Chapter 23: Gauss's Law

Homework:

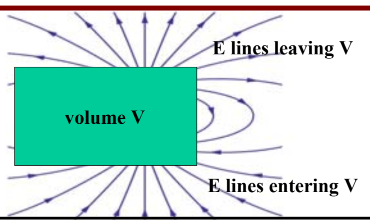
- Read Chapter 23
- Questions 2, 5, 10
- Problems 1, 5, 32
- **Quiz Thursday on Chapters 23, 24.**

Gauss's Law

- **Gauss's Law is the first of the four *Maxwell Equations* which summarize all of electromagnetic theory.**
- **Gauss's Law gives us an *alternative* to Coulomb's Law for calculating the electric field due to a given distribution of charges.**

Gauss's Law: The General Idea

The net number of electric field lines which leave any volume of space is proportional to the net electric charge in that volume.



Flux

The ***flux Φ*** of the ***field E*** through the ***surface S*** is **defined** as

$$\Phi = \int_S \vec{E} \cdot d\vec{A}$$

The **meaning** of flux is just the **number of field lines** passing through the surface.

Best Statement of Gauss's Law

The outward flux of the electric field through any closed surface equals the net enclosed charge divided by ε_0.

Gauss's Law: The Equation

$$\oint_S \vec{E} \cdot d\vec{A} = Q_{enc} / \varepsilon_0$$

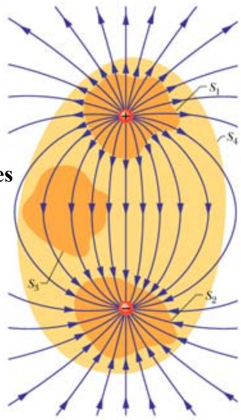
- S is any **closed** surface.
- Q_{enc} is the **net** charge **enclosed** within S.
- dA is an element of area on the surface of S.
- $d\vec{A}$ is in the direction of the **outward normal.**

$$\varepsilon_0 = 8.85 \times 10^{-12} \ SI \ units$$

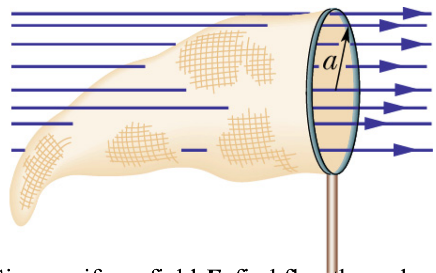
Flux Examples

Assume two charges, +q and –q. Find fluxes through surfaces. Remember flux is negative if lines are entering closed surface.

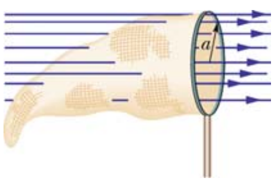
$\Phi_1 = +q/\varepsilon_0$
$\Phi_2 = -q/\varepsilon_0$
$\Phi_3 = 0$
$\Phi_4 = (q - q)/\varepsilon_0 = 0$



Another Flux Example



Given uniform field ***E***, find flux through net.



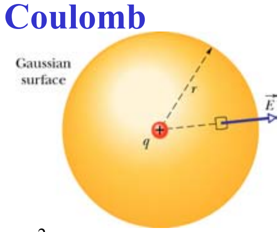
Must have ***closed surface,*** so let ***S*** be (net+circle). Then $Q_{enc} = 0$ so Gauss's law says $\Phi_S = 0$.

$$But \quad \Phi_{circle} = \pi a^2 E$$

$$\therefore \quad \Phi_{net} = \underline{-\pi a^2 E}$$

Gauss ⇒ Coulomb

Given a point charge, draw a concentric sphere and apply Gauss's Law:



$$\vec{E} = E(r)\hat{r}$$

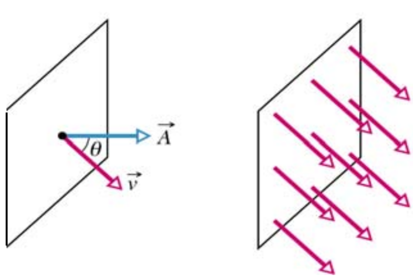
$$\Phi = \oint \vec{E} \cdot d\vec{A} = E(r)\, 4\pi r^2$$

Gauss ⇒ $\Phi = q/\varepsilon_0$

$$\therefore \quad E(r) = q/(4\pi r^2 \varepsilon_0) = \boxed{kq/r^2} \ !$$

If a vector field v passes through an area A at an angle θ, what is the name for the product $\vec{v} \cdot \vec{A} = vA\cos\theta$?

Q.23-1

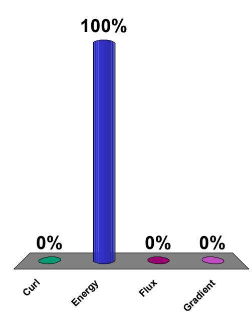


(1) curl (2) energy (3) flux (4) gradient

Q.23-1

What is the name for the product $\vec{v} \cdot \vec{A} = vA\cos\theta$?

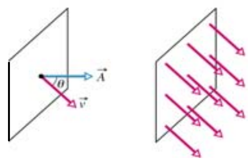
1. Curl
2. Energy
3. Flux
4. Gradient



Q.23-1

$$\Phi = \vec{v} \cdot \vec{A} = vA\cos\theta$$

$=$ ***flux*** from the Latin "to flow".



