Capacitance and Dielectrics

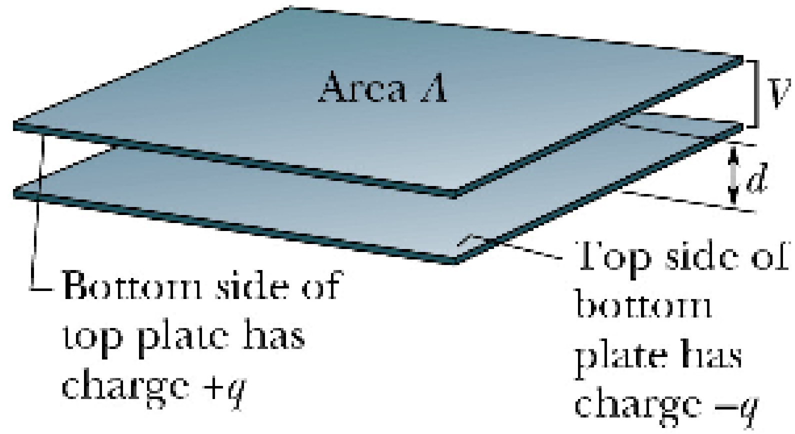
Capacitance

General Definition:

$$C = q / V$$

Special case for parallel plates:

$$C = \frac{\varepsilon_0 A}{d}$$



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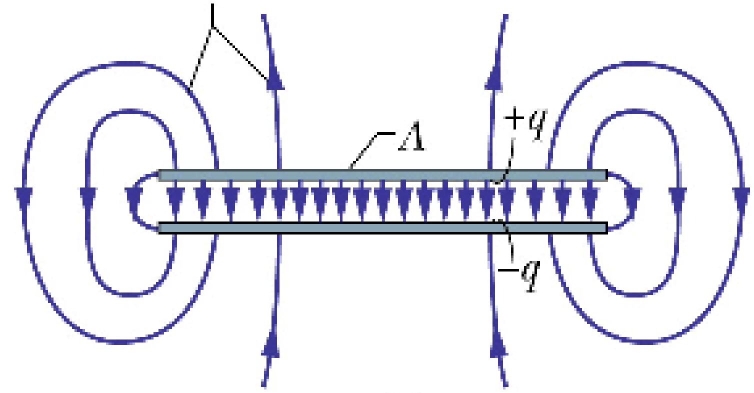

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Potential Energy

- I must do work to charge up a capacitor.

- This energy is stored in the form of ***electric potential energy.***

- We showed that this is $$U = \frac{Q^2}{2C}$$

- Then we saw that this energy is stored in the electric field, with a volume ***energy density***

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Potential difference and Electric field

Since potential difference is work per unit charge,

$$\Delta V = \int_a^b E\,dx$$

For the *parallel-plate* capacitor E is uniform, so

$$V = Ed$$

Also for parallel-plate case Gauss's Law gives

$$E = \sigma / \varepsilon_0 = \frac{Q}{\varepsilon_0 A} = Vd \quad so \quad C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

Spherical example

A spherical capacitor has inner radius a = 3mm, outer radius b = 6mm. The charge on the inner sphere is q = 2 C. What is the potential difference?

From Gauss's Law or the Shell Theorem, the field inside is

$$E = \frac{kq}{r^2}$$

From definition of potential difference

$$V = \int_a^b \frac{kq}{r^2} dr = kq\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$= 9\times10^{9} \times 2\times10^{-9}\left[\frac{1}{3\times10^{-3}} - \frac{1}{6\times10^{-3}}\right] = 18\times10^{3}\left(\frac{1}{3} - \frac{1}{6}\right) = \underline{3\times10^{3}\ V}$$

What is the capacitance?

$$C = Q/V = (2\ C)/(3000\ V) = \underline{6.7\times10^{-4}\ F}$$

Q.25-1

A capacitor has capacitance $C = 6\ \mu F$ and charge $Q = 2\ nC$. If the charge is increased to 4 nC what will be the new capacitance?

