Electrostatics (RECAP)
Electric Current

Key ideas

- **Electric charge:** conserved and quantized
- **Electric field:**
  - force per unit charge, field lines, adding vectors
- **Flux:** amount of field passing through an area
- **Electric potential:**
  - energy per unit charge, integral of field
  - new unit: electron volt (eV)
- **Dipole moment:** paired + and - charges
- **Capacitance:** device to store charge and energy
- **Dielectrics:** polarization, dielectric constant

Fundamental Laws

Coulomb's Law:

1. $F = kQq/r^2$
2. $E = kQ/r^2$
3. $V = kQ/r$

Gauss's Law:

The outward flux of the electric field through any closed surface equals the net enclosed charge divided by ε_0.

Relations between potential and field:

The potential difference between A and B is the work required to carry a unit positive charge from A to B.

$$\Delta V = -\int E_x dx \qquad E_x = -\frac{dV}{dx} \quad etc.$$

Terminology

Words whose precise definitions you must know:

Field

Flux

Potential

Potential difference

Dipole moment

Capacitance

Dielectric constant

And of course the SI units for all these things.

Q. 25 - 3

$$C = \frac{2\pi\varepsilon_0}{\log(b/a)}$$

In the text, this formula is derived for the capacitance per unit length of a long cylindrical capacitor, such as a coaxial cable.

In this derivation, the potential difference was calculated by means of an integral over the electric field. What was that integral?

$$(1)\ \int r\,dr \quad (2)\ \int r^2\,dr \quad (3)\ \int \frac{dr}{r} \quad (4)\ \int \frac{dr}{r^2}$$

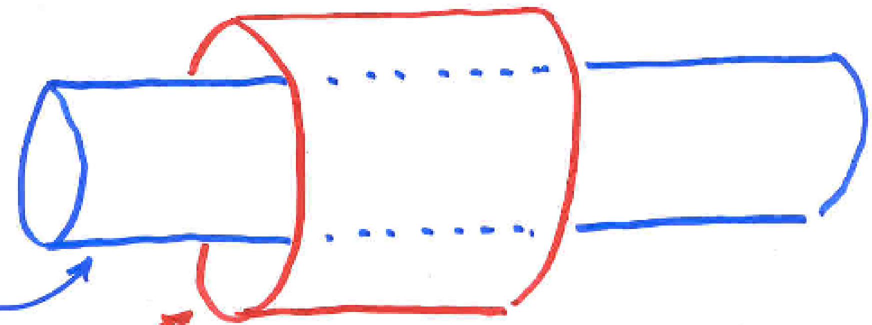
Q.25-3

What was the integral needed for the capacitance of a coaxial cable?

1) $\int r \, dr$
2) $\int r^2 \, dr$
3) $\int (1/r) \, dr$
4) $\int (1/r^2) \, dr$

Long Line of Charge

Gauss's Law for cylindrical symmetry



LONG cylinder, uniform charge density λ
(Coulombs / meter)

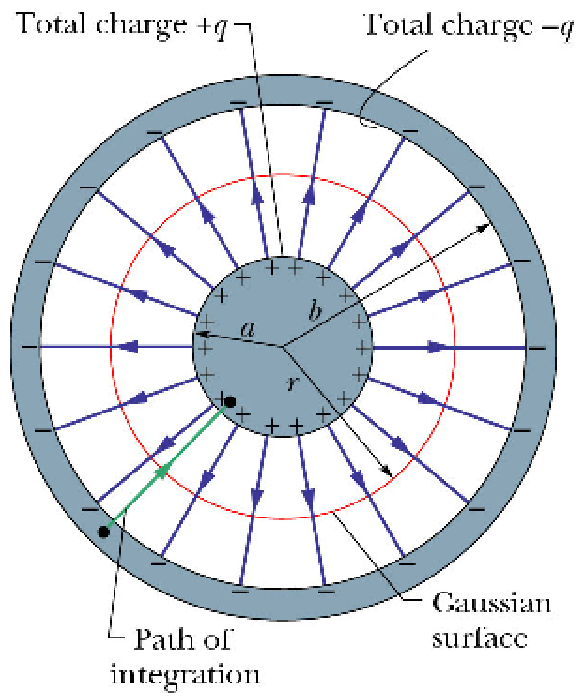
Concentric imaginary gaussian surface
of radius r, length ℓ.

$$\left.\begin{aligned}\oint \vec{E}\cdot d\vec{A} &= EA = 2\pi r \ell E \\ Q_{in} &= \ell\lambda\end{aligned}\right\}$$

Gauss's Law $\Rightarrow$ $\boxed{E(r) = \dfrac{\lambda}{2\pi\varepsilon_0 r}}$

Q. 25 - 3

Fig. 25-6:



Gauss's Law: $E = 2k\lambda / r$

Potential difference: $V = \int_a^b E dr = 2k\lambda \int_a^b \frac{dr}{r}$

(1) $\int r\, dr$ (2) $\int r^2\, dr$ (3) $\boxed{\int \frac{dr}{r}}$ (4) $\int \frac{dr}{r^2}$

Review: The electron-volt

- **One eV is the energy to move an electron through a potential difference of one volt.**
- **Note this is a unit of energy, not potential.**
- **This is not an SI unit but is used in all processes involving electrons, atoms, etc.**

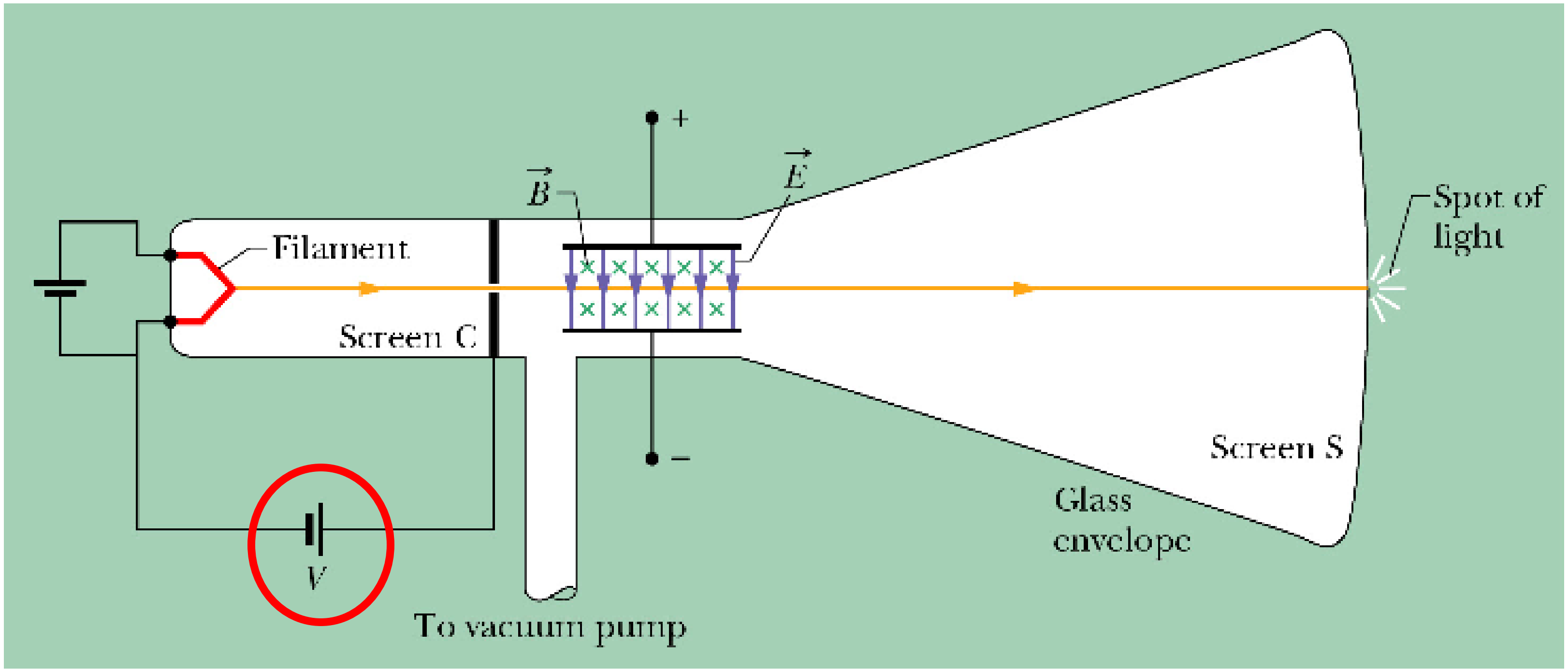
$$U = qV$$

So if V=1 and q=e then:

$$U = 1\,eV = e \times V$$

$$= (1.6 \times 10^{-19}\ C) \times (1\ V) = \underline{1.6 \times 10^{-19}\ J}$$

