Electric Potential

- Quiz Thursday on Chapters 23, 24.

Outline

- **Potential as energy per unit charge.**
- **Third form of Coulomb's Law.**
- **Relations between field and potential.**

Potential Energy per Unit Charge

Just as the ***field*** is defined as ***force*** per unit charge, the ***potential*** is defined as ***potential energy*** per unit charge:

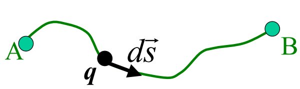
$$\vec{F} = q\vec{E} \quad and \quad U = qV$$

The SI unit for potential is the ***volt***. (1V=1J/C)

- Potential is often casually called "voltage".
- As with potential energy, it is really the ***potential difference*** which is important.

Potential Energy Difference

If a charge q is originally at point A, and we then move it to point B, the ***potential energy*** will increase by the amount of work we have done in carrying the charge from A to B.



$$dW = \vec{F}_{ext} \cdot d\vec{s}$$

$$U_B - U_A = W_{AB} = \int_A^B \vec{F}_{ext} \cdot d\vec{s}$$

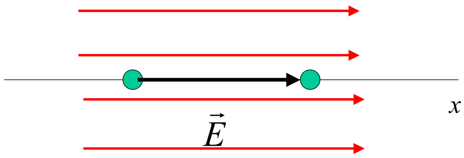
Potential Difference

The ***potential difference*** between point A and point B is this work ***per unit charge.***

$$W_{AB} = qV_{AB} = \int_A^B \vec{F}_{ext} \cdot d\vec{s} = -\int_A^B q\vec{E} \cdot d\vec{s}$$

$$\therefore \quad V_{AB} = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

Relation between E and V



$$\Delta V = -\int E_x dx$$

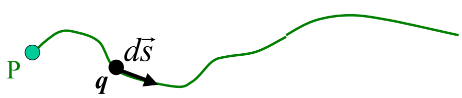
$$And \quad E_x = -\frac{dV}{dx} \quad etc.$$

Potential Relative to Infinity

- **We have defined potential difference ΔV. But to have a value for V itself we need to decide on a zero point.**
- **In circuits, we define the *earth* to have V=0, and often *ground* the circuit.**
- **In electrostatics we normally define V=0 far away from all charges. Then at point P we write V(P) to mean V(P) - V(∞).**

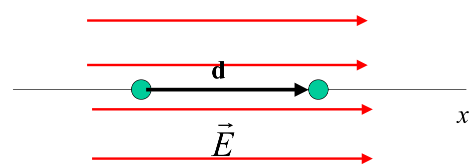
Potential at a point

The potential at point P is the work required to bring a one-coulomb test charge from far away to the point P.



$$V(P) = \int_{\infty}^{P} \frac{\vec{F}\cdot d\vec{s}}{q} = \int_{P}^{\infty} \vec{E}\cdot d\vec{s}$$

Example 1: Uniform Field



$$\Delta V = -\int Edx = -Ed$$

$$V(0) - V(d) = Ed$$

Work required to push a 1-coulomb test charge against the field from x=d to x=0.

"Work = force X distance"

Q.24-1

Suppose, using an xyz coordinate system, in some region of space, we find the electric potential is

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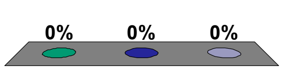
$V(x) = Ax^2$ **where A is a constant.**

What is the x-component of the electric field in this region?

A. $E_x = Ax$

B. $E_x = Ax^2$

C. $E_x = -2Ax$



Q. 24-1

Given $V(x) = Ax^2$

where A is has the constant value $A = 25\, V/m^2$.

What is the x-component of the electric field?

Solution:

$$E_x = -\frac{d}{dx}V = -A\frac{dx^2}{dx} = -2Ax$$

(1) $E_x = Ax$ **(2)** $E_x = Ax^2$ **(3)** $E_x = -2Ax$

Furthermore

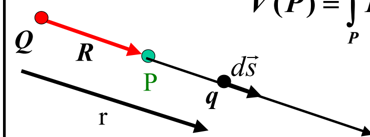
What are the y and z components of the electric field in the previous question?

$$E_y = -\frac{d}{dy}V = -A\frac{dx^2}{dy} = 0$$

and also $$E_z = -\frac{d}{dz}V = 0$$

Case of a Single Point Charge

$$V(P) = \int_P^\infty \vec{E}\cdot d\vec{s} = \int_R^\infty E(r)dr$$



$$V(R) = \int_R^\infty E(r)dr = kQ\int_R^\infty \frac{dr}{r^2}$$

$$= \left[-\frac{kQ}{r}\right]_R^\infty = -kQ\left[\frac{1}{\infty} - \frac{1}{R}\right] = \frac{kQ}{R}$$

Coulomb's Law for V

So we now have a ***third form of Coulomb's Law:***

1. $F = kQq/r^2$
2. $E = kQ/r^2$
3. $V = kQ/r$

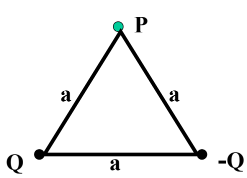
Potential is not a Vector

Adding forces and fields means ***adding vectors:*** finding the ***resultant vector.***

Adding potentials means ***adding numbers,*** and taking account of their signs. But it is much simpler than adding vectors.

