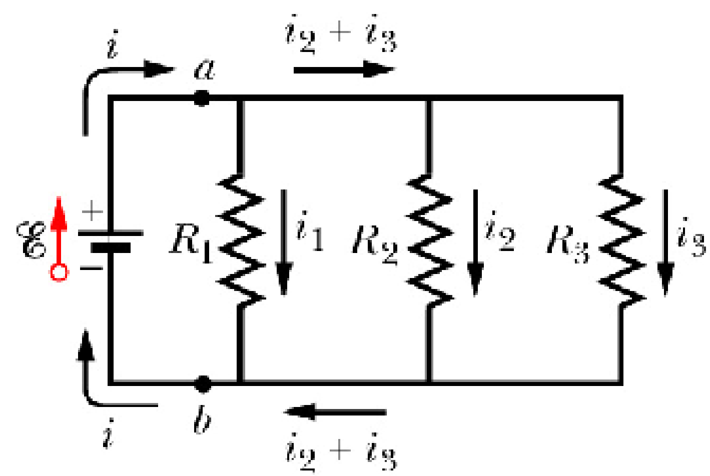
$$R = R_1 + R_2 + R_3$$



Why?

Parallel:

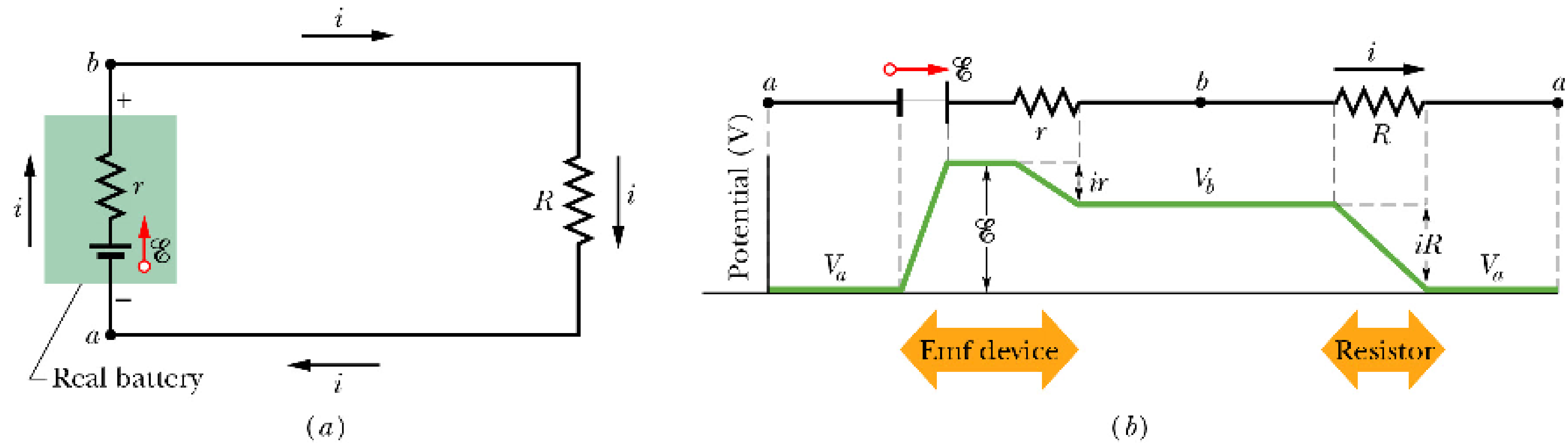
$$\mathbf{1/R = 1/R_1 + 1/R_2 + 1/R_3}$$



Why?

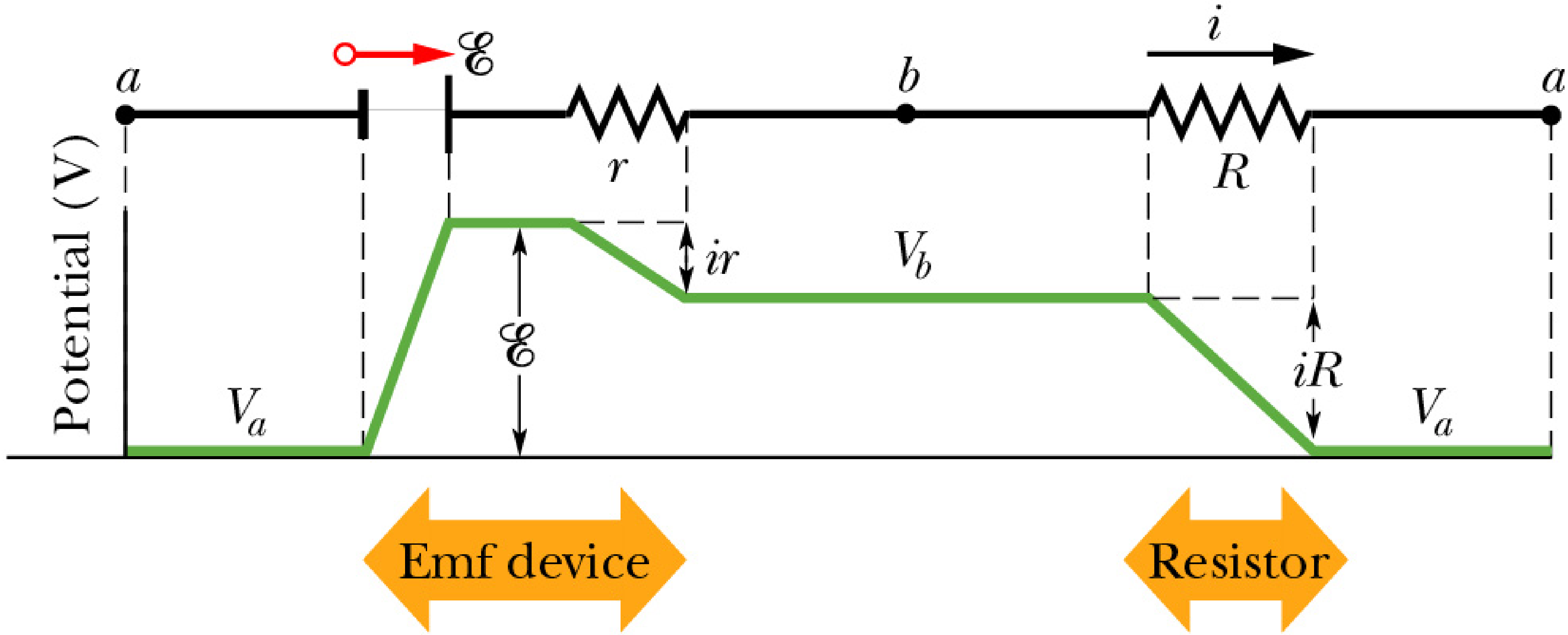
Following the Potential

Study Fig. 27-4 in the text to see how the potential changes from point to point in a circuit.



Note the net change around the loop is zero.

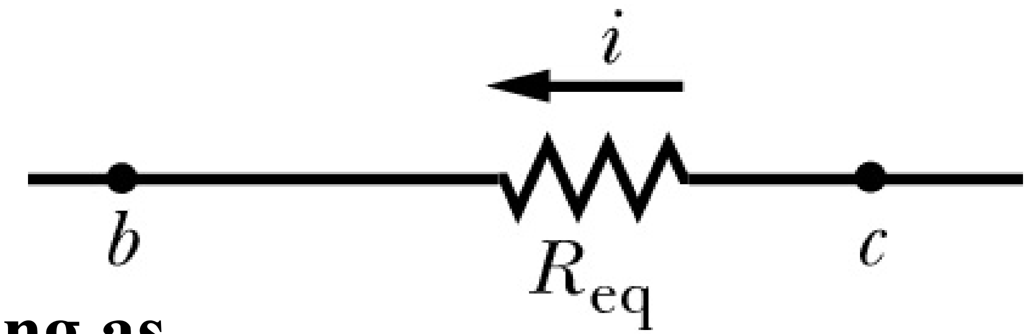
Following the Potential



(b)

Note the net change around the loop is zero.

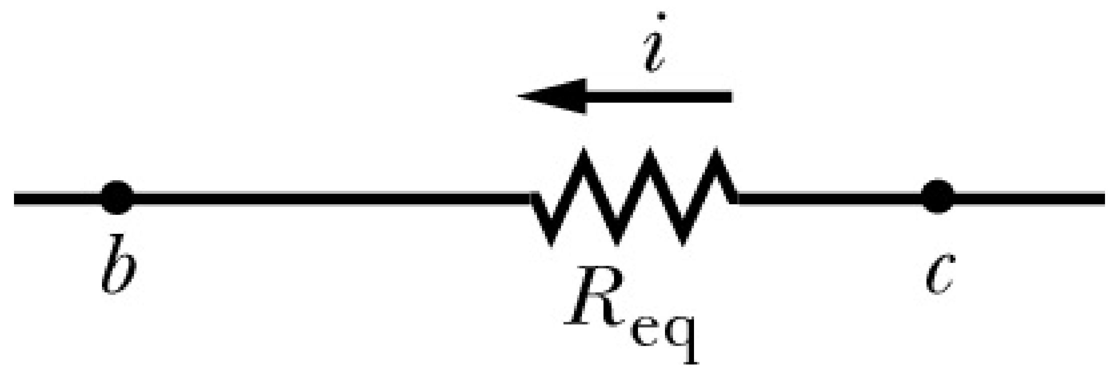
Q.27-1



With the current *i* flowing as shown, which is at the *higher potential*, point *b* or point *c*?