Cathode Ray Tube



Electron gun: potential V gives electron energy in eV.

$$K = \frac{1}{2}mv^2 = qV$$

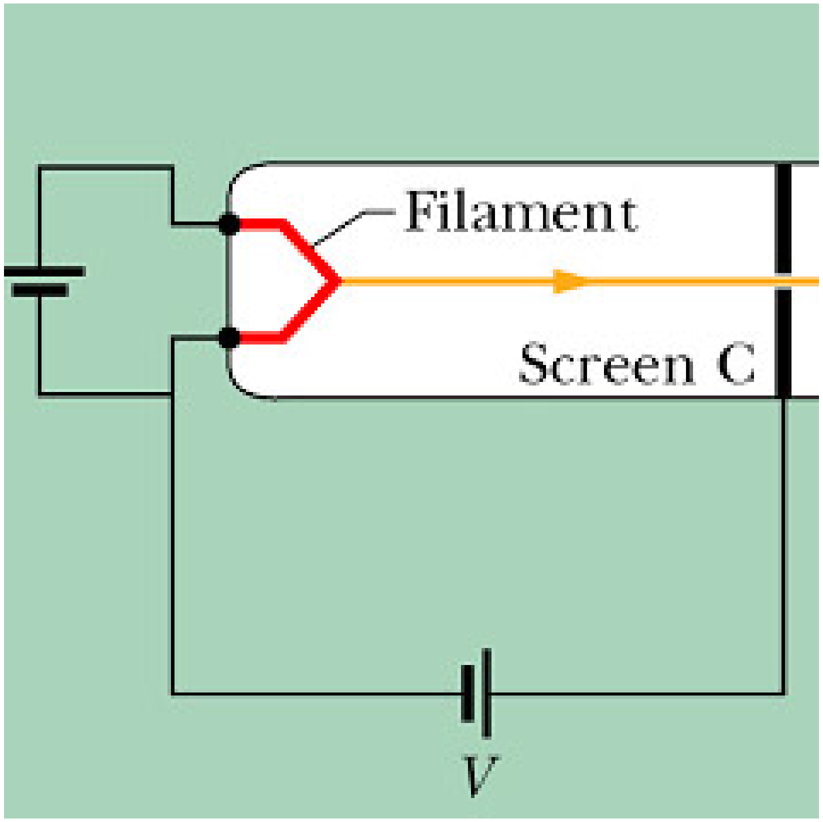
So if V = 500 volts, electron energy is K = 500 eV

High energies

- For X-ray machines, accelerate electrons with potentials of thousands of volts, so we speak of kinetic energies in ***keV***. $10^3\ eV$
- In nuclear physics, accelerators produce beams of particles with energies in ***MeV***. $10^6\ eV$
- In elementary-particle physics, high-energy particles beams have energies measured in giga-volts: ***GeV***. $10^9\ eV$

Q. 25-4

Which direction will the electric field lines point in this electron gun?



(1) To the right

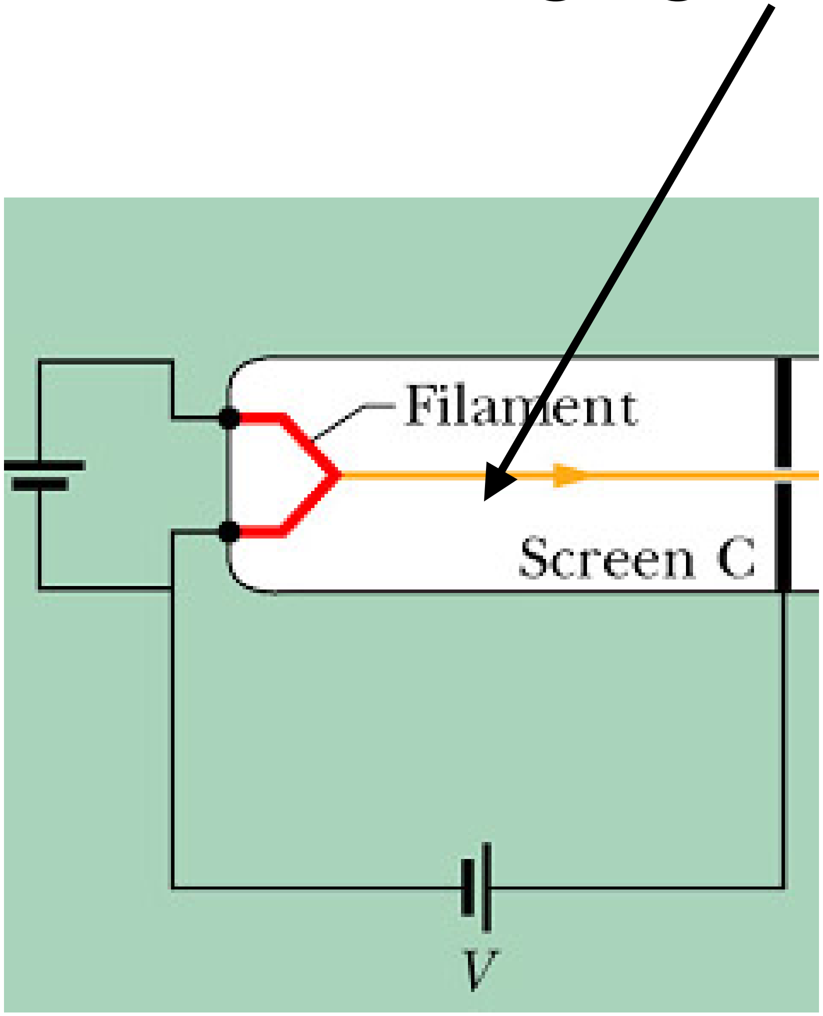
(2) To the left

(3) Upward

(4) Downward

Q.25-4

Which way does E point in the accelerating region?



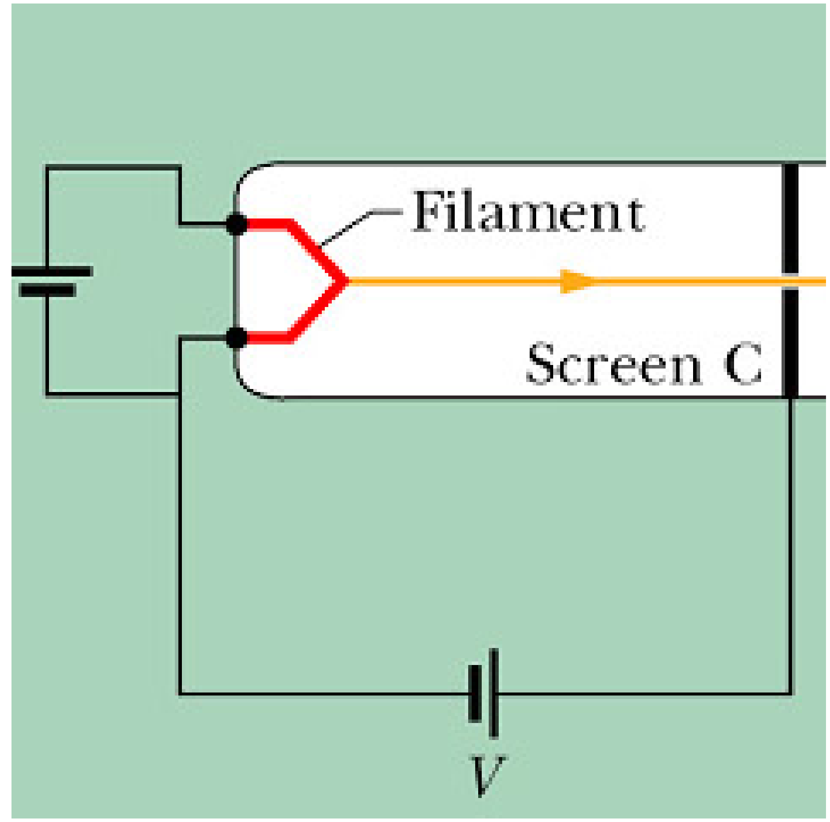
1) To the right.
2) To the left.
3) Upward.
4) Downward.