(1) 3 μF

(2) 6 μF

(3) 12 μF

(4) 24 μF

Q. 25 - 1

A capacitor has capacitance C = 6 μF and a charge Q = 2 nC. If the charge is increased to 4 nC what will be the new capacitance?

Solution:

Capacitance depends on the structure of the capacitor, not on its charge.

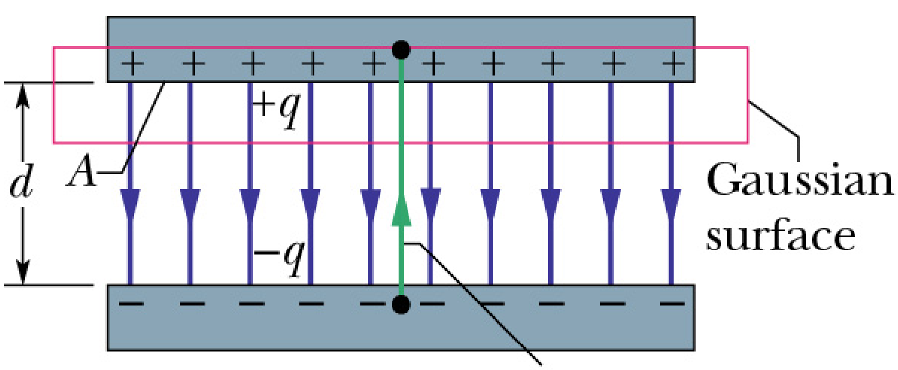
(1) 3 μF (2) 6 μF (3) 12 μF (4) 24 μF

Q.25-2

A parallel-plate capacitor has plates of area $0.1\ m^2$ and carries a charge of 9 nC.

What is the electric field between the plates?

1) **10,000 V/m**
2) **900 V/m**
3) **90 nV/m**

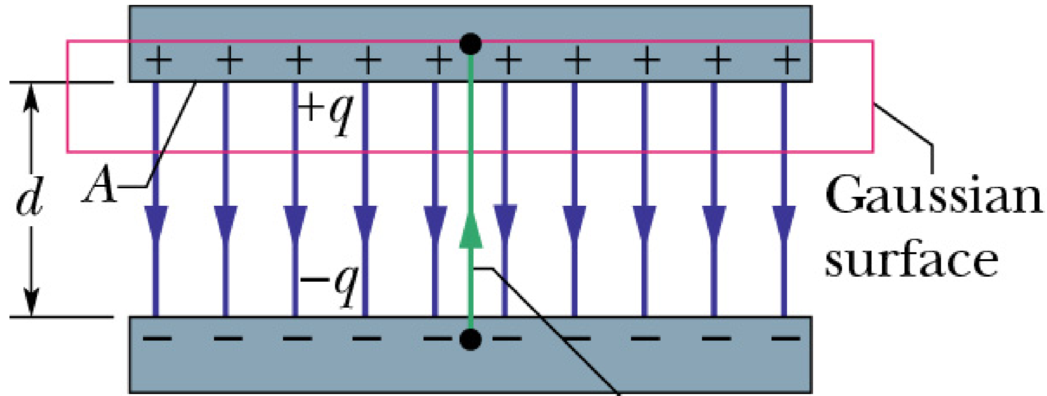


Q. 25 - 2

$A = 0.1\, m^2$

$Q = 9 \times 10^{-9}\, C$

$E = ?$



$$E = \sigma / \varepsilon_0$$

Solution:

Field at surface of conductor:

