Chapters 22, 23: The Electric Field

NOTE! Plan B for registering the RF clickers: If you have not already done so, go to your WileyPlus account "My Profile" and change your student ID to your clicker number. Wiley+

Review: Electric Fields

- Definition of *electric field:* $\vec{F} = q\vec{E}$
  SI unit: newton per coulomb (N/C)
- Coulomb's Law for a point charge *Q:*
  $F = kQq/r^2$ OR $E = kQ/r^2$
- Principle of superposition (*vector* addition):
  $$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$

Electric Field Lines

We visualize the field by drawing **field lines.** These are defined by three properties:

- Lines point in the same direction as the field.
- Density of lines gives the magnitude of the field.
- Lines begin on + charges; end on – charges.

From these properties it is easy to see that

- Field lines never cross

Coulomb's Law for the Field

Coulomb's law for the *force* on *q* due to *Q:*

$$\vec{F} = k\frac{qQ}{r^2}\hat{r} = q\vec{E}$$

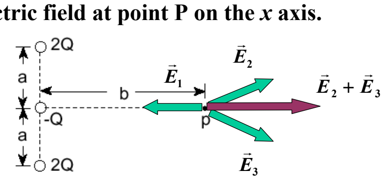
Coulomb's law for the *field* *E* due to *Q:*

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

Example

Three point charges are placed on the *y* axis as shown. Find the electric field at point P on the *x* axis.

$$E_1 = \frac{kQ}{b^2}$$



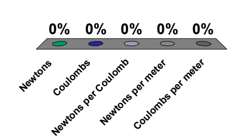
$$E_2 = E_3 = k\frac{2Q}{a^2+b^2} \qquad \vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \qquad E_y = 0$$

$$(\vec{E}_2 + \vec{E}_3)_x = 2E_2\cos\Theta = 2E_2 b/\sqrt{a^2+b^2}$$

$$E_x = (\vec{E}_2 + \vec{E}_3)_x - E_1 = 4kQb/(a^2+b^2)^{3/2} - kQ/b^2$$

Q.22-1 What is the SI unit for the electric field?

1. Newtons
2. Coulombs
3. Newtons per Coulomb
4. Newtons per meter
5. Coulombs per meter



Q.22-1

What is the SI unit for the electric field?

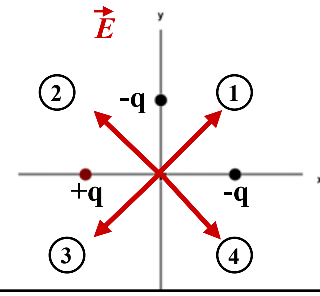
Field is force per unit charge:

$$\vec{F} = q\vec{E}$$

1. Newtons
2. Coulombs
3. Newtons per Coulomb
4. Newtons per meter
5. Coulombs per meter

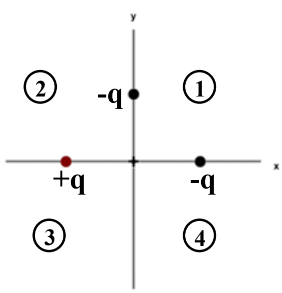
Q.22-2

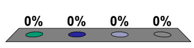
Three charges (one + and two -) are placed on the *x* and *y* axes as shown. What is the *approximate direction* of the electric field *at the origin*? Will it be pointing toward point 1, 2, 3, or 4?



Q.22-2

1. (1)
2. (2)
3. (3)
4. (4)

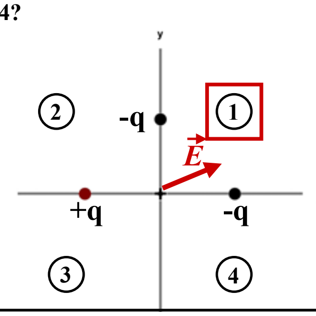




Q.22-2

Three charges (one + and two -) are placed on the *x* and *y* axes as shown. What is the *approximate direction* of the electric field *at the origin*? Will it be pointing toward point 1, 2, 3, or 4?

Solution. Imagine a positive test charge placed at the origin. It will be atracted to the –q charges and repelled by the +q charge.



Linear charge distribution

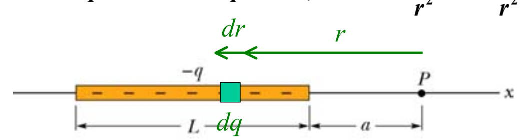
- *Linear charge density* = charge per unit length
- If a rod of length 2.5 m has a *uniform* linear charge density $\lambda = 3$ C/m, then the total charge on the rod is (2.5 m) × (3 C/m) = 7.5 C.
- If a rod of length L carries a *non-uniform* linear charge density $\lambda(x)$, then adding up all the charge produces an *integral:*

$$Q = \int_a^b dq = \int_a^b \lambda(x)\, dx$$

Example: Ch. 22 Prob. 27

Find the electric field at point P.