Q. 25-4

Which direction will the electric field lines point in this electron gun?

$$\vec{F} = q\vec{E} = -e\vec{E}$$

So for force to the right, we need field to the left!



(1) To the right

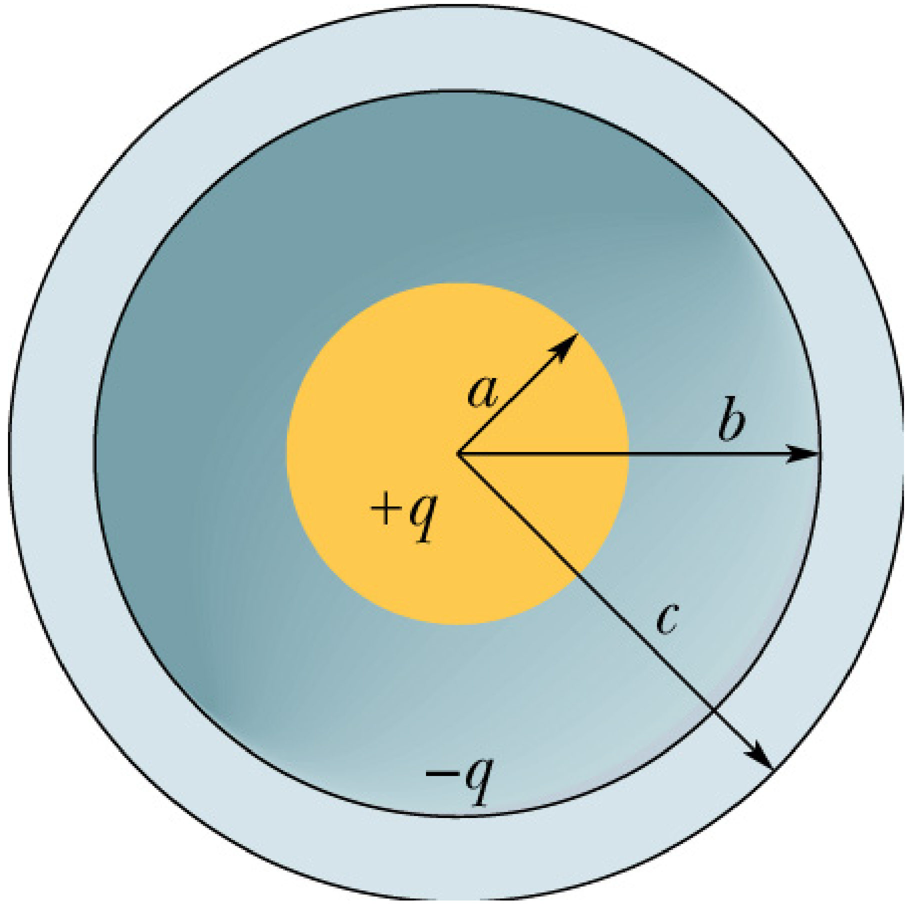
(2) To the left

(3) Upward

(4) Downward

Text problem 23-49

- Nonconducting sphere of radius a with charge q uniformly distributed
- Concentric metal shell of inner radius b, outer radius c, with total charge –q.

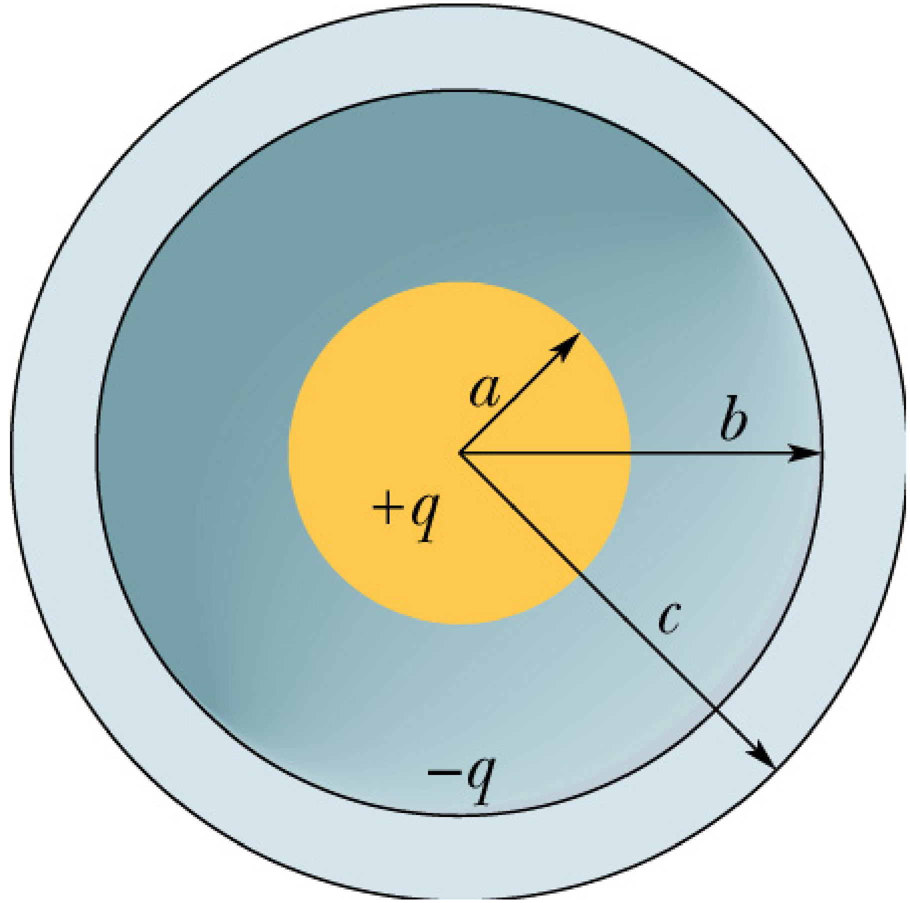


(a) Find field everywhere.

(b) How is charge distributed on shell?

Text problem 49(a)

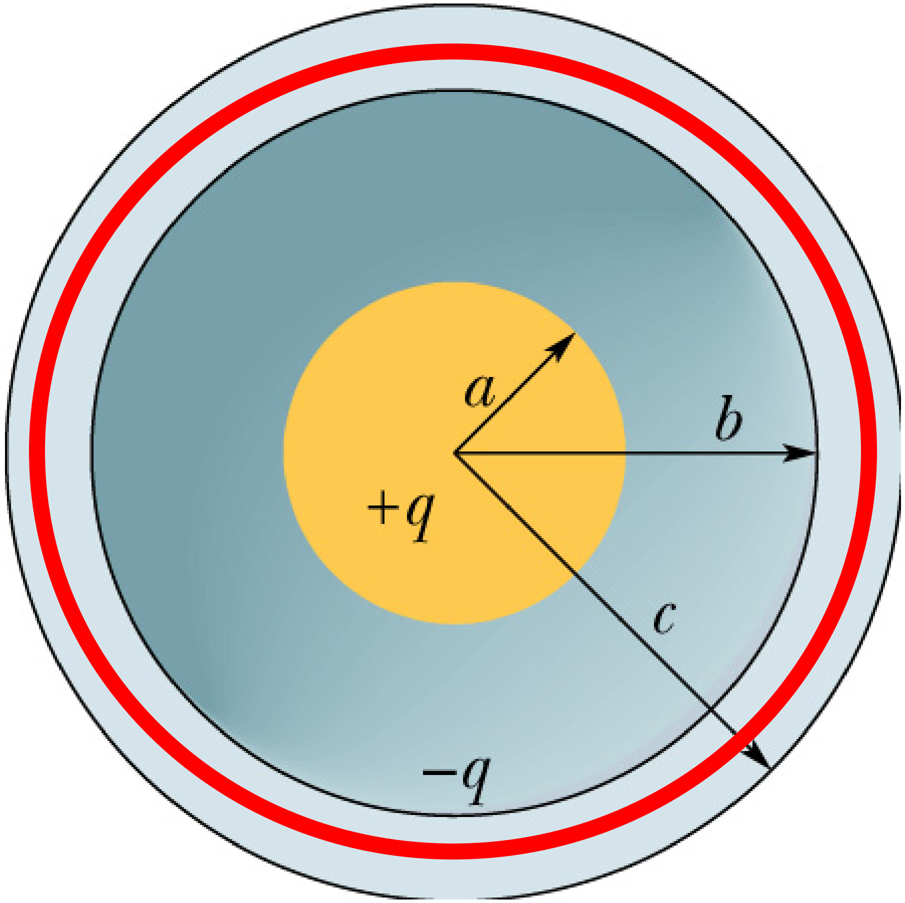
- For r<a, have previous solution: $$E = \frac{\rho r}{3\varepsilon_0}$$
- For a<r<b, shell theorem gives: $$E = \frac{kq}{r^2}$$
- For b<r<c, we're inside metal, so **E = 0.**
- For c<r, shell theorem gives **E = 0.**