1) **B is higher**
2) **C is higher**
3) **They are the same**
4) **Not enough information**

Solution



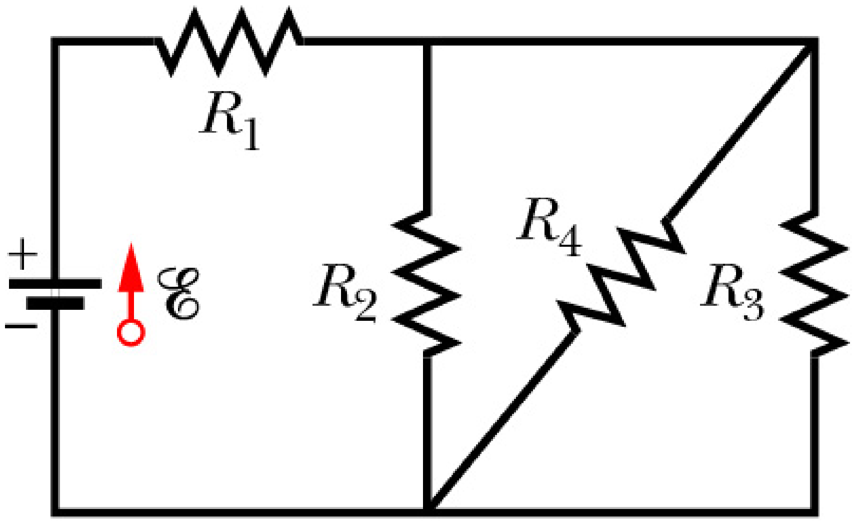
With the current ***i*** flowing as shown, which is at the ***higher potential,*** point ***b*** or point ***c***?

Solution: Current flows from high to low potential just like water flows down hill.

(1) *b* is higher **(2) *c* is higher**

(3) they're the same **(4) not enough info**

Example: Problem 27-30



$$\mathcal{E} = 6.0\,V \quad R_1 = 100\,\Omega$$

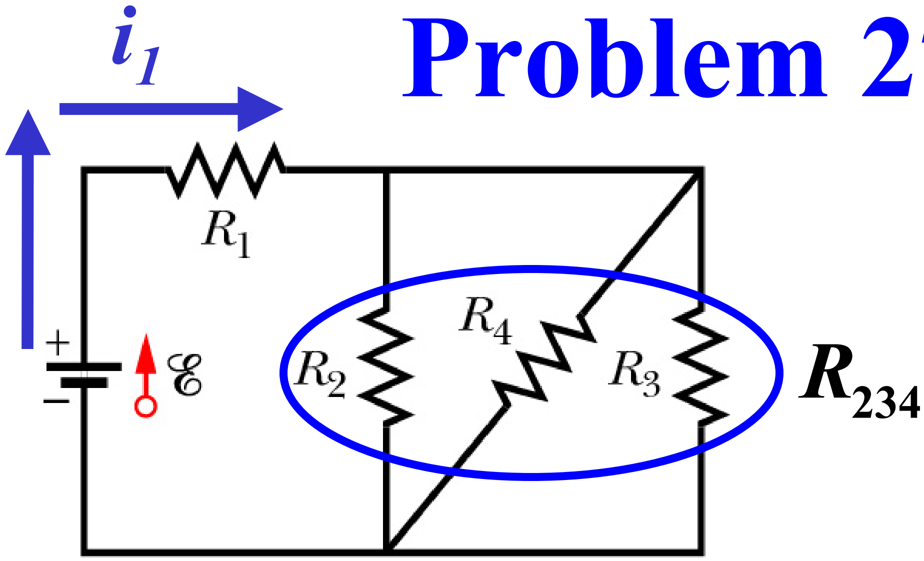
$$R_2 = R_3 = 50\,\Omega$$

$$R_4 = 75\,\Omega$$

(a) Find the equivalent resistance of the network.

(b) Find the current in each resistor.

Problem 27-30 (part a)



$$\mathcal{E} = 6.0\,V \quad R_1 = 100\,\Omega$$

$$R_2 = R_3 = 50\,\Omega$$

$$R_4 = 75\,\Omega$$

(a) Find the equivalent resistance of the network.

$$1/R_{234} = 1/R_2 + 1/R_3 + 1/R_4 = 16/300$$

$$So \quad R_{234} = 300/16 = 19\,\Omega$$

Now R_1 and R_{234} are in series so

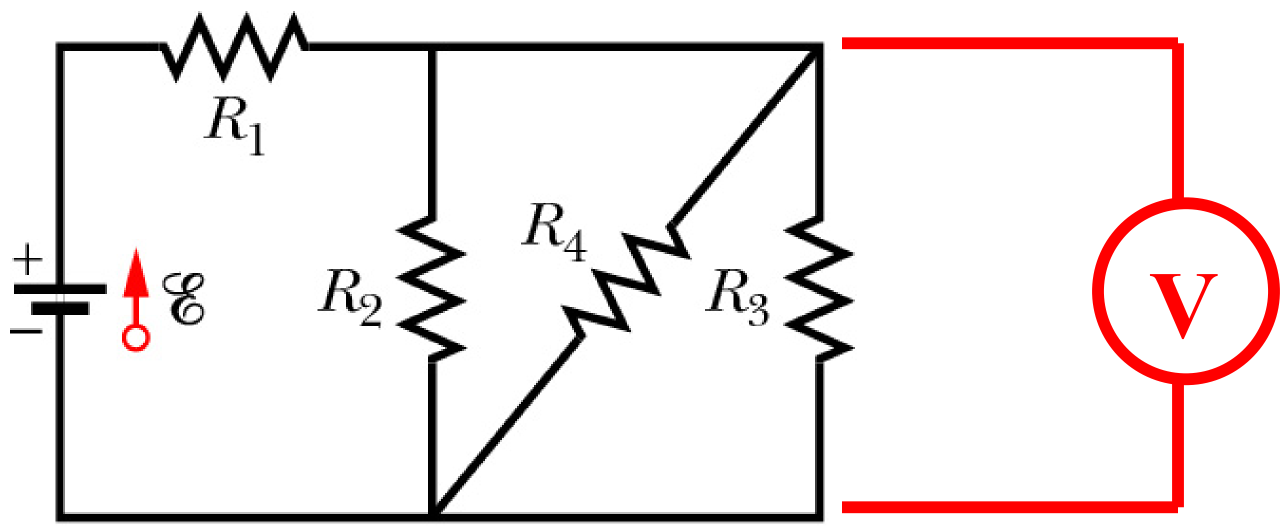
$$R_{eq} = R_1 + R_{234} = 100\,\Omega + 19\,\Omega = \underline{119\,\Omega}$$

(b) Now current $\quad i_1 = \mathcal{E}/R_{eq} = 50\,mA$

Problem 27-30 (part b)

(b) Find the current in each resistor.

First note that $i_2R_2 = i_3R_3 = i_4R_4$.



$$V = iR_{234}$$
$$= .050\,A \times 19\,\Omega$$
$$= 0.95\,V$$

So

$$i_2 = V / R_2 = .95/50 = 19\,mA$$
$$i_3 = V / R_3 = .95/50 = 19\,mA$$
$$i_4 = V / R_4 = .95/75 = 12\,mA$$

Check: These three add up to i_1 = 50 mA.