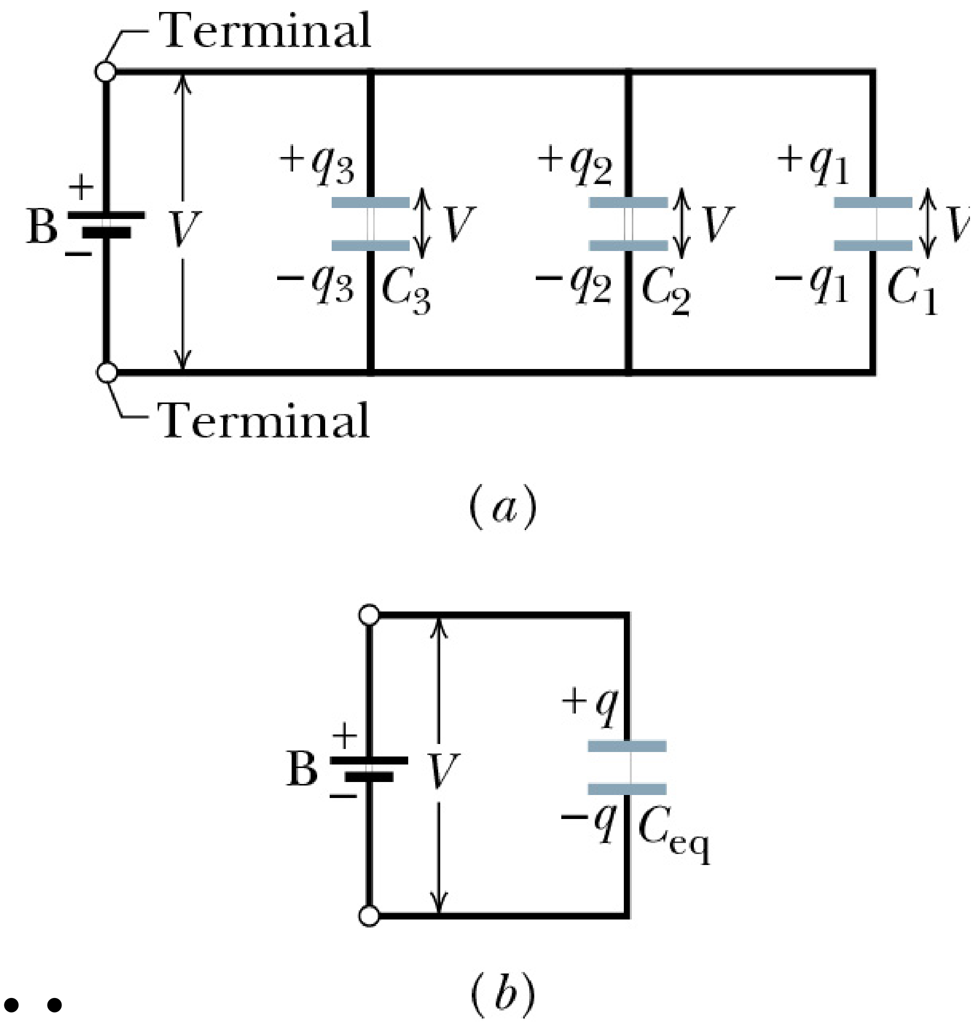
$$\sigma = Q / A = \frac{9 \times 10^{-9}}{1 \times 10^{-1}} = 9 \times 10^{-8}$$

$$E = \frac{9 \times 10^{-8}}{9 \times 10^{-12}} = 1 \times 10^{4}\, V/m$$

(1) $1 \times 10^{4}\, V/m$ (2) $900 V/m$ (3) $9 \times 10^{-8}\, V/m$

Capacitors in Parallel

If several capacitors are connected in parallel as shown, we can consider the arrangement as a single capacitor.

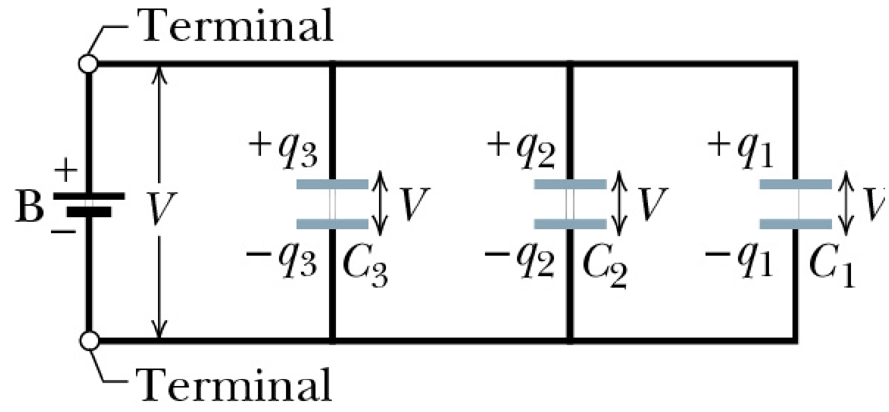


By charge conservation:

$$q_{tot} = q_1 + q_2 + \cdots$$

The wires are good conductors, so:

$$V_{tot} = V_1 = V_2 = \cdots$$



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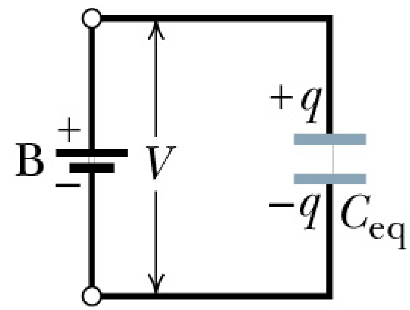

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Therefore:

$$C_1 + C_2 + \cdots = \frac{q_1}{V_1} + \frac{q_2}{V_2} + \frac{q_3}{V_3} + \cdots$$

$$= \frac{q_1 + q_2 + q_3 + \cdots}{V_{tot}} = q_{tot} / V_{tot} = C_{tot}$$

Capacitors in Series

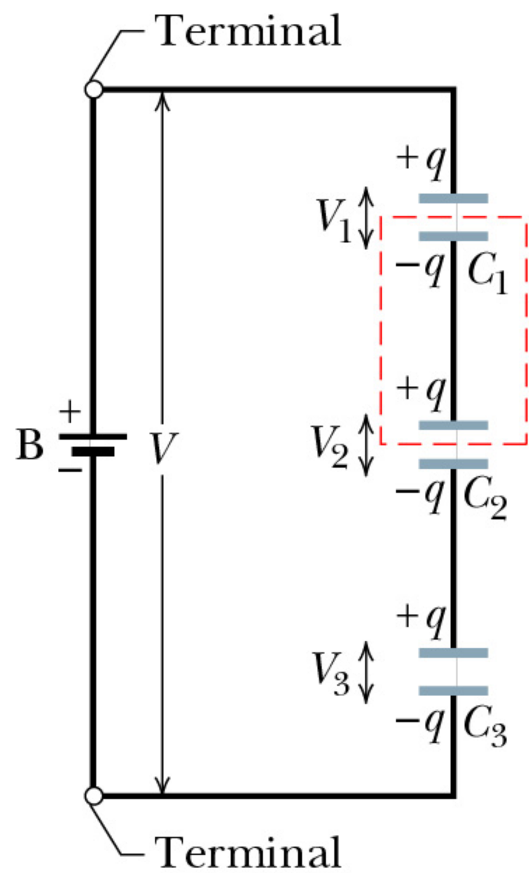
What if several capacitors are connected in ***series*** as shown?

Now charge conservation gives:

$$q_{tot} = q_1 = q_2 = \cdots$$

And now it is the voltages that add:

$$V_{tot} = V_1 + V_2 + \cdots$$



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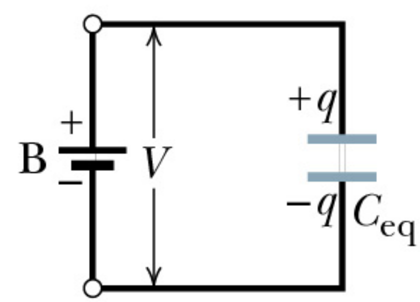