In all cases, field is radially outward.

Text problem 49(b)

- Draw ***gaussian sphere of radius r*** with b<r<c.
- Because we're inside a metal, E = 0.
- **Therefore** flux = 0.
- **Therefore** enclosed charge = 0.



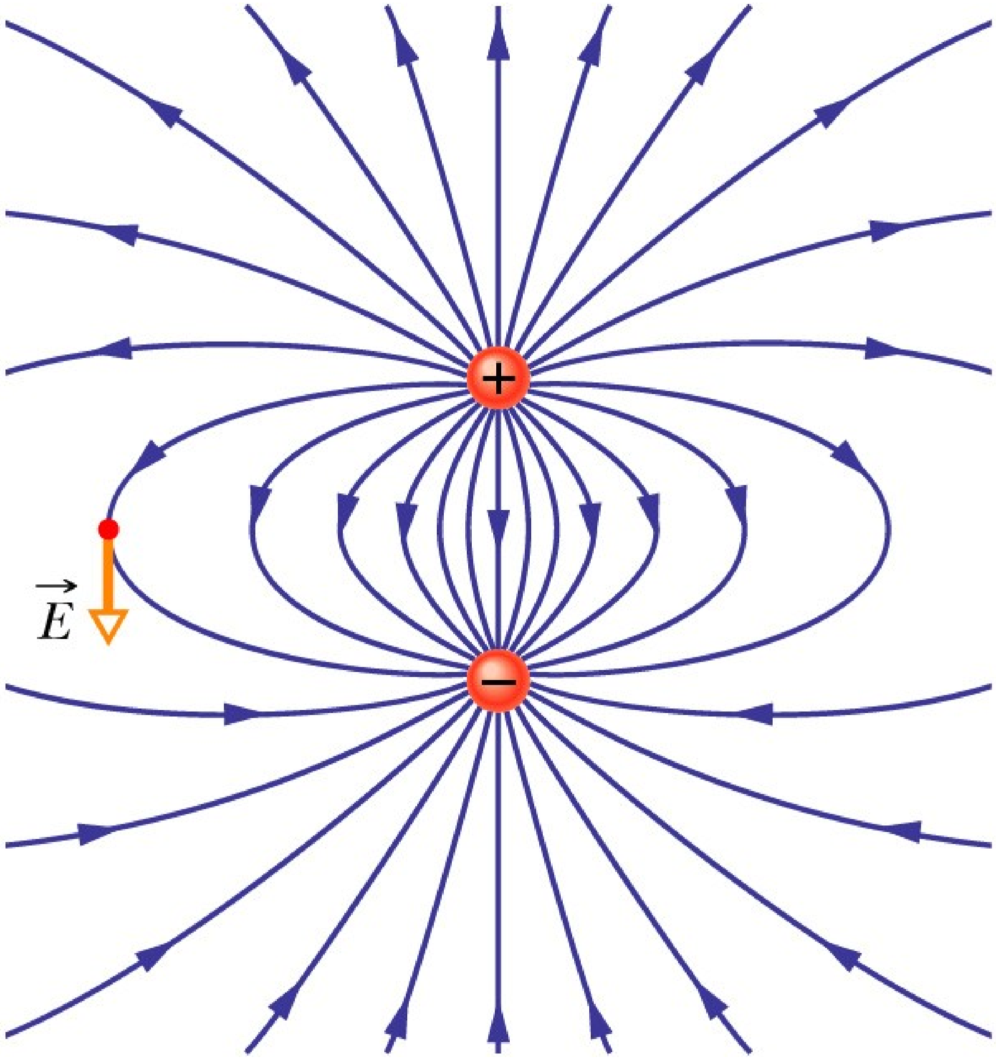
- **Therefore** there is **–q** on inner surface of shell.
- **Therefore** there is **no** charge on outer surface.

Electric Dipole

- **The combination of two charges of equal but opposite sign is called a dipole.**
- **If the charges +q and –q are separated by a distance d, then the** *dipole moment* $\vec{p}$ **is defined as a vector pointing from –q to +q of magnitude** $p = qd$**.**

$$-q \quad \vec{p} \quad +q$$

Electric Field Due to a Dipole



Potential due to a dipole

Exact potential at P:

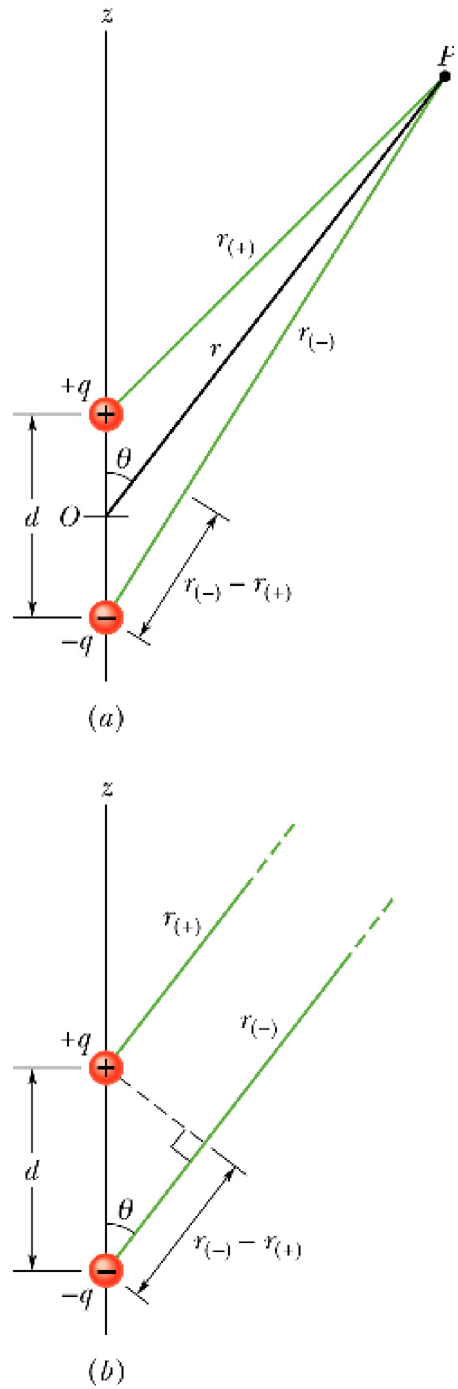
$$V = kq/r_{+} - kq/r_{-}$$

$$r_{+} = \sqrt{r^2 + (d/2)^2 - rd\cos\theta}$$

$$r_{-} = \sqrt{r^2 + (d/2)^2 + rd\cos\theta}$$

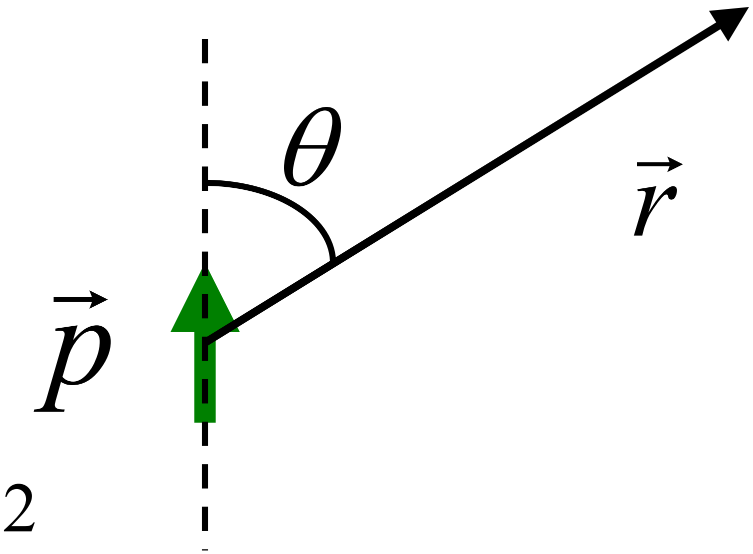
Approx. potential at P, r>>d:

$$V \approx kp\cos\theta / r^2$$



Result: the dipole potential

So we have found that for large r, the ***potential produced by a dipole*** is:

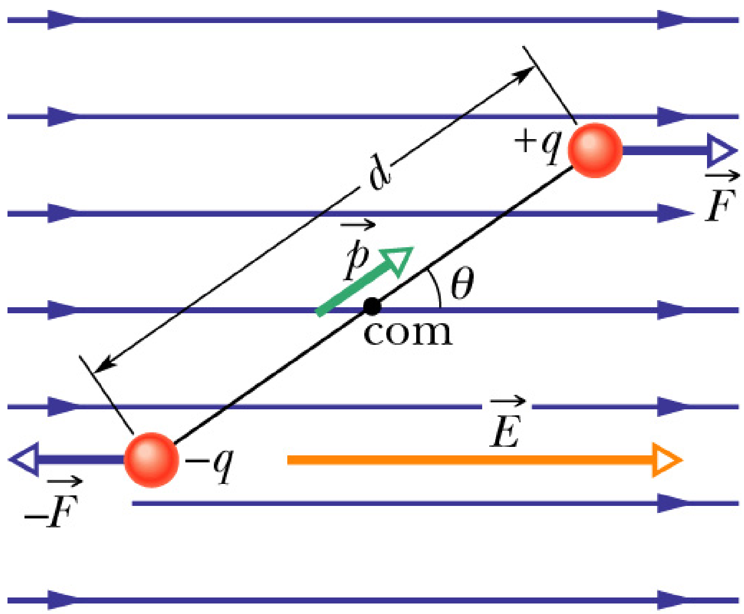


$$V(\vec{r}) = kp\cos\theta / r^2$$

Note this can also be written:

$$V(\vec{r}) = k\frac{\vec{r}\cdot\vec{p}}{r^3}$$

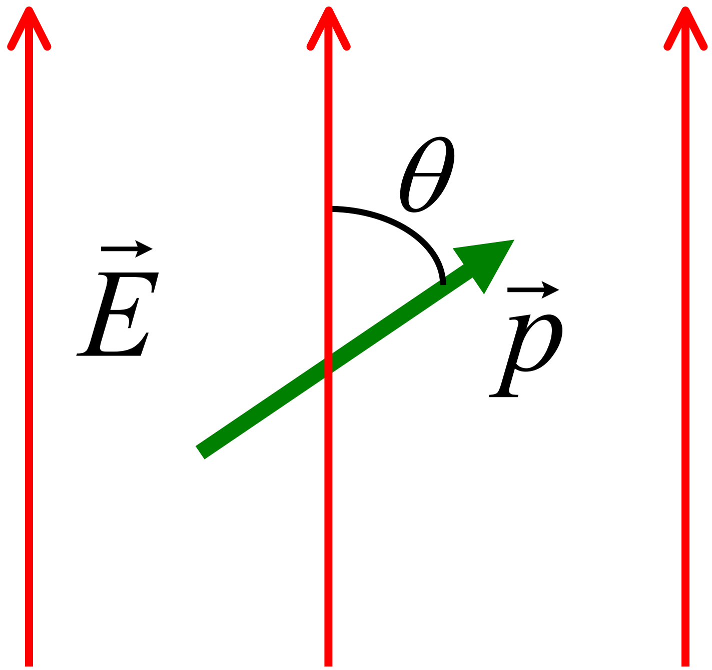
Torque on a Dipole in a Field



$$\tau = 2 \times F \times (\frac{d}{2}\sin\theta) = qE \times d\sin\theta = pE\sin\theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

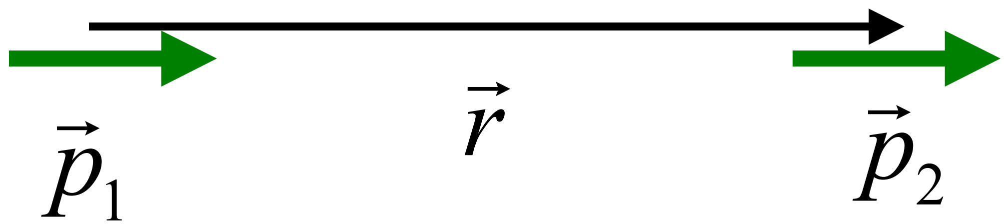
Energy of dipole in given field



$$U = -2Q(E\cos\theta)(d/2)$$
$$= -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

So dipole tends to ***align with*** an applied field.

Interaction of two dipoles



$$U = qV(r + d/2) - qV(r - d/2)$$

$$= \frac{kqp}{(r + d/2)^2} - \frac{kqp}{(r - d/2)^2} = -k\frac{p_1 p_2}{r^3}$$

Attractive potential: work required to pull apart.

Current and Resistance

Chapters 26, 27

TODAY:

- Ohm's Law
- Ideal Meters
- Sources of Voltage and Power
- Resistors in series and parallel

Current is rate of flow of charge

- If you watch closely at a fixed point as current is flowing along a wire, the current is the amount of charge that passes by per unit time.
- SI unit is the Ampere: 1A = 1C/s.
- Example: In a cathode-ray tube, $n = 3 \times 10^{15}$ electrons leave the electron gun per second. What is the current of this electron beam?
- Solution:

$$ne = 3 \times 10^{15} \times 1.6 \times 10^{-19} = 5 \times 10^{-4} C/s = 0.5\ mA$$

Current and Resistance

- **When a current flows through a *perfect* conductor (*e.g.* copper wire) there is no change in potential.**
- **When current flows through an *imperfect* conductor (*resistor*) there must be a field E to make it flow and thus a change in potential V = Ed for a distance d.**
- **If the current I is proportional to the field E which is causing it, then it will be proportional to the potential change V. This is Ohm's "Law" and the proportionality constant is called the *resistance* R.**

$$\boxed{V = IR}$$

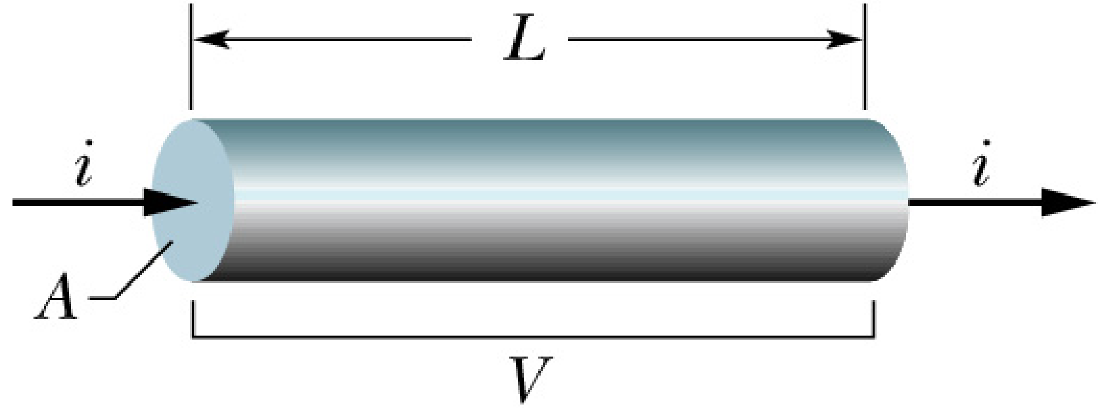
Notes on Ohm’s Law

- **This is not a fundamental law of nature like Gauss’s Law. It’s just a proportionality which is approximately true for some materials under some conditions.**