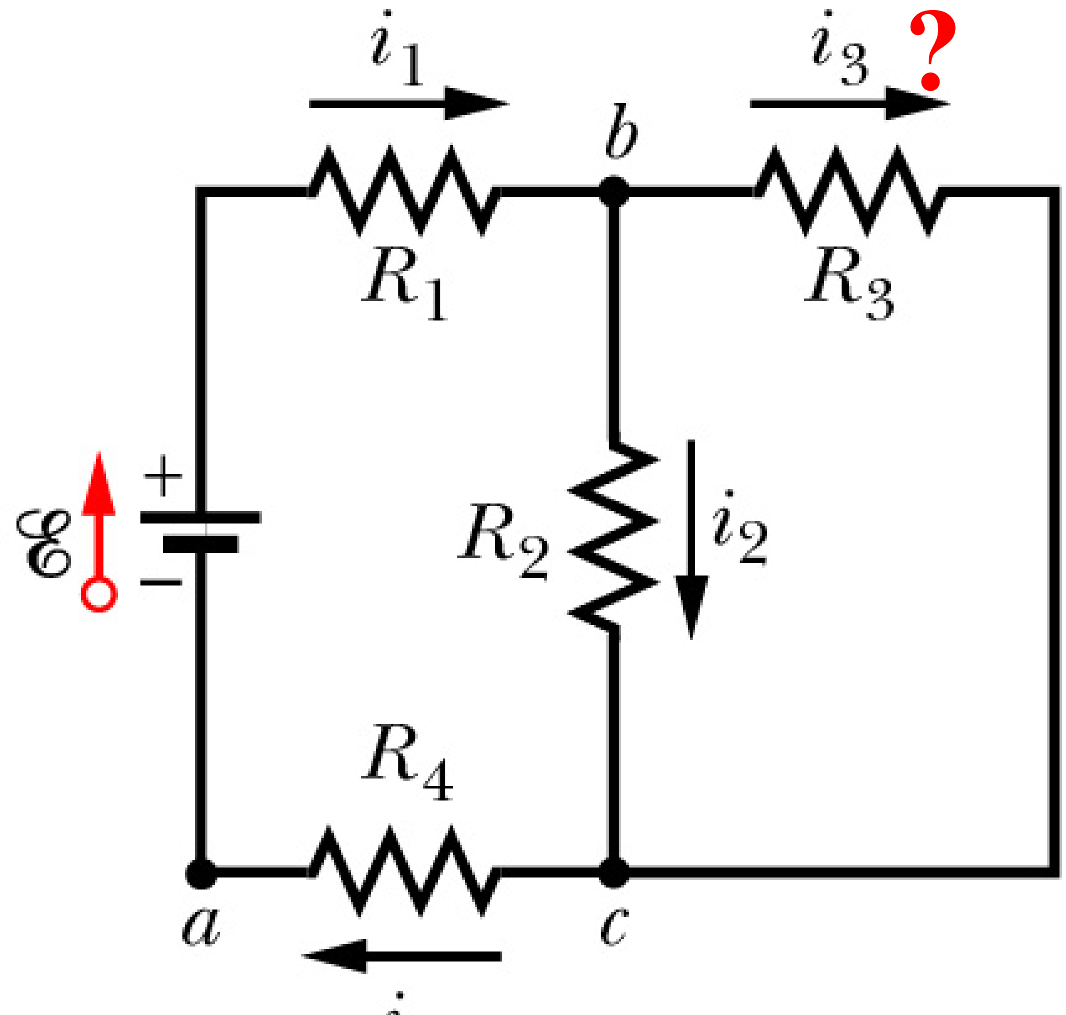
Q.27-2

$R_2 = 2\Omega$

$R_3 = 3\Omega$

$i_2 = 6\,A$

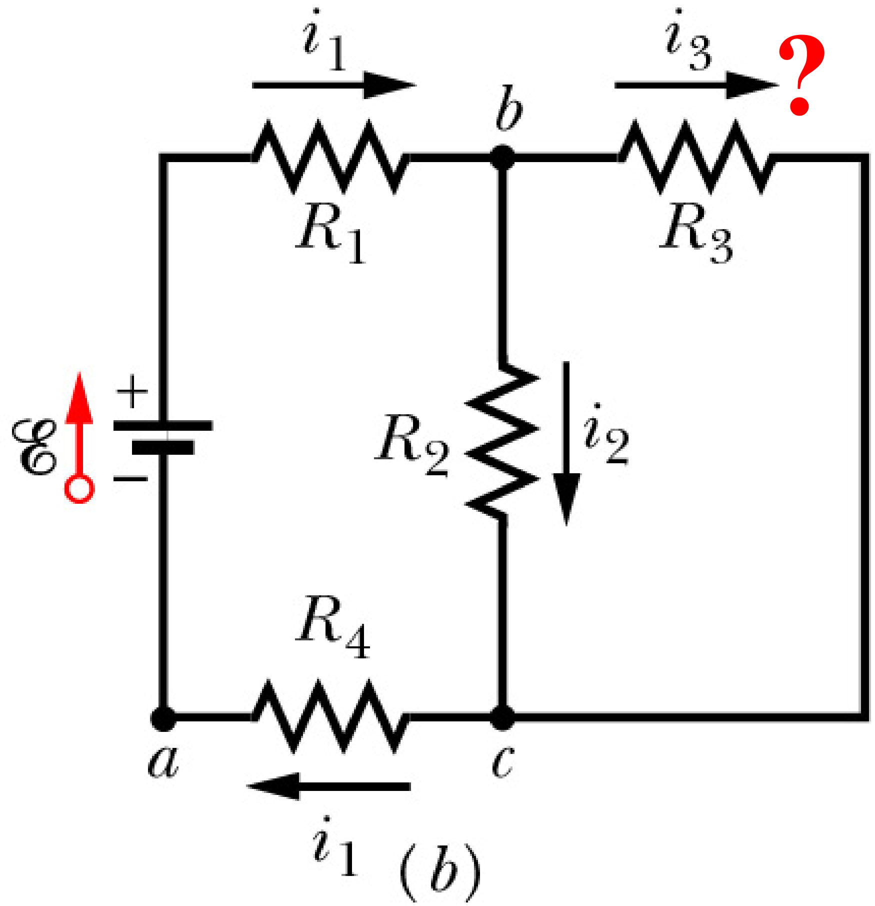
$i_3 = ?$



(b)

Calculate i_3, find the closest single-digit number (0-9).

Q.27-2



$$R_2 = 2\Omega$$

$$R_3 = 3\Omega$$

$$i_2 = 6\ A$$

$$i_3 = ?$$

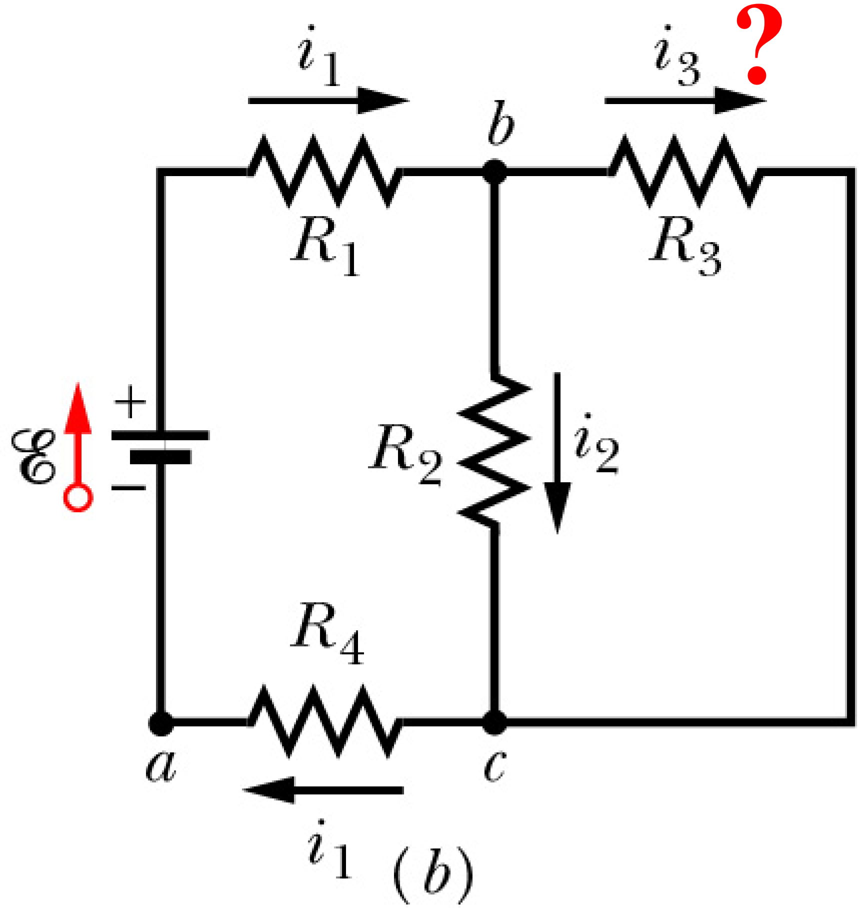
1) 1 A
2) 2 A
3) 3 A
4) 4 A
5) 5 A
6) 6 A
7) 7 A
8) 8 A
9) 9 A

Q.27-2

$$R_2 = 2\,\Omega$$

$$R_3 = 3\,\Omega$$

$$i_2 = 6\,A$$



$$V_b - V_c = i_2 R_2 = i_3 R_3$$

$$\therefore i_3 = \frac{i_2 R_2}{R_3} = \frac{2}{3} i_2 = 4\,A$$

$$\boxed{4}$$

More Complicated Circuits

How do we solve a problem with more than one emf and several loops? We can’t do it just by series and parallel resistor combinations.

Rules for Multiloop Circuits

- ***The net voltage change around any loop is zero.***

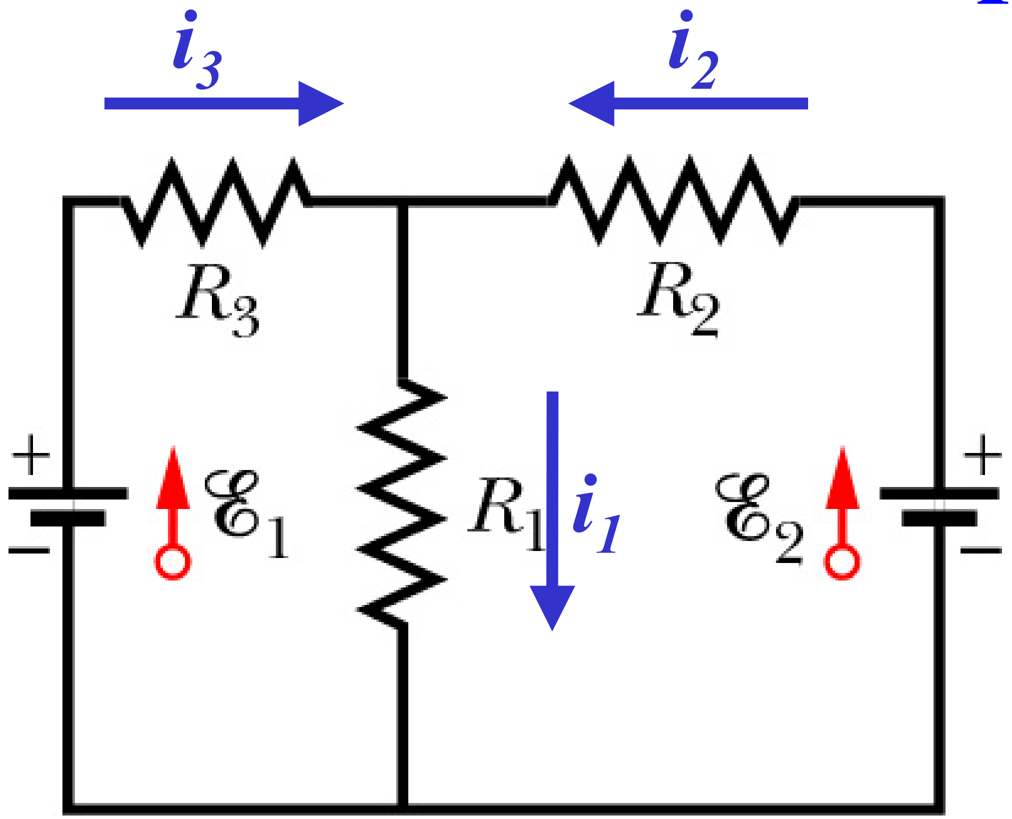
  "Energy conservation"

- ***The net current into any junction is zero.***

  "Charge conservation"

Using these two rules we can always get enough equations to solve for the currents if we are given the emfs and resistances.