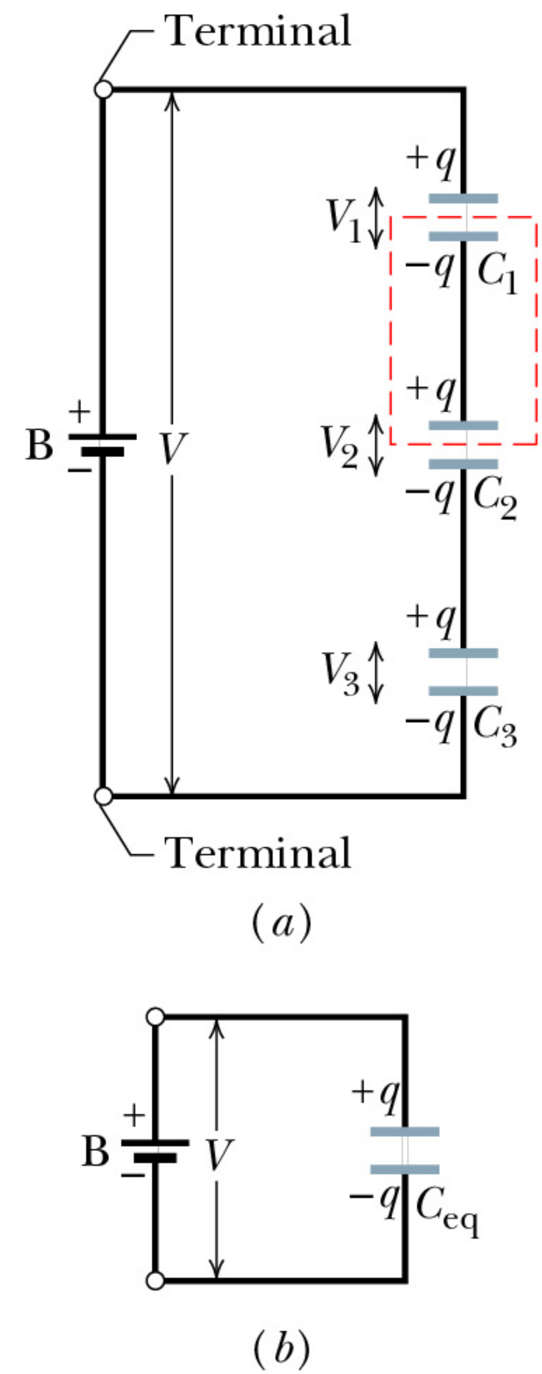
(b)

So now the voltage sum, when written in terms of the capacitances, gives the result for the series case:

$$V_{tot} = V_1 + V_2 + \cdots$$

$$q_{tot} / C_{tot} = q_1 / C_1 + q_2 / C_2 + \cdots$$

$$1 / C_{tot} = 1 / C_1 + 1 / C_2 + \cdots$$



Results for Capacitor Combinations

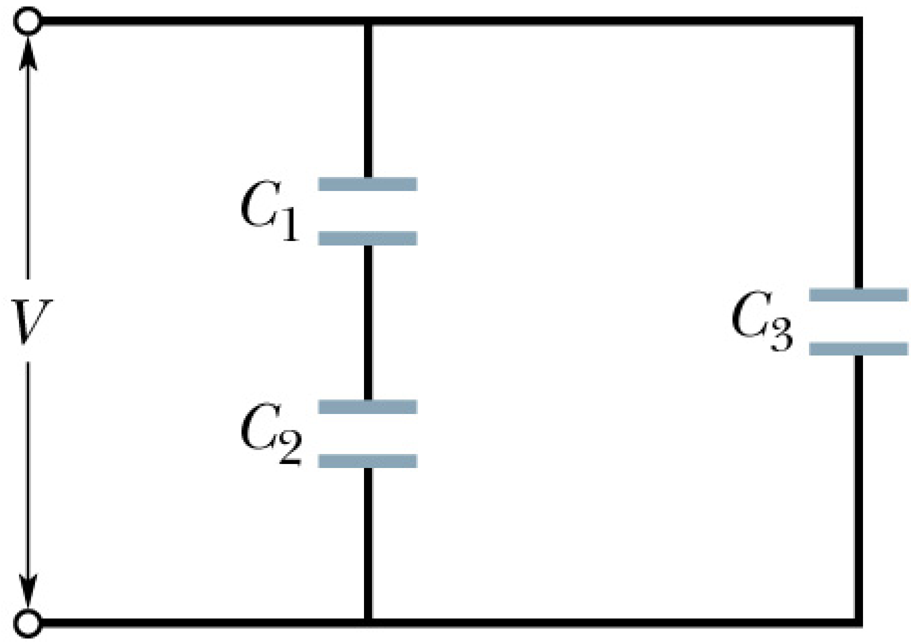
- **Capacitors in parallel:**

$$C_{tot} = C_1 + C_2 + \cdots$$

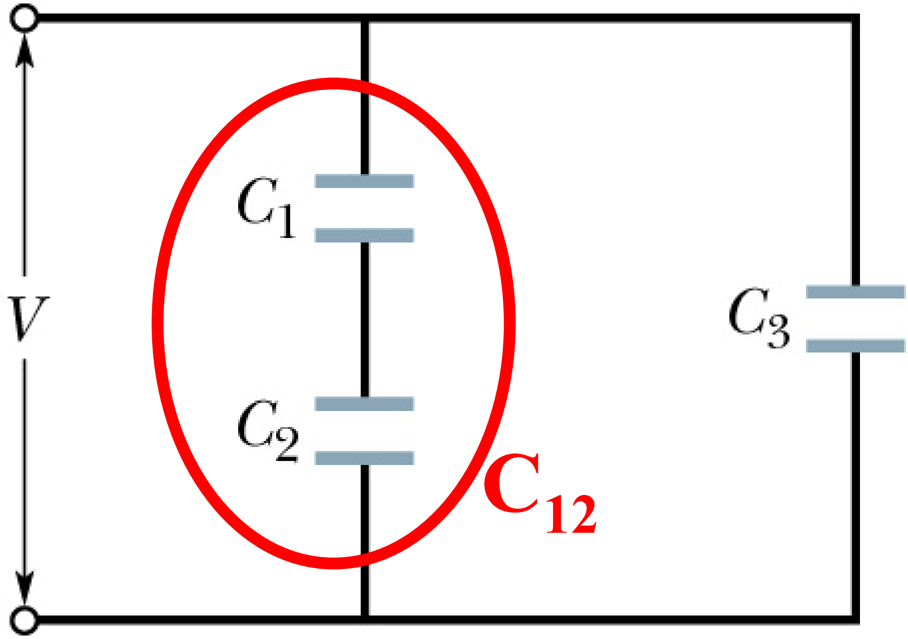
- **Capacitors in series:**

$$1/C_{tot} = 1/C_1 + 1/C_2 + \cdots$$

Example: Problem 25-8



What is effective capacitance of the device?



$$C_1 = 10\,\mu F$$
$$C_2 = 5.0\,\mu F$$
$$C_3 = 4.0\,\mu F$$

$$1/C_{12} = 1/C_1 + 1/C_2 = 1/10 + 1/5 = 3/10$$

$$C_{12} = 10/3 = 3.3\,\mu F$$

$$C_{tot} = C_{12} + C_3 = 3.3 + 4 = \underline{7.3\,\mu F}$$

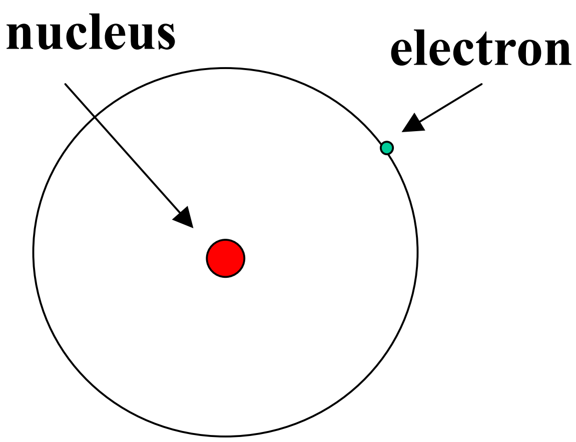
Dielectric Materials

Effect of placing a dielectric between the plates of a charged capacitor:

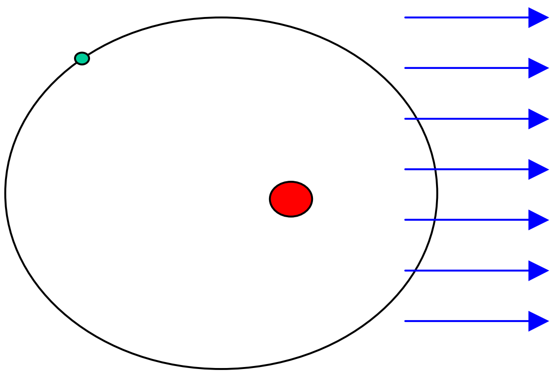
- **Applied electric field polarizes material**
- **This produces an induced surface charge**
- **This reduces field within material**
- **This reduces potential difference**
- **This increases capacitance**

Polarization of atoms and molecules

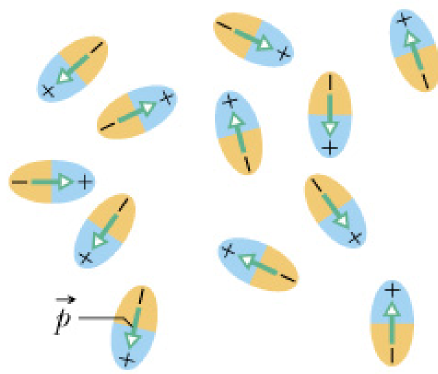
Non-polar molecules: induced dipoles



E=0



Polar molecules: preferential orientation



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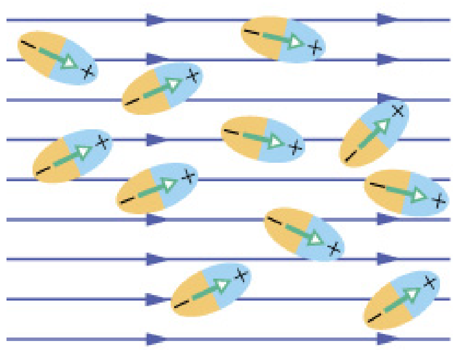


(b)

Dielectric materials

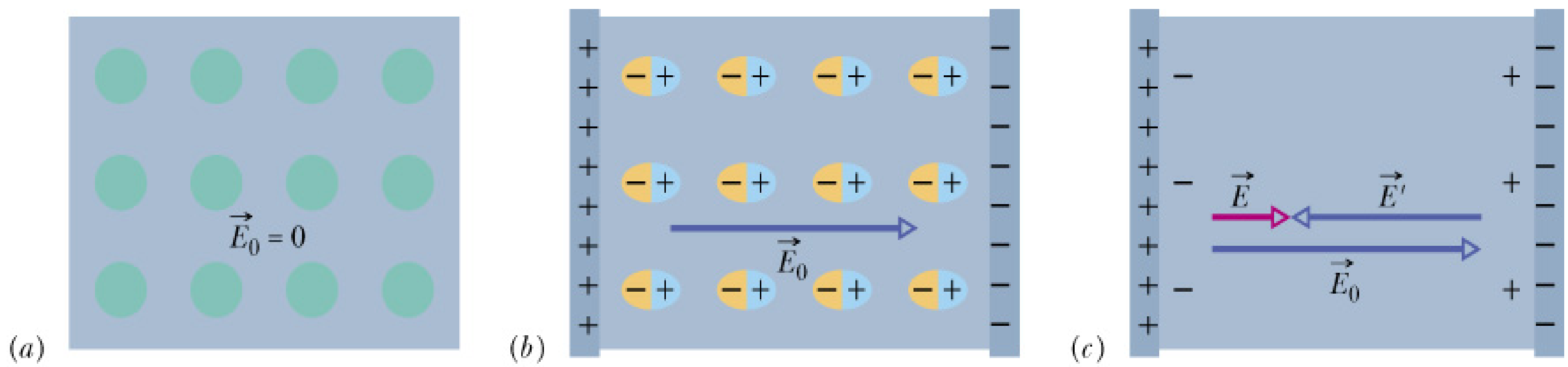
Dielectric materials polarize under external electric field E_0