- **The entire electronics industry is based on materials which *violate* Ohm’s Law.**

- **Ohm’s Law should really be written as a *potential drop* $\Delta V = -IR$ because if you follow the current the potential decreases.**

Microscopic Form



$$E = V / L = IR / L = JRA / L = J\rho$$

So we define ***resistivity ρ***: $\boldsymbol{R = \rho\ L / A}$

and ***current density j:*** $\boldsymbol{J = I / A}$

So the ***microscopic form*** of Ohm's Law is

$$\boldsymbol{E = J\rho}$$

Resistance

- **So for a given object (resistor) we can measure its resistance R = V/I.**
- **The SI unit of resistance is the ohm (Ω).**
- **Clearly 1Ω = 1V/A (ohm = volt/amp).**
- **Thus resistivity** ρ **has units Ω-m.**
- **But remember Ohm's Law is only an approximation.** *For example, resistance normally changes with temperature.*

Example: Problem 26-15

- **A wire is 2 m long, with a diameter of 1 mm.**
- **If its resistance is 50 mΩ, what is the resistivity of the material?**

$$R = \rho l / A$$

$$\rho = RA / l = \frac{\left(50 \times 10^{-3}\ \Omega\right) \times \pi \times \left(0.5 \times 10^{-3}\ m\right)^2}{2}$$

$$= 2 \times 10^{-8}\ \Omega m$$

(Note this is consistent with table on page 689.)

Voltage and Power Sources

When a battery or any other voltage source delivers a current i at a potential difference V it is supplying *power* $P = iV$.

$$P = \frac{U}{t} = \frac{qV}{t} = iV$$

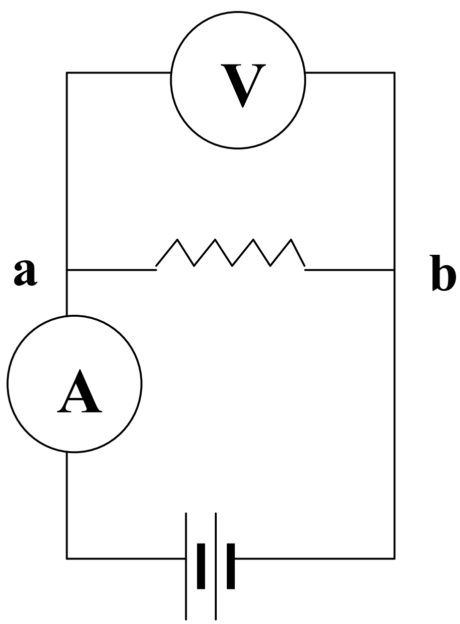
When a current i flows through a resistor with a voltage drop V, a friction-like process called *Joule heating* converts this power $P=iV$ into heat.

Note on P = I V

This is the easiest equation in the world to remember, IF you know what the three quantities mean:

$$voltage \times current = \frac{energy}{charge} \times \frac{charge}{time} = \frac{energy}{time} = power$$

Ideal Meters



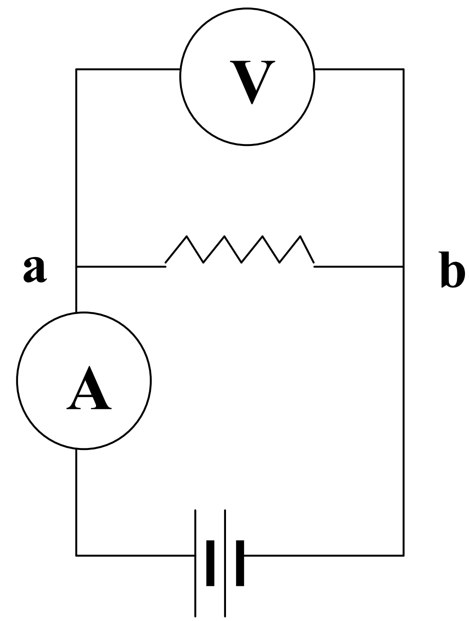
"Ideal ammeter"
Measures current and has zero potential drop (zero R)

"Ideal voltmeter"
Measures $V_{ab}=V_a-V_b$ but draws zero current (infinite R)

"Ideal wires"
V is constant along any wire (zero R)

Example

Meter A reads i = 2A flowing upward as shown, and meter V reads V = 6V.



(a) What is resistance R?

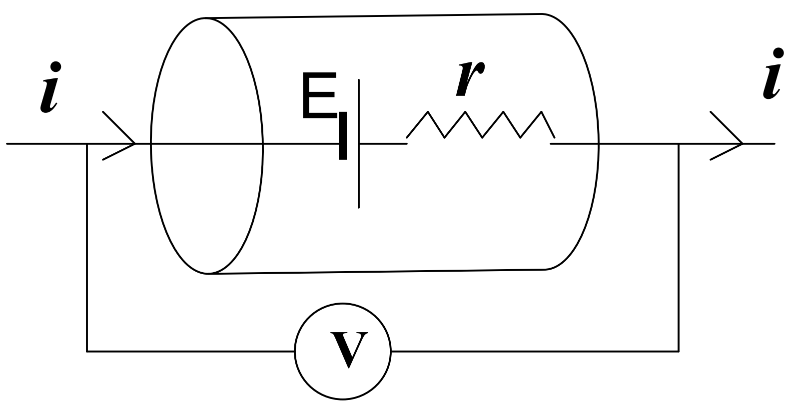
$$R = V / i = 6V / 2A = 3\Omega$$

(b) Which point is at the higher potential, ***A*** or ***B***?

Point ***A*** because potential ***drops*** by 6 volts.

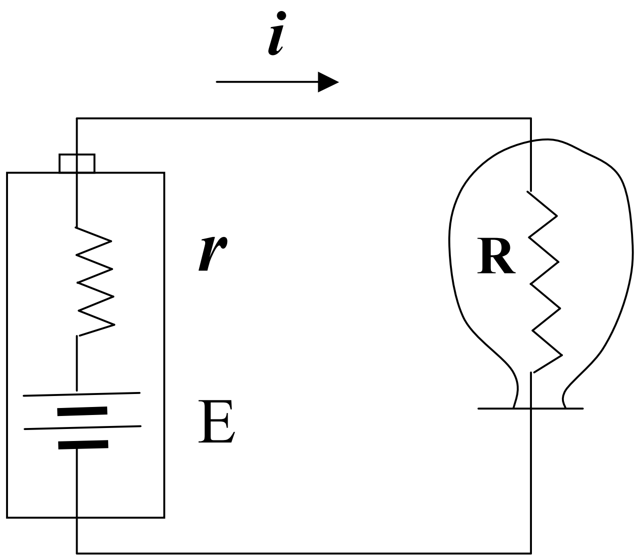
EMF

- Electromotive force (emf) is not a force: it's the potential difference provided by a power supply.
- For a battery providing a current i, the terminal voltage V is less than the emf E because there is an internal resistance r:



$$V = E - ir$$

Power in a Simple Circuit



Dry cell **Light bulb**

$$\Delta V_{loop} = -ir + \mathcal{E} - iR = 0$$

$$\therefore \quad \mathcal{E} = i(r + R)$$

ε

$$P_{chem} = i\mathcal{E} = i^2 r + i^2 R$$

$$= P_{heating\ battery} + P_{heating\ filament}$$

Example

For a battery with internal resistance r = 25 Ω, what load resistor R will get maximum power?