Example



$$\mathcal{E}_1 = 24\,V$$

$$\mathcal{E}_2 = 12\,V$$

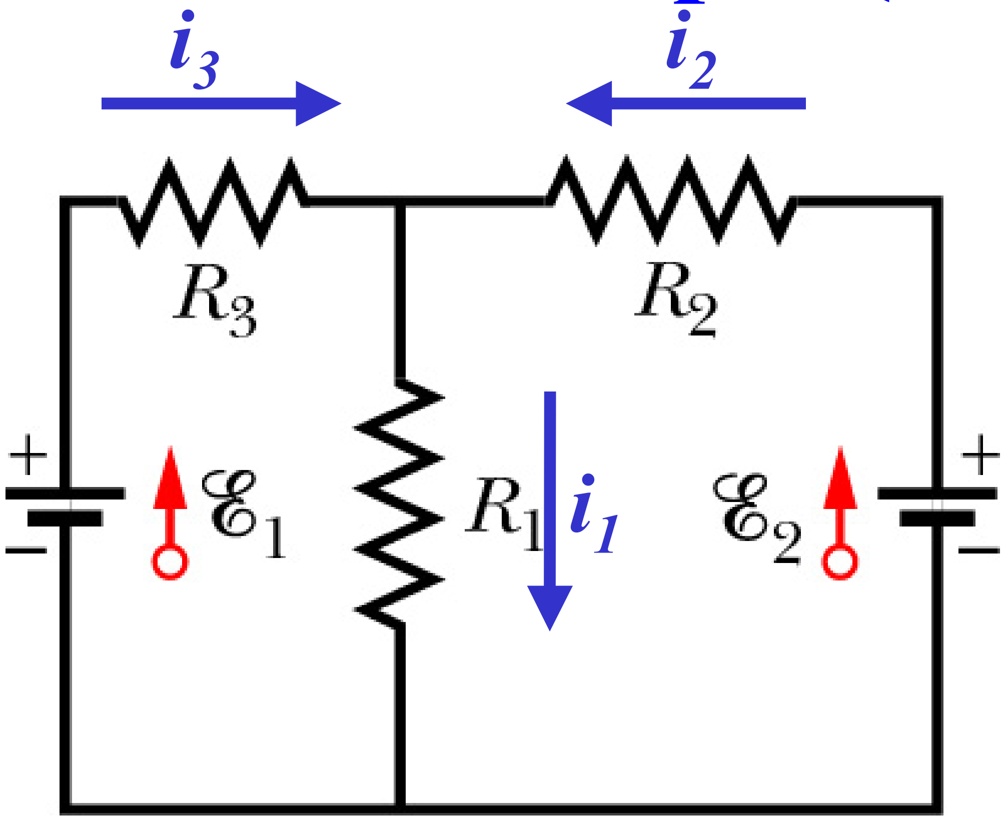
$$R_1 = 5\,\Omega$$

$$R_2 = R_3 = 30\,\Omega$$

Find all currents!

First define unknowns: i_1, i_2, i_3

Example (continued)



Left-hand loop:

$$\mathcal{E}_1 - i_3 R_3 - i_1 R_1 = 0$$

Right-hand loop:

$$\mathcal{E}_2 - i_2 R_2 - i_1 R_1 = 0$$

Junction:

$$i_1 = i_2 + i_3$$

Algebra: solve 3 equations for 3 unknowns i_1, i_2, i_3

Loop and junction equations:

$$\mathcal{E}_1 - i_3 R_3 - i_1 R_1 = 0 \qquad i_1 = i_2 + i_3$$

$$\mathcal{E}_2 - i_2 R_2 - i_1 R_1 = 0$$

Put in the given numbers and also replace i_1 by i_2+i_3:

$$5i_1 + 30i_3 = 5i_2 + 35i_3 = 24$$

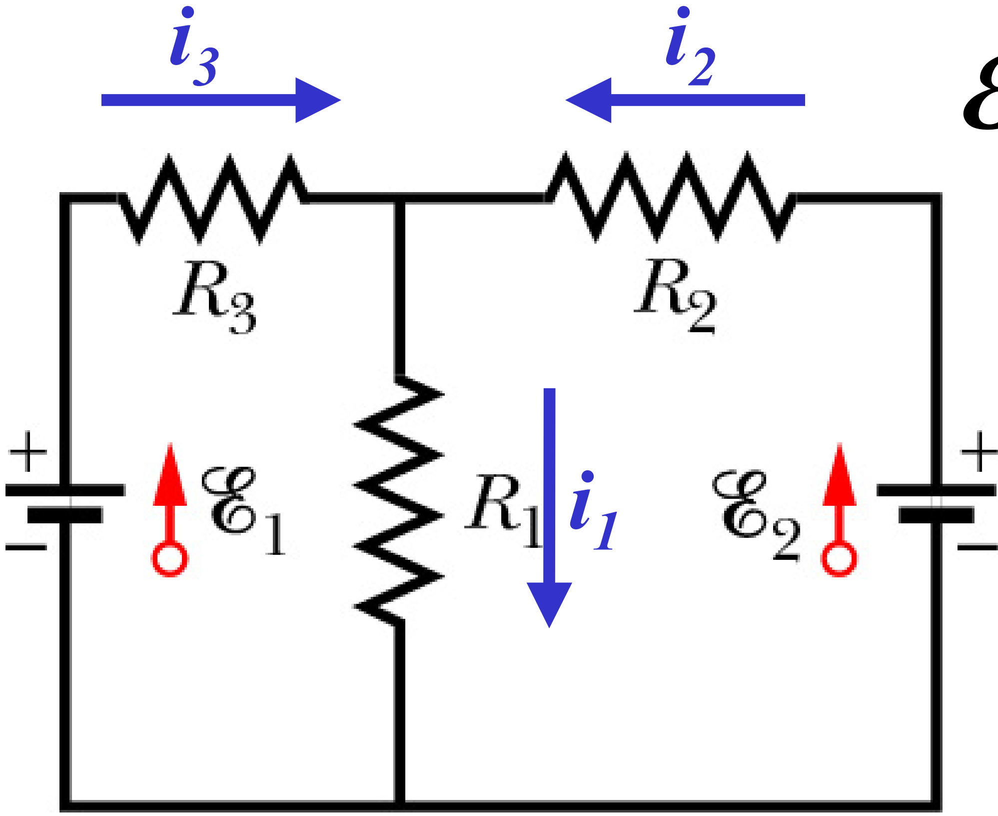
$$5i_1 + 30i_2 = 35i_2 + 5i_3 = 12$$

Solve two equations in two unknowns to get:

$$i_2 = 250\,mA \qquad i_3 = 650\,mA$$

Add to get $\quad i_1 = i_2 + i_3 = 900\,mA$

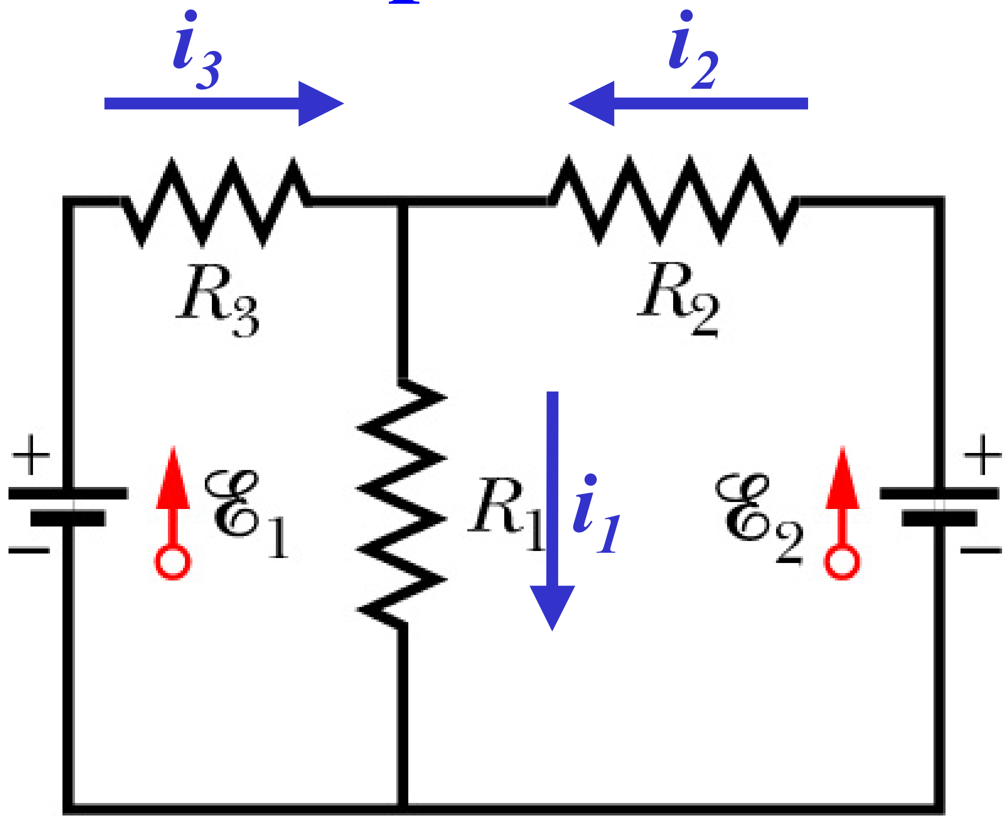
Check by using outer loop:



$$\mathcal{E}_1 - i_3 R_3 + i_2 R_2 - \mathcal{E}_2$$
$$= 0\ ?$$

$$24 - 30(.65 - .25) - 12$$
$$= 12 - 30 \times .40$$
$$= 12 - 12$$
$$= 0$$

Repeat with a different R_1



$$\mathcal{E}_1 = 24V$$

$$\mathcal{E}_2 = 12V$$

$$\boxed{R_1 = 40\Omega}$$

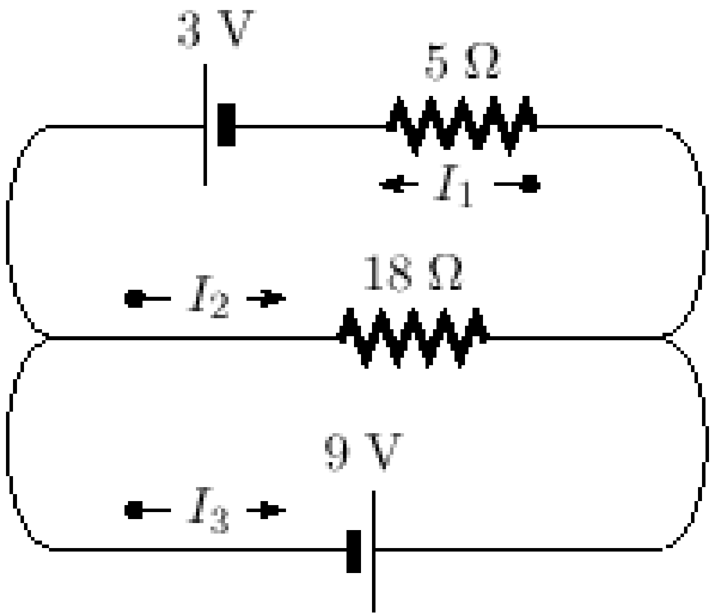
$$R_2 = R_3 = 30\Omega$$

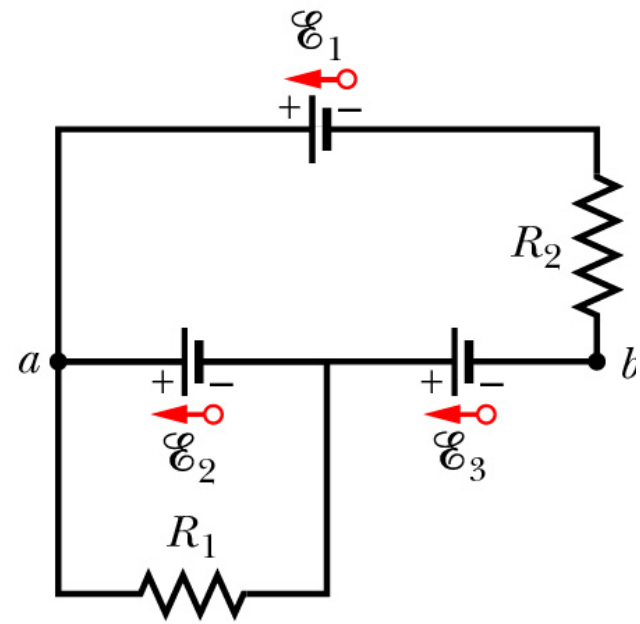
Exercise for the student: Same equations give *negative* i_2 in this case! This means *current going downward* through right-hand battery.

Back to Basics

- Examples that don’t involve so much algebra, but focus on the ideas of current and voltage.
- Even though you have a multiloop circuit so you need to write down the equations from the loop rule and the junction rule, you may not have to actually solve simultaneous equations.

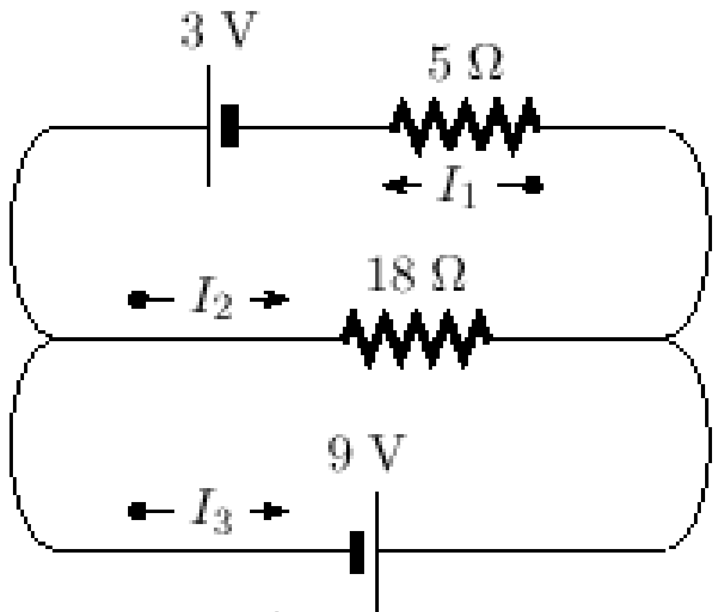
Simpler Examples





Textbook homework problem 27-19

Both these problems can be solved for ***one unknown at a time,*** without messy algebra.



$$9 + 18 \times I_2 = 0 \qquad I_2 = -0.5\ A$$

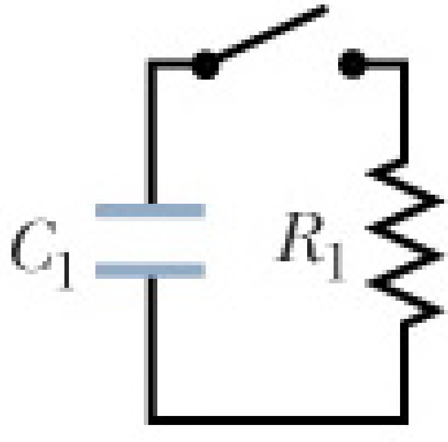
$$9 - 5 \times I_1 + 3 = 0 \qquad I_1 = \frac{12}{5} = 2.4\ A$$

$$I_3 = I_1 - I_2 = 2.9\ A$$

Check:

$$P_{in} = 9I_3 + 3I_1 = 33.3\ W \qquad P_{out} = 5I_1^{\,2} + 18I_2^{\,2} = 33.3W$$

Discharging a Capacitor



Capacitor has charge Q_0.

At time t=0, close switch.

What is charge q(t) for t>0?

Obviously q(t) is a function which decreases gradually, approaching zero as t approaches infinity.

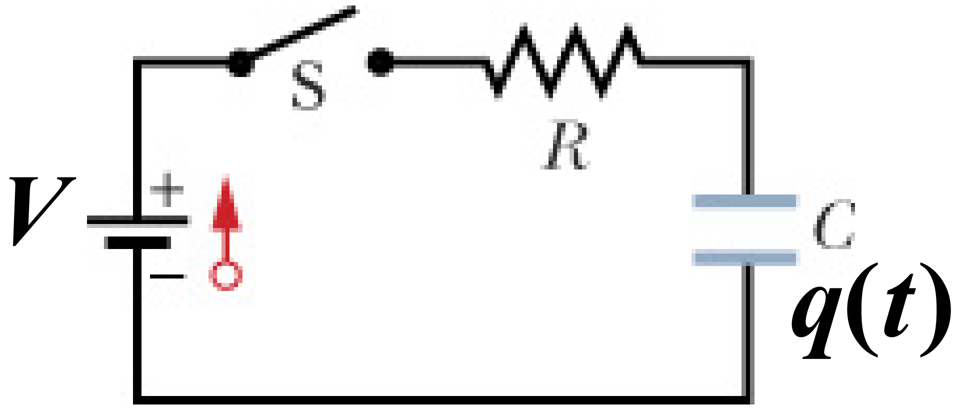
What function would do this?

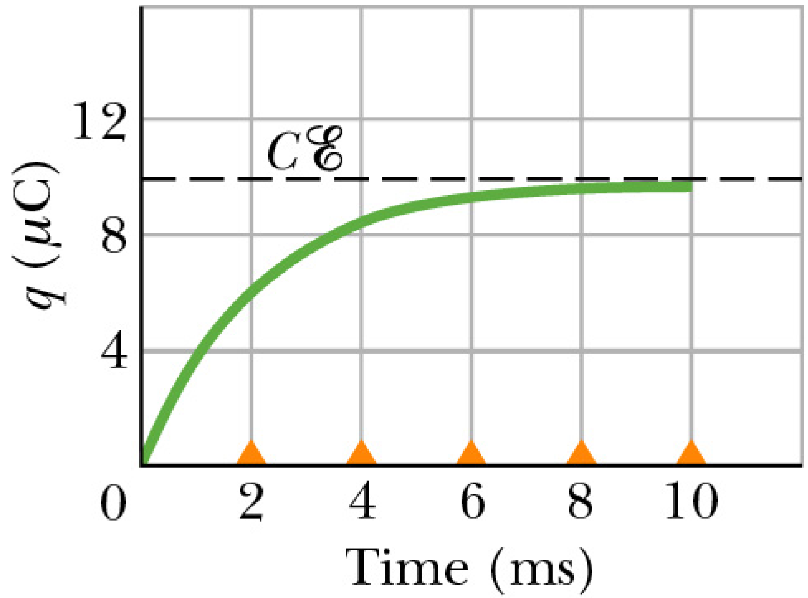
$$q(t) = Q_0 e^{-t/\tau}$$

But what is the time constant ♦?