Polarization creates the opposing electric field: $-E'$

Total field $E = E_0 - E' = E_0/\kappa < E_0$

κ is called **dielectric constant**

It shows how the field in a dielectric is weaker than in vacuum



Dielectric Constant

- So the effect of a dielectric is to ***decrease*** the field and ***increase*** the capacitance.
- The factor by which E is decreased and C is increased is defined to be the ***dielectric constant*** κ.

$$E = E_0 / \kappa$$

$$C = \kappa C_0$$

Example: Problem 25-48

Given a parallel-plate capacitor, dielectric-filled, with area $A = 100\ cm^2$, charge $Q = 890$ nC, and electric field $E = 1.4$ kV/mm.

(a) Find dielectric constant of the material.

(b) Find the induced charge.

(a) If there were no dielectric the field between the plates would be given by Gauss's Law as

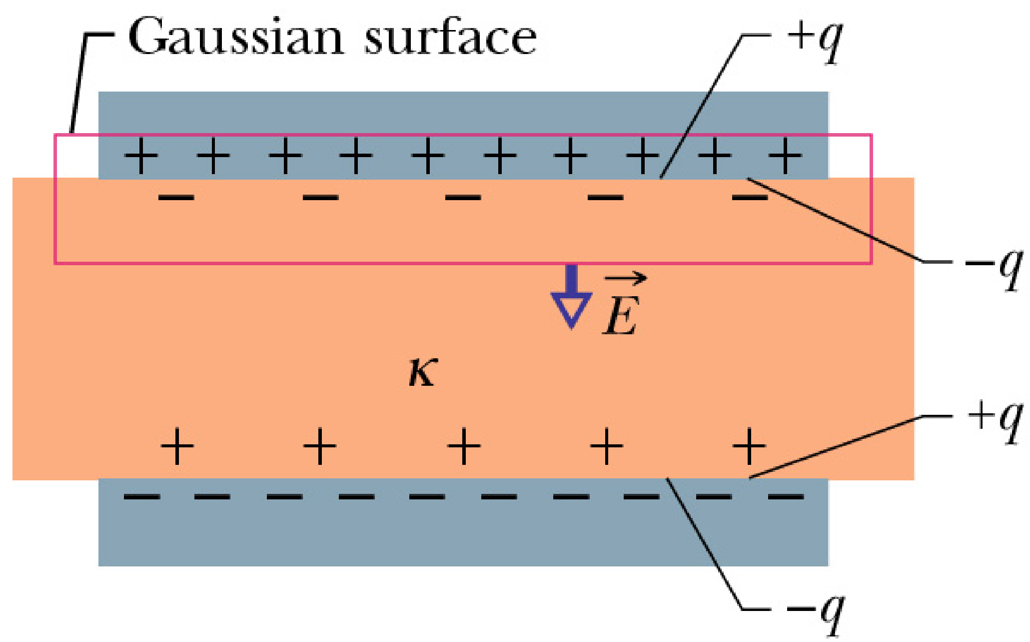
$$E_0 = (q/A)/\varepsilon_0$$

$$E_0 = \frac{8.9\times10^{-7}/10^{-2}}{8.85\times10^{-12}} = 1.01\times10^{7}\ V/m$$

But by definition: $E = E_0/\kappa$

$$\textit{So}\ \kappa = E_0/E = 1.01\times10^{7}/1.4\times10^{6} = 7.2$$

(b)



$$Gauss \Rightarrow$$

$$E = \frac{q_{tot}/A}{\varepsilon_0}$$

$$So \quad q_{tot} = \varepsilon_0 EA$$

$$= 8.85\times10^{-12}\times1.4\times10^{6}\times10^{-2} = 12.4\times10^{-8}C = 124nC$$

$$But \quad q_{tot} = q - q'$$

$$so \quad q' = q - q_{tot} = 890\,nC - 124\,nC = \boxed{766\,nC}$$