(1) curl **(2) energy** **(3) flux** **(4) gradient**

Q.23-2

Two charges Q_1 and Q_2 are inside a *closed cubical box* of side a. What is the *net outward flux* through the box?

(1) $\Phi = 0$

(2) $\Phi = (Q_1 + Q_2)/\varepsilon_0$

(3) $\Phi = k(Q_1 + Q_2)/a^2$

(4) $\Phi = \dfrac{Q_1 + Q_2}{4\pi\varepsilon_0 a^2}$

(5) $\Phi = \dfrac{Q_1 - Q_2}{4\pi\varepsilon_0 a^2}$

Q.23-2

Two charges Q_1 and Q_2 are inside a *closed cubical box* of side a. What is the *net outward flux* through the box?

1. $\Phi = 0$ — 0%
2. $\Phi = (Q_1 + Q_2)/\varepsilon_0$ — 100%
3. $\Phi = k(Q_1 + Q_2)/a^2$ — 0%
4. $\Phi = \dfrac{Q_1 + Q_2}{4\pi\varepsilon_0 a^2}$ — 0%
5. $\Phi = \dfrac{Q_1 - Q_2}{4\pi\varepsilon_0 a^2}$ — 0%

Q.32-2

Two charges Q_1 and Q_2 are inside a *closed cubical box* of side a. What is the *net outward flux* through the box?

$\Phi = 0$

$\Phi = (Q_1 + Q_2)/\varepsilon_0$

$\Phi = k(Q_1 + Q_2)/a^2$

$\Phi = \dfrac{Q_1 + Q_2}{4\pi\varepsilon_0 a^2}$

$\Phi = \dfrac{Q_1 - Q_2}{4\pi\varepsilon_0 a^2}$

Gauss:

The outward flux of the electric field through any closed surface equals the net enclosed charge divided by ε_0.

Application of Gauss's Law

- **We want to compute the electric field at the surface of a charged metal object.**
- **This gives a good example of the application of Gauss's Law.**
- **First we establish some facts about good conductors.**
- **Then we can get a neat useful result:**

$$E = \sigma / \varepsilon_0$$

Fields in Good Conductors

Fact: **In a steady state the electric field inside a good conductor must be zero.**

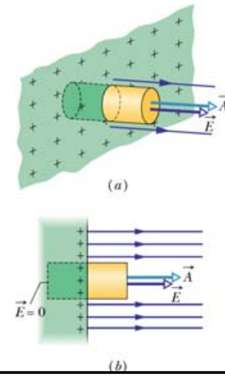
Why? **If there were a field, charges would move. Charges will move around until they find the arrangement that makes the electric field zero in the interior.**

Charges in Good Conductors

Fact: In a steady state, any net charge on a good conductor must be entirely on the surface.

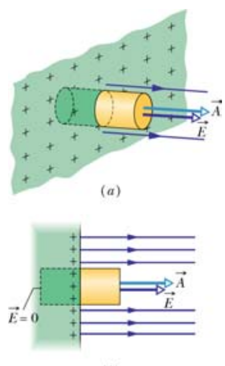
Why? If there were a charge in the interior, then by Gauss's Law there would be a field in the interior, which we know cannot be true.

Field at the Surface of a Conductor



- Construct closed gaussian surface, sides perpendicular to metal surface, face area = A.
- Flux through left face is zero because E=0.
- Field is perpendicular to surface or charges would move, therefore flux through sides is 0.
- So net outward flux is EA.

Field at the Surface of a Conductor



- Let σ stand for the surface charge density (C/m²)
- Then $Q_{enc} = \sigma A$.
- Now Gauss's Law gives us

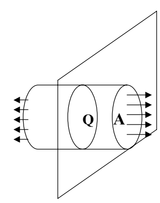
$$\Phi = Q_{enc} / \varepsilon_0$$

$$EA = \sigma A / \varepsilon_0$$

$$E = \sigma / \varepsilon_0$$

Large Sheet of Charge

σ = charge /area = surface charge density



$$\Phi = \frac{Q}{\varepsilon_0}$$ **Gauss' law**

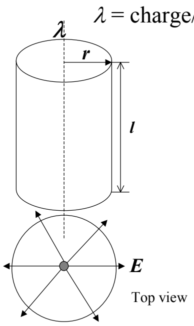
$$Q = \sigma A$$

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = 2EA$$

$$2EA = \frac{\sigma A}{\varepsilon_0} \qquad E = \frac{\sigma}{2\varepsilon_0}$$

Long Line of Charge

λ = charge/length = linear charge density



$$\Phi = \frac{Q}{\varepsilon_0}$$ **Gauss' law**

$$Q = l\lambda$$

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = EA$$

$$A = 2\pi r l$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

Summary for different dimensions

Point charge, d=0

$$\Phi = 4\pi r^2 E = Q / \varepsilon_0$$

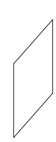
$$E = Q / (4\pi\varepsilon_0 r^2)$$

Line charge, d=1

$$E \propto \frac{1}{r^{2-d}}$$

$$\Phi = 2\pi r l E = \lambda l / \varepsilon_0$$

$$E = \lambda / (2\pi\varepsilon_0 r)$$



Surface charge, d=2

$$\Phi = 2AE = \sigma A / \varepsilon_0$$

$$E = \sigma / (2\varepsilon_0)$$