Analyze circuit equation: find $\tau = RC$

Charging a Capacitor





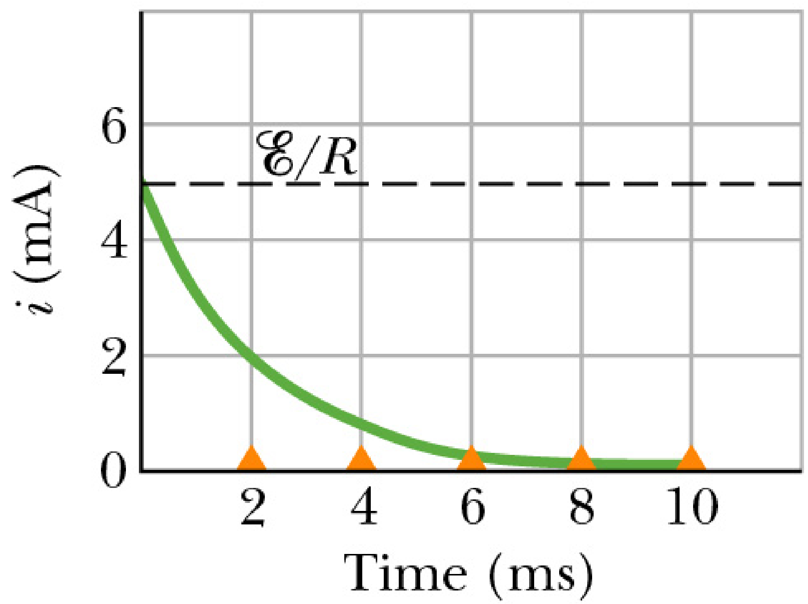
(a)

For small t, q=0 and i=V/R.

For large t, q=CV and i=0.

$$q(t) = CV\left[1 - e^{-t/\tau}\right]$$

$$\tau = RC$$



(b)

DC Circuits II

- Circuits Review
- RC Circuits
- Exponential growth and decay

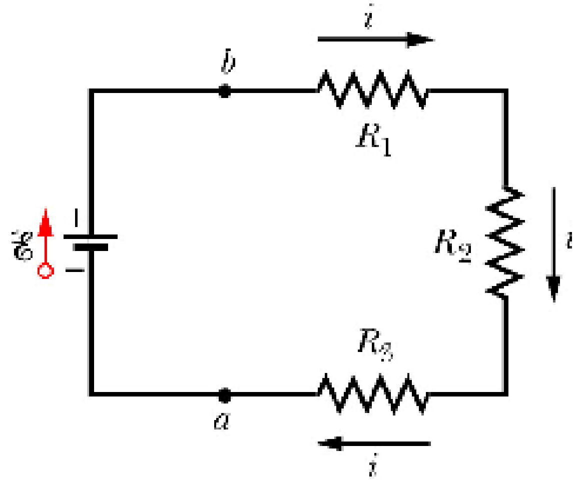
Circuits review so far

- **Resistance and resistivity** $R = \rho l / A$
- **Ohm's Law and voltage drops** $\Delta V = -iR$
- **Power and Joule heating** $P = iV$
- **Resistors in series and parallel**
- **Loop and junction rules**

Review: Series and Parallel Resistors

Series:

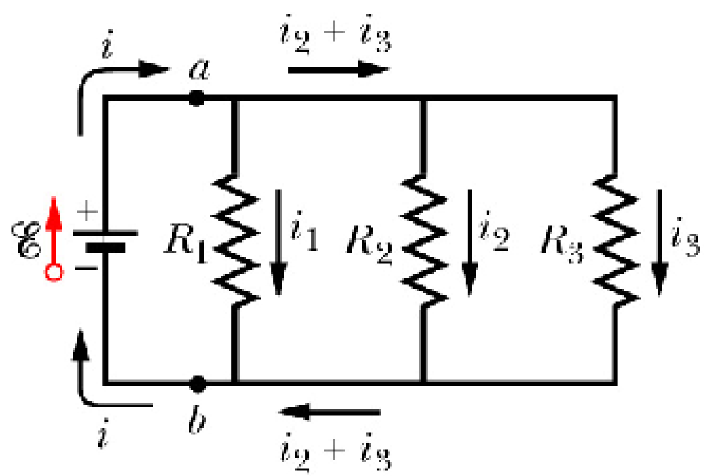
$$R = R_1 + R_2 + R_3$$



Why?

Parallel:

$$\mathbf{1/\mathit{R} = 1/\mathit{R}_1 + 1/\mathit{R}_2 + 1/\mathit{R}_3}$$



Why?

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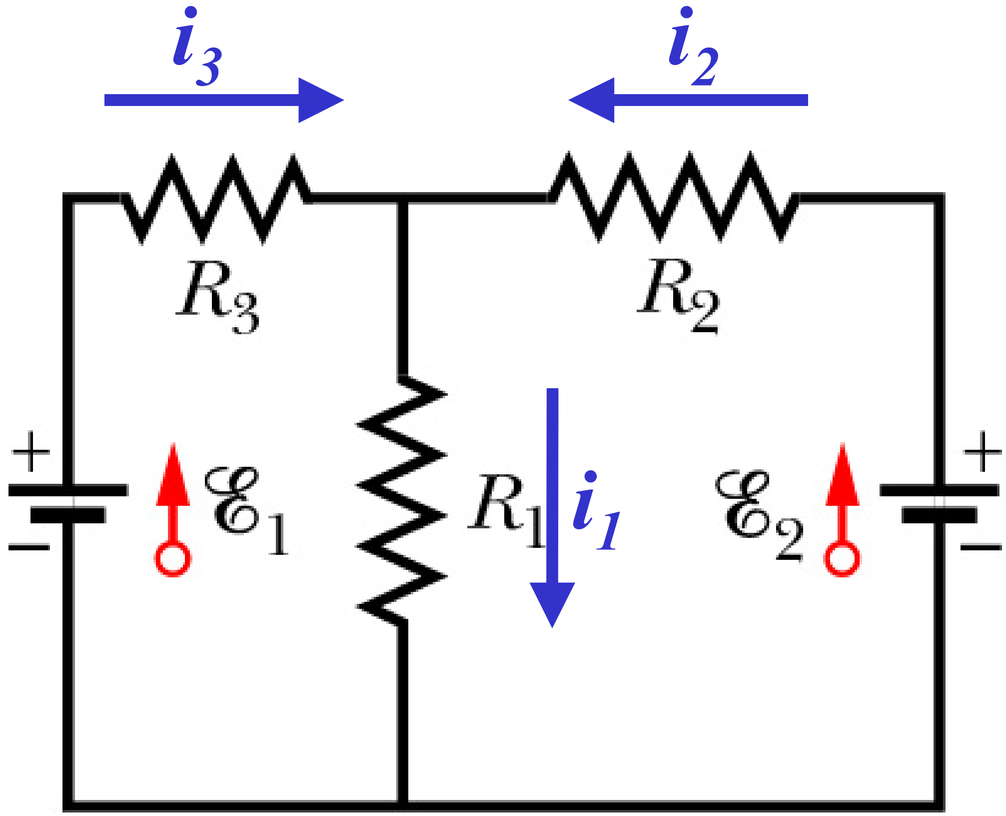
Using these two rules we can always get enough equations to solve for the currents if we are given the emfs and resistances.

Review example



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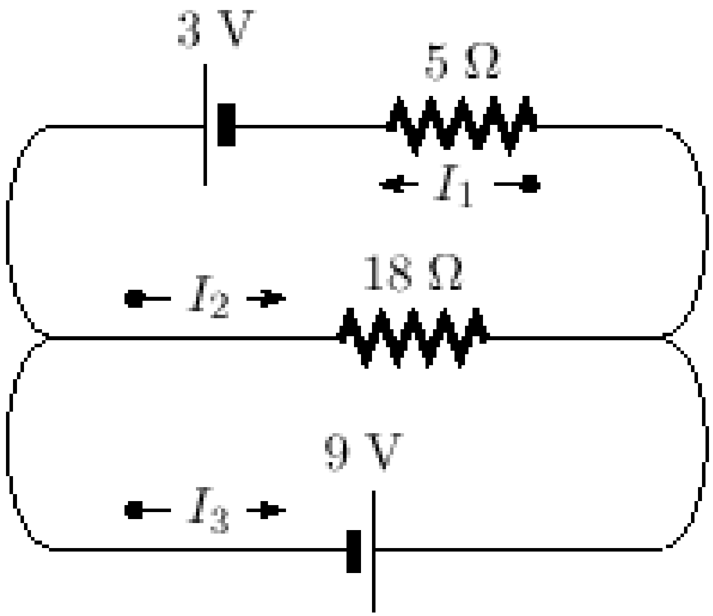
Junction:

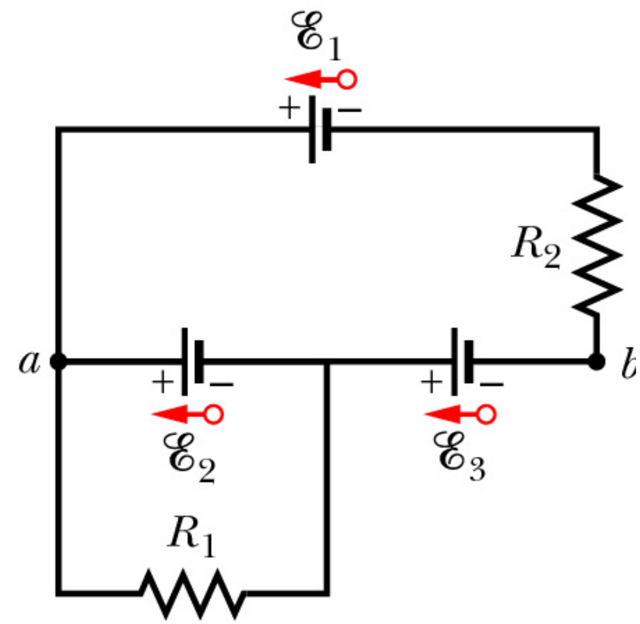
$$i_1 = i_2 + i_3$$

Algebra: solve 3 equations for 3 unknowns i_1, i_2, i_3

If any $i < 0$, current flows the opposite direction.