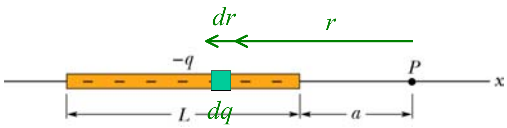
$$\text{Let } \lambda = -q/L. \quad \text{Then } dq = \lambda\, dr, \quad dE = k\frac{dq}{r^2} = k\lambda\frac{dr}{r^2}$$



$$E = \int dE = k\lambda \int_a^{a+L} \frac{dr}{r^2} = k\lambda\left[-\frac{1}{r}\right]_a^{a+L} = k\lambda\left(\frac{1}{a} - \frac{1}{a+L}\right)$$

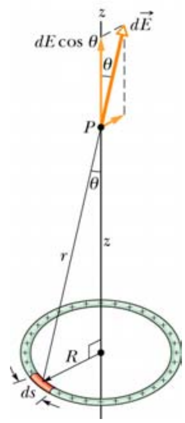
Suppose density is not uniform

Still true that $dq = \lambda\, dr, \quad dE = k\dfrac{dq}{r^2} = k\lambda\dfrac{dr}{r^2}$



$$E = \int dE = k\int_a^{a+L} \lambda(r)\frac{dr}{r^2}$$

Field of a charged ring

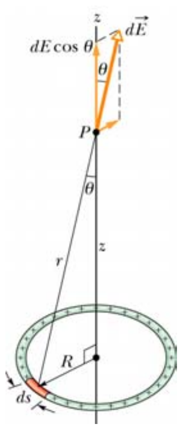


Uniform linear charge density so $dq = \lambda ds$ and $dE = kdq/r^2$

By symmetry, E_x=E_y=0 and so $E = E_z$ = sum of all dE_z, and $dE_z = \cos\theta\, dE$.

$$E = \int \frac{k\cos\theta\, dq}{r^2} = k\cos\theta\frac{Q}{r^2}$$

Charged ring continued

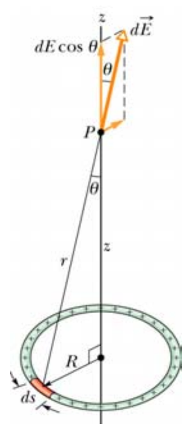


$$E = \int \frac{k\cos\theta\, dq}{r^2} = k\cos\theta\frac{Q}{r^2}$$

But $\cos\theta = z/r$ *so*

$$E = \frac{kQz}{r^3} = \frac{kQz}{(z^2+R^2)^{3/2}}$$

Charged ring result



$$E = \int \frac{k\cos\theta\, dq}{r^2} = k\cos\theta\frac{Q}{r^2}$$
$$= \frac{kQz}{r^3} = \frac{kQz}{(z^2+R^2)^{3/2}}$$

NOTE: This is a good example of a special result, which is the answer to an *example problem*, not a fundamental principle to be memorized. It is the *process* we are supposed to be learning, *not* the result!

NOTE

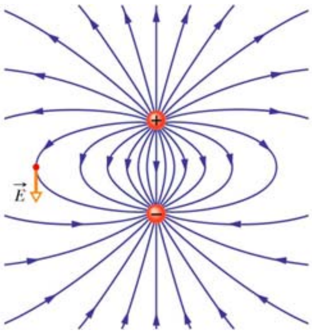
This result for the field on the axis of a charged ring can be derived more easily in Chapter 24 using the idea of ***electric potential.***

Electric Dipole

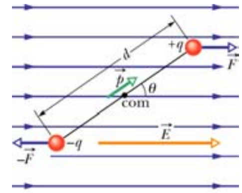
- The combination of two charges of equal but opposite sign is called a dipole.
- If the charges +q and –q are separated by a distance d, then the ***dipole moment*** $\vec{p}$ is defined as a vector pointing from –q to +q of magnitude $p = qd$.

$-q \quad \vec{p} \quad +q$

Electric Field Due to a Dipole



Torque on a Dipole in a Field



$$\tau = 2 \times F \times (\frac{d}{2} \sin\theta) = qE \times d \sin\theta = pE \sin\theta$$

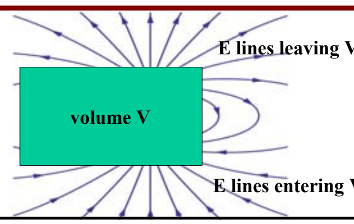
$$\vec{\tau} = \vec{p} \times \vec{E}$$

Gauss's Law

- Gauss's Law is the first of the four ***Maxwell Equations*** which summarize all of electromagnetic theory.
- Gauss's Law gives us an ***alternative*** to Coulomb's Law for calculating the electric field due to a given distribution of charges.

Gauss's Law: The General Idea

The net number of electric field lines which leave any volume of space is proportional to the net electric charge in that volume.



Flux

The ***flux Φ*** of the ***field E*** through the ***surface S*** is **defined** as

$$\Phi = \int_S \vec{E} \cdot d\vec{A}$$

The **meaning** of flux is just the **number of field lines** passing through the surface.

Best Statement of Gauss's Law

The outward flux of the electric field through any closed surface equals the net enclosed charge divided by ε_0.

Gauss's Law: The Equation

$$\oint_S \vec{E} \cdot d\vec{A} = Q_{enc} / \varepsilon_0$$

- S is any **closed** surface.
- Q_{enc} is the **net** charge **enclosed** within S.
- dA is an element of area on the surface of S.
- $d\vec{A}$ is in the direction of the **outward normal.**

$$\varepsilon_0 = 8.85 \times 10^{-12} \; SI \; units$$

Chapters 22, 23: The