$$P_{load} = i^2 R = \left(\frac{\mathcal{E}}{r+R} \right)^2 R = \mathcal{E}^2 \left(\frac{R}{(r+R)^2} \right)$$

$$\frac{d}{dR} \left(\frac{R}{(r+R)^2} \right) = \frac{(r+R)^2 - 2R(r+R)}{(r+R)^4} = 0$$

$$(r+R)^2 = 2R(r+R)$$

$$r + R = 2R$$

$$R = r = 25\Omega$$

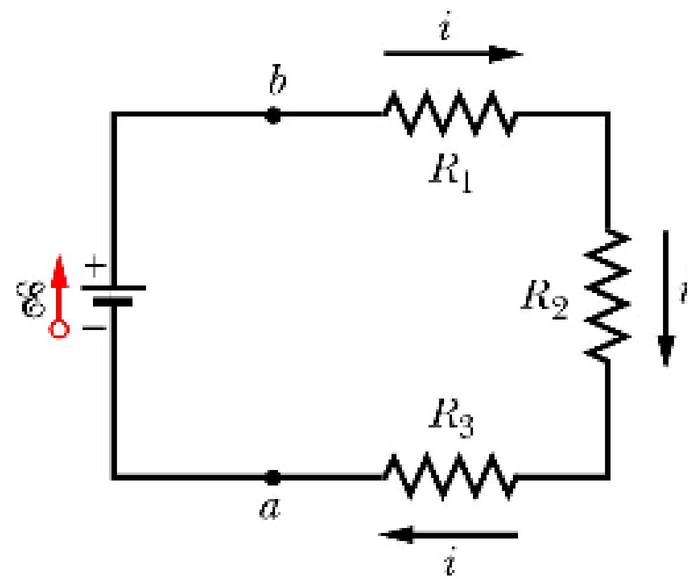
Resistors in Series

- Voltage drops add
- Currents are equal.

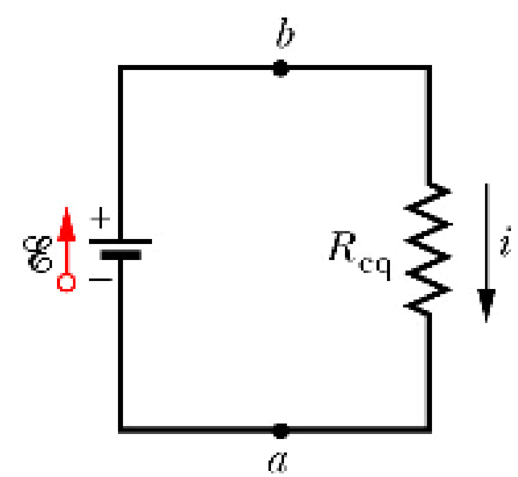
$$V_b - V_a = \mathcal{E} = iR_1 + iR_2 + iR_3$$

But we want $\mathcal{E} = iR_{eq}$

$$\boxed{\therefore \quad R_{eq} = R_1 + R_2 + R_3}$$



(a)



(b)

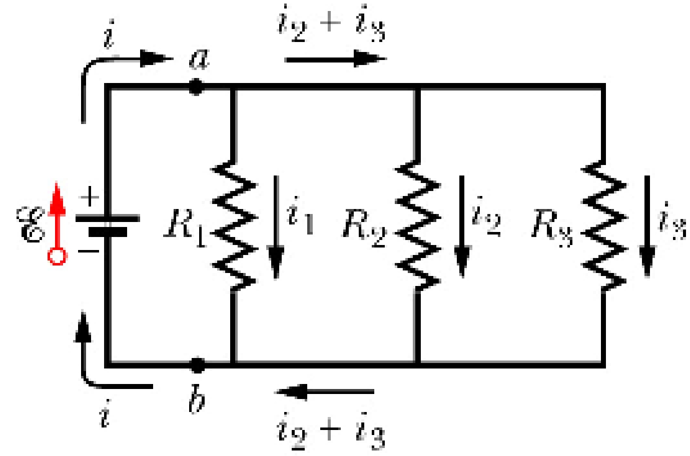
Resistors in Parallel

- **Currents add**
- **Voltage drops are equal.**

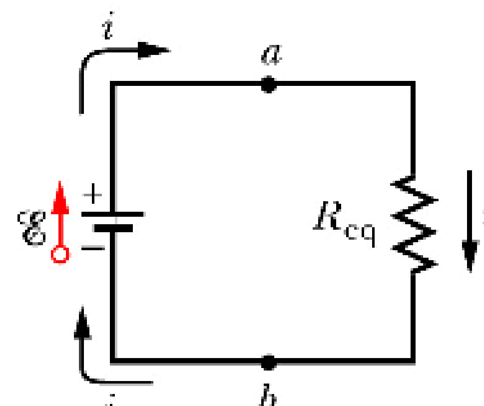
$$i = i_1 + i_2 + i_3$$

$$\frac{\mathcal{E}}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$But \quad \mathcal{E} = V_1 = V_2 = V_3$$



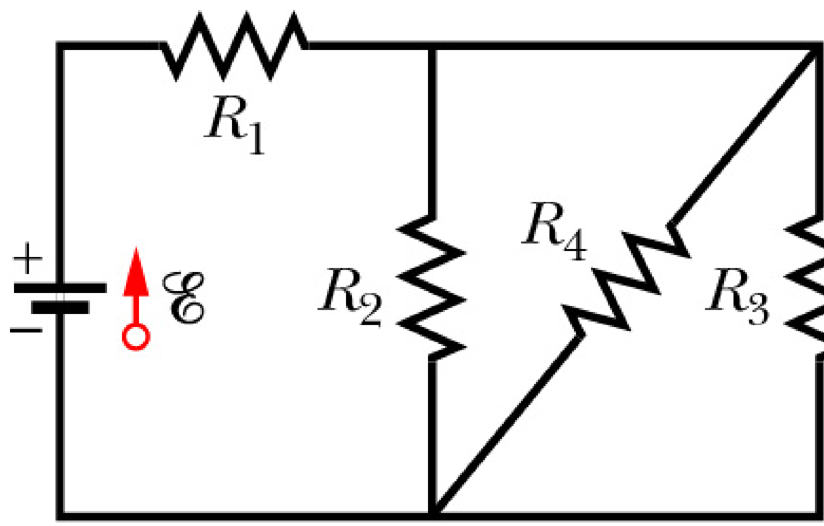
(a)



(b)

$$\therefore \quad 1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$$

Example: Problem 27-30

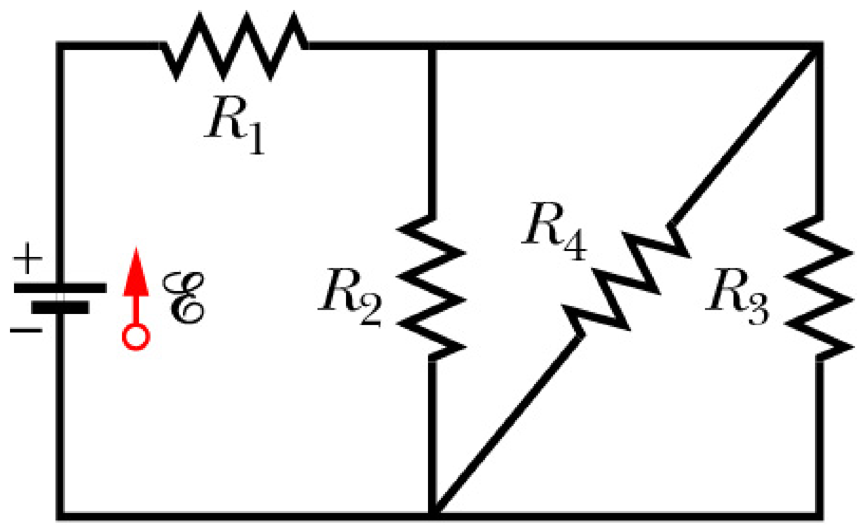


$$\mathcal{E} = 6.0\,V \quad R_1 = 100\,\Omega$$
$$R_2 = R_3 = 50\,\Omega$$
$$R_4 = 75\,\Omega$$

(a) Find the equivalent resistance of the network.

(b) Find the current in each resistor.