Simpler Examples





Textbook homework problem 27-19

Both these problems can be solved for ***one unknown at a time,*** without messy algebra.

Q.27-3

Resistors R_1 and R_2 are connected in series. If $R_2 > R_1$, what can you say about the resistance R of the combination?

1. $R > R_2$
2. $R_2 > R > R_1$
3. $R_1 > R$
4. None of the above

Q.27-3

- Resistors R_1 and R_2 are connected in series.
- $R_2 > R_1$.
- What can you say about the resistance R of this combination?

Solution:

$$R = R_1 + R_2 \quad so \quad R > R_2$$

$\boxed{(1)\ R > R_2}$ (2) $R_2 > R > R_1$

(3) $R_1 > R$ (4) *None of the above*

Q.27-4

Resistors R_1 and R_2 are connected <u>in parallel</u>. If $R_2 > R_1$, what can you say about the resistance R of the combination?

1. $R > R_2$
2. $R_2 > R > R_1$
3. $R_1 > R$
4. None of the above

Q.27-4

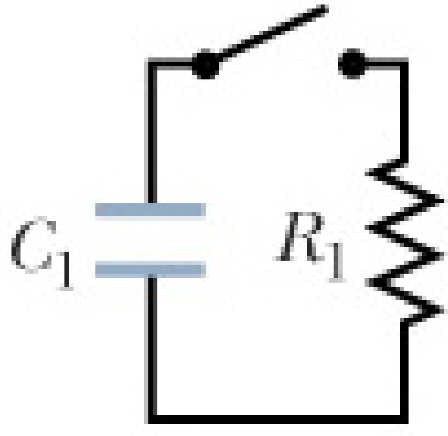
- Resistors R_1 and R_2 are connected in parallel.
- $R_2 > R_1$.
- What can you say about the resistance R of this combination?

Solution: $$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2} \quad so \quad \frac{1}{R}>\frac{1}{R_1}$$

(1) $R > R_2$ (2) $R_2 > R > R_1$

$\boxed{(3)\ R_1 > R}$ (4) *None of the above*

Discharging a Capacitor



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At time t=0, close switch.

What is charge q(t) for t>0?

Obviously q(t) is a function which decreases gradually, approaching zero as t approaches infinity.

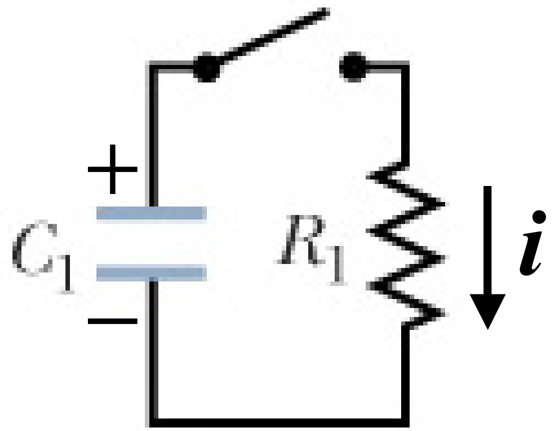
What function would do this?

$$q(t) = Q_0 e^{-t/\tau}$$

But what is the time constant ♦?

Analyze circuit equation: find $\tau = RC$

Discharging a Capacitor



Sum voltage changes around loop:

$$Q/C - iR = 0\ ,\ i = \frac{Q}{RC}$$

But

$$i = -\frac{dQ}{dt}$$

Get ***differential equation*** for *Q(t)*:

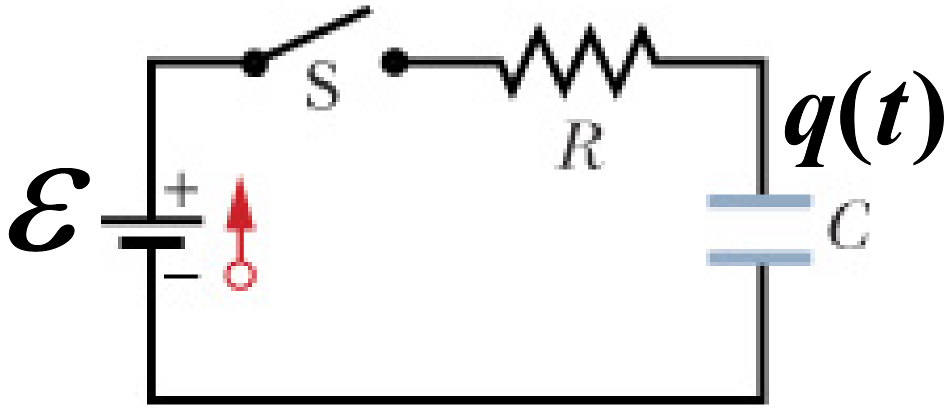
$$\frac{dQ}{dt} = -\frac{Q}{RC}$$

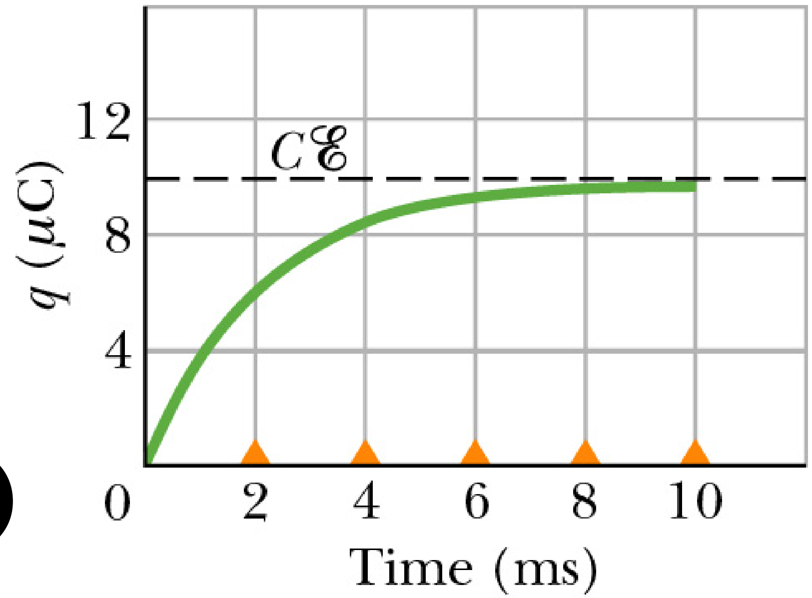
Solution:

$$\boxed{Q(t) = Q_0 e^{-t/\tau}}$$

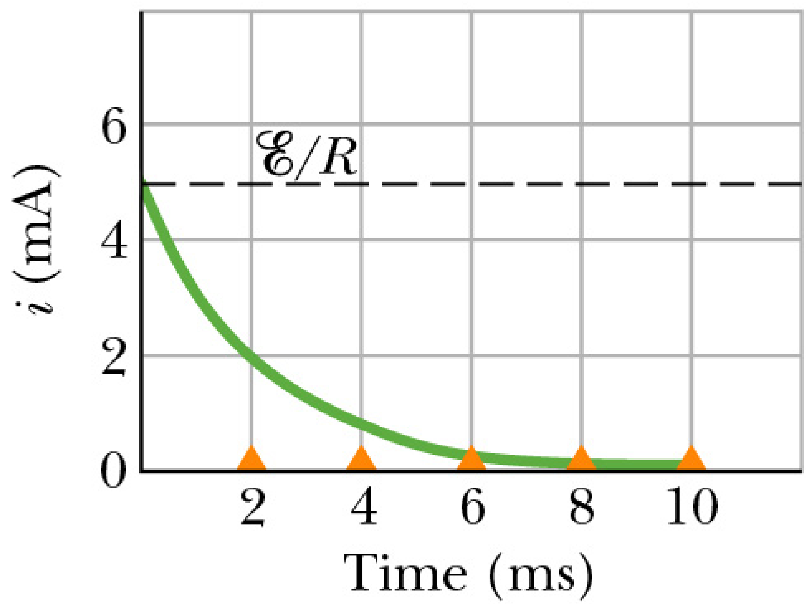
Where ♦ is the time constant $\tau = RC$

Charging a Capacitor





(a)



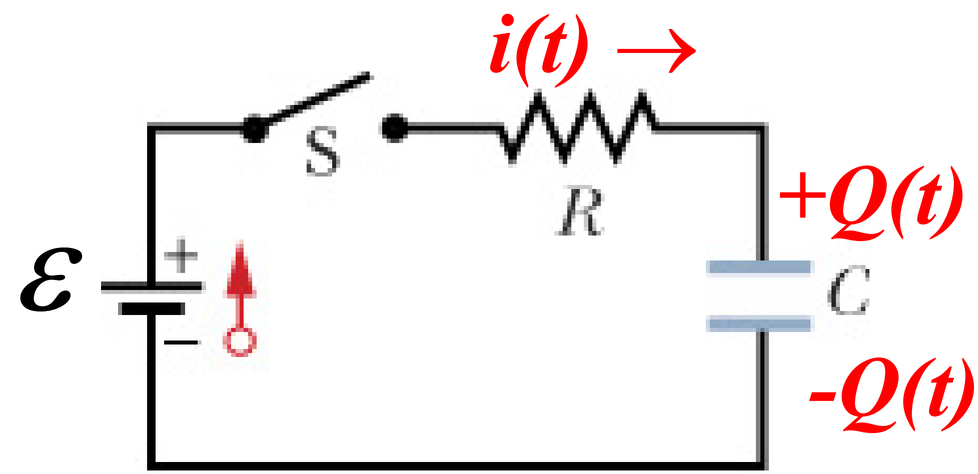
(b)

For small t, q=0 and $i = \mathcal{E} / R$

For large t, i=0 and $q = C\mathcal{E}$

But what are $q(t), i(t)$?