Thus the third form of Coulomb's Law is the simplest!

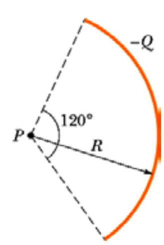
Example 1: Adding Potentials



$$V(P) = V_1 + V_2 = \frac{kQ}{a} + \left(-\frac{kQ}{a}\right) = 0$$

Note that the ***field*** at point P is ***not*** zero!

Q. 24-2



Uniformly charged rod with charge of –Q bent into arc of 120° with radius R.

What is V(P), the electric potential at the center?

$$(1) +\frac{kQ}{R} \quad (2) -\frac{kQ}{R} \quad (3) \frac{kQ}{2R} \quad (4) -\frac{kQ}{2R}$$

Q.24-2

Uniformly charged rod with charge of –Q bent into arc of 120° with radius R.

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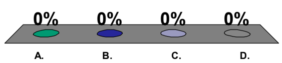
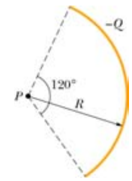
What is V(P), the electric potential at the center?

A. +kQ/R
B. -kQ/R
C. +kQ/2R
D. -kQ/2R



Q. 24-2



What is V(P), the electric potential at the center?

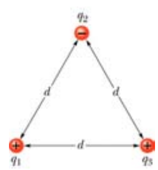
Solution: All bits of charge are at the same distance from P. Thus

$$V = \int \frac{kdq}{r} = \frac{k}{R}\int dq = \frac{k}{R}(-Q)$$

$$(1) + \frac{kQ}{R} \quad (2) - \frac{kQ}{R} \quad (3) \frac{kQ}{2R} \quad (4) - \frac{kQ}{2R}$$

Potential Energy of Some Charges

The potential energy U of a group of charges is the work W required to assemble the group, bringing each charge in from infinity.



We can show that the result is

$$U = U_{12} + U_{13} + U_{23} + \cdots$$

Where the potential energy of each ***pair*** is of the form

$$U_{12} = kq_1q_2 / r_{12}$$

Binding Energy

If the total potential energy ***U*** of a group of charges is ***negative*** that means we have to do work to pull them apart. The magnitude of this negative potential energy is called the ***binding energy.***

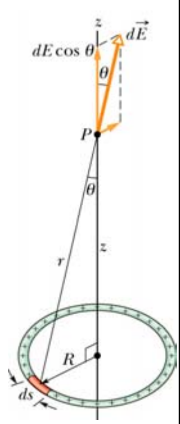
Examples:

- Removing an electron from an atom to form a positive ion.
- Removing a space probe from earth's gravitational field.

Example 3: Charged Ring

- In Ch. 22 the *E* field of a charged ring is calculated.
- Here we compute *V* first and then use it to get *E*.
- Get *V* at *P* on axis of circular ring, a distance *z* from center:

$$V(P) = \int \frac{kdQ}{r}$$



Example 3 Continued

Key point: All bits of charge are the same distance from point P!

$$V(P) = \int \frac{kdQ}{r} = \frac{k}{r}\int dQ = \frac{kQ}{\sqrt{R^2 + z^2}}$$

Note we have no struggling with angles or adding many little vectors as we do if we compute *E*.

Using V to get E

Now that we have V on the axis we can get E_z on the axis by differentiation: $E_z = -\frac{dV}{dz}$

This is slightly messy, but we just need to remember that $\frac{df^n}{dz} = nf^{n-1}\frac{df}{dz}$

where here $f^n = (R^2 + z^2)^{-1/2}$

So finally we get

$$E(z) = -\frac{d}{dz}\frac{kQ}{\sqrt{R^2 + z^2}} = \frac{kQz}{(R^2 + z^2)^{3/2}}$$

which is exactly the result the textbook gets in Ch. 22 by direct integration of the field.

And also: $E_y = -\frac{d}{dy}V = -A\frac{dx^2}{dy} = 0$

$E_z = -\frac{d}{dz}V = 0$

Furthermore

What are the y and z components of the electric field in the previous question?

$$E_y = -\frac{d}{dy}V = -A\frac{dx^2}{dy} = 0$$

and also $E_z = -\frac{d}{dz}V = 0$

Summary of Basics

- **U = qV**
- **V = kQ/r**
- $W_{AB} = q(V_B - V_A)$

$$\Delta V = -\int E_x dx$$

$$E_x = -\frac{dV}{dx} \quad etc.$$

Channel Setting Instructions for ***ResponseCard RF***

1. Press and release the "GO" button.

2. While the light is flashing red and green, enter the 2 digit channel code (ie. channel 1 = 01, channel 21 = 21).

3. After the second digit is entered, Press and release the "GO" button.

4. Press and release the "1/A" button. The light should flash yellow to confirm.