Problem 27-30 (part a)

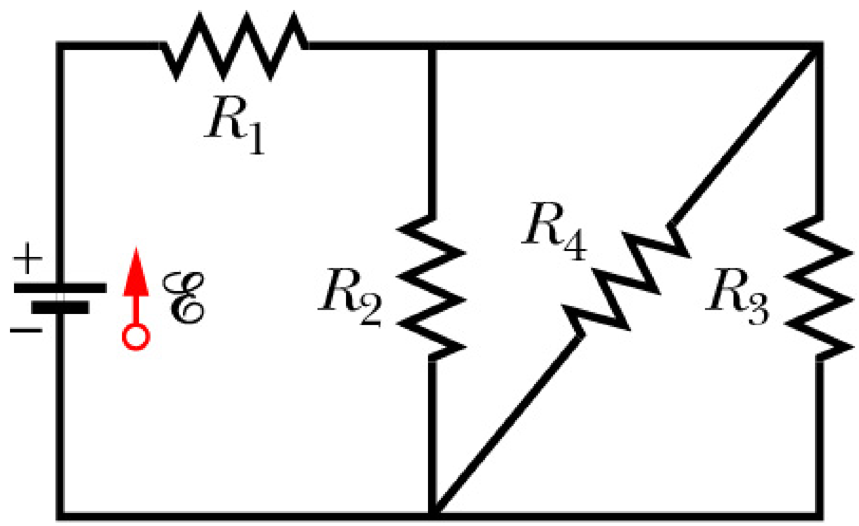


$$\mathcal{E} = 6.0\,V \quad R_1 = 100\,\Omega$$
$$R_2 = R_3 = 50\,\Omega$$
$$R_4 = 75\,\Omega$$

(a) Find the equivalent resistance of the network.

$$1/R_{234} = 1/R_2 + 1/R_3 + 1/R_4$$
$$= 1/50 + 1/50 + 1/75 = 16/300$$
$$So \quad R_{234} = 300/16 = 19\,\Omega$$

Problem 27-30 (part a cont'd)



$$\mathcal{E} = 6.0\,V \quad R_1 = 100\,\Omega$$

$$R_2 = R_3 = 50\,\Omega$$

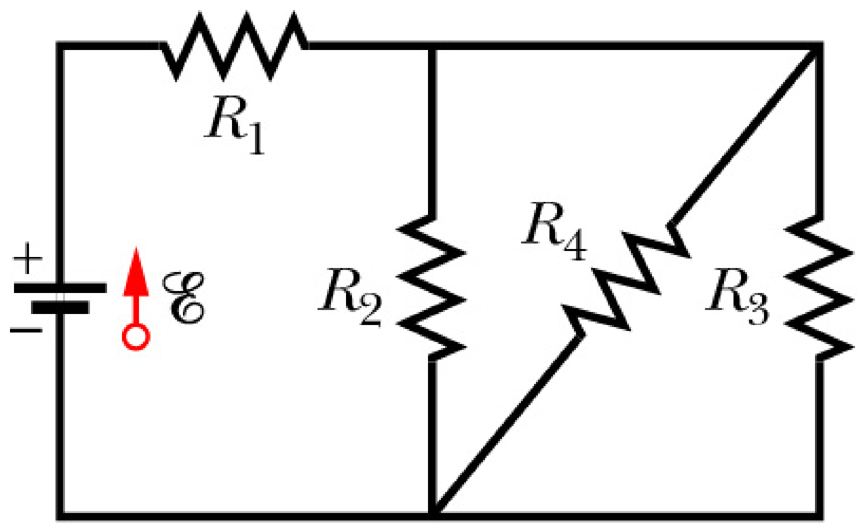
$$R_4 = 75\,\Omega$$

(a) Find the equivalent resistance of the network.

$$\textit{Now } R_1 \textit{ and } R_{234} \textit{ are in series so}$$

$$R_{eq} = R_1 + R_{234} = 100\,\Omega + 19\,\Omega = \boxed{119\,\Omega}$$

Problem 27-30 (part b)



$$\mathcal{E} = 6.0\,V \quad R_1 = 100\,\Omega$$

$$R_2 = R_3 = 50\,\Omega$$

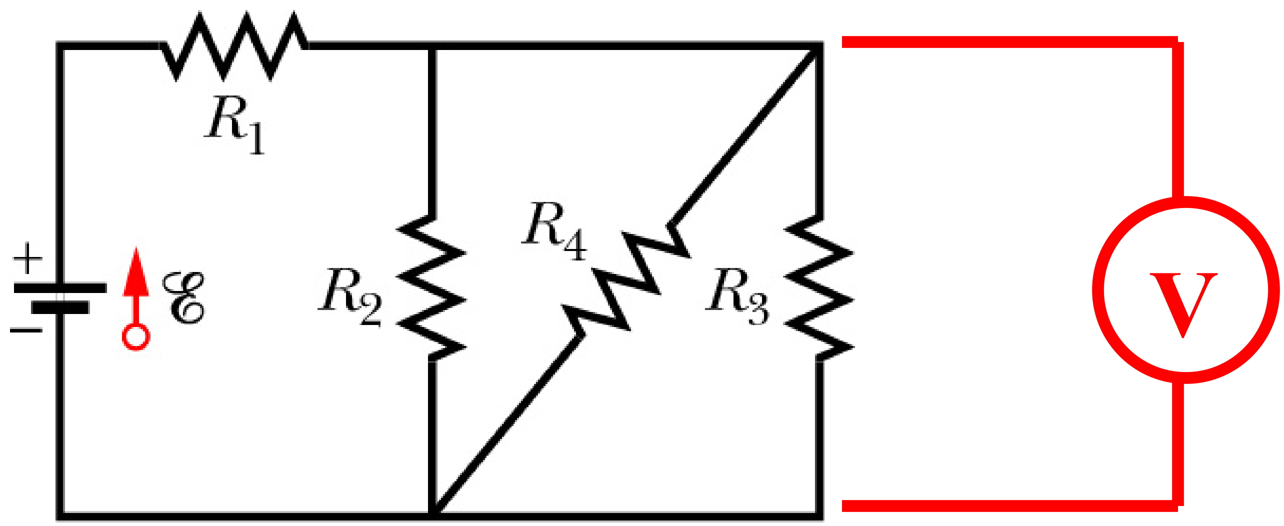
$$R_4 = 75\,\Omega$$

(b) Find the current in each resistor.

First get the total current from the battery, which is also the current through R_1:

$$\boldsymbol{i_1 = \mathcal{E} / R_{eq} = 6.0 / 119 = .050\,A = 50\,mA}$$

Problem 27-30 (part b cont'd)

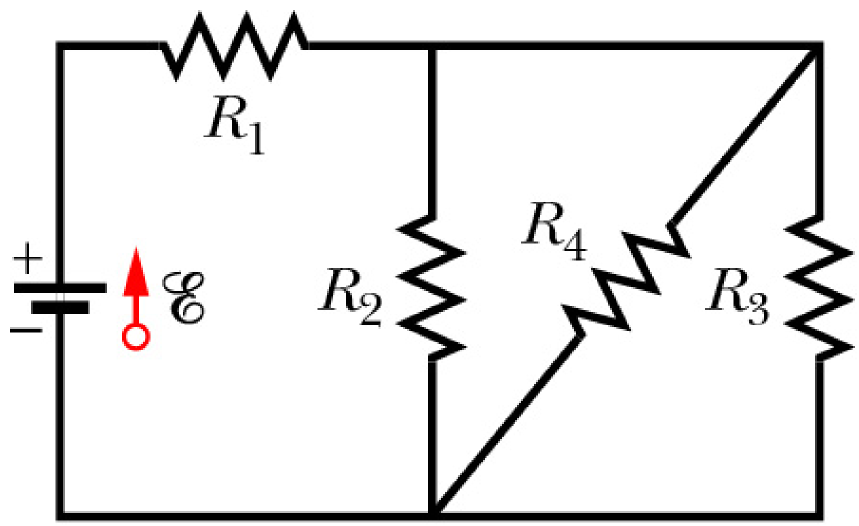


$$V = iR_{234}$$
$$= .050\,A \times 19\,\Omega$$
$$= 0.95\,V$$

(b) Find the current in each resistor.

$$i_2 = V / R_2 = .95/50 = 19\,mA$$
$$i_3 = V / R_3 = .95/50 = 19\,mA$$
$$i_4 = V / R_4 = .95/75 = 12\,mA$$

Problem 27-30 (check)



$$i = i_1 = 50\,mA$$
$$i_2 = i_3 = 19\,mA$$
$$i_4 = 12\,mA$$

Check by adding the currents in the three branches:

$$i_2 + i_3 + i_4 = 19 + 19 + 12 = 50\,mA$$
$$= i_1$$