Charging a Capacitor



Sum voltage changes:

$$\mathcal{E} - iR - Q/C = 0$$

$$i = \frac{dQ}{dt}$$

Get diff. eq.: $$\frac{dQ}{dt} = \frac{\mathcal{E}}{R} - \frac{Q}{RC}$$

Solution $$Q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$$\tau = RC$$

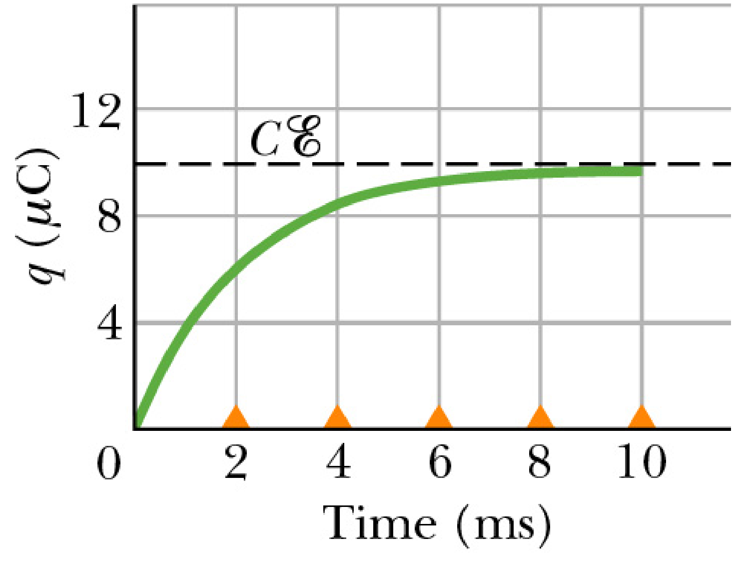
Charging a Capacitor

See solution gives desired behavior:

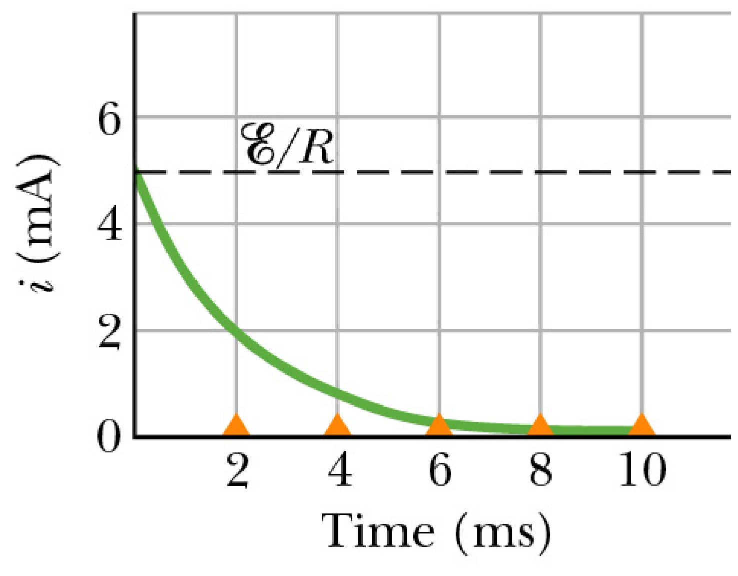
$$Q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$$\rightarrow 0 \quad as\ t \rightarrow 0$$

$$\rightarrow C\mathcal{E} \quad as\ t \rightarrow \infty$$



(a)



(b)

Exponential Growth and Decay

This ***simple differential equation*** occurs in many situations:

$$\boxed{\frac{dQ}{dt} = (Const.)Q}$$

If dQ/dt = +KQ, we have the "snowball" equation: growth rate proportional to size. ***Population growth.***

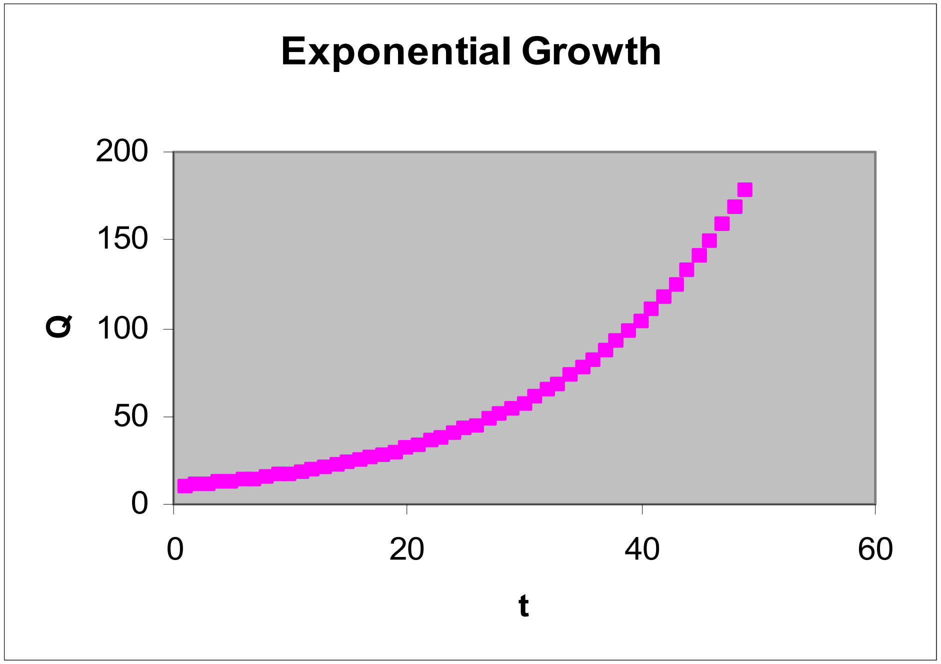
$$\frac{dQ}{dt} = +KQ \longrightarrow Q = Q_0 e^{+Kt}$$

If dQ/dt = -KQ, we have rate of ***decrease*** proportional to size. For example ***radioactive decay.***

$$\frac{dQ}{dt} = -KQ \longrightarrow Q = Q_0 e^{-Kt}$$

Try Exponential Solution

We know we want a result which increases faster and faster. One function which does this is the exponential function. So try that:



$$\textit{To solve} \quad \frac{dQ}{dt} = KQ \quad \textit{try} \quad Q(t) = Q_0 e^{t/\tau}$$

Questions:

1. Is this a solution?
2. If so, what is the ***"time constant"*** τ ?

To solve $\dfrac{dQ}{dt} = KQ$ *try* $Q(t) = Q_0 e^{t/\tau}$

$$Q(t) = Q_0 e^{t/\tau}$$

$$\frac{dQ}{dt} = Q_0 \frac{d}{dt} e^{t/\tau} = \frac{Q_0}{\tau} e^{t/\tau} = \frac{Q}{\tau}$$

But we want $\dfrac{dQ}{dt} = KQ$

So we **DO** have a solution **IF** $\boxed{\tau = 1/K}$

Doubling Time

If $Q(t) = Q_0 e^{t/\tau}$

how long does it take for Q to double?

$$\frac{Q(t+\Delta t)}{Q(t)} = \frac{e^{(t+\Delta t)/\tau}}{e^{t/\tau}} = e^{\Delta t/\tau}$$

$$And \quad e^{\Delta t/\tau} = 2 \quad if \quad \Delta t/\tau = \ln(2) = 0.693$$

$$So \quad \Delta t \cong 0.7\,\tau$$

Radioactive Decay

For an unstable isotope, a certain fraction of the atoms will disintegrate per unit time.

$$For \quad \frac{dQ}{dt} = -KQ \quad use \quad Q(t) = Q_0 e^{-t/\tau}$$

Now τ is called the mean life, and the half-life is $T_{1/2} = \tau \ln(2)$ = time for half the remaining atoms to disintegrate, and

$$T_{1/2} \cong 0.7\,\tau$$

Discharge of a Capacitor

Back to electricity. From the loop rule we got

$$\frac{dQ}{dt} = -\frac{Q}{RC} = -KQ$$

So the solution is $Q(t) = Q_0 e^{-t/\tau}$

But what are Q_0 and τ?

Initial condition: $Q_0 = Q(0)$

Time constant: $\tau = 1/K = \boxed{RC}$

Example

A 40 pF capacitor with a charge of 20 nC is discharged through a 50 MΩ resistor.

(a) What is the time constant?

$$\tau = RC = 40\times10^{-12}\times50\times10^{6} = 2.0\times10^{-3}\ s$$

(b) At what time will ½ the charge remain?

$$T_{1/2} = 0.7\tau = 0.7\times2.0\times10^{-3}\ s = 1.4\ ms$$

(c) How much charge will remain after 5 ms?

$$Q(t) = Q_0 e^{-t/\tau} = 20\times e^{-2.5} = 1.64\,nC$$

Circuits Summary

Things to remember about DC circuits:

- Resistance and resistivity
- Ohm's Law and voltage drops
- Power and Joule heating
- Resistors in series and parallel
- Loop and junction rules
- RC circuits: charging and discharging a capacitor
- RC time constant

$$R = \rho l / A$$

$$\Delta V = -iR$$

$$P = iV$$

$$Q(t) = Q_0 e^{-t/\tau} \quad \tau = RC$$

Quiz tomorrow on Chapters 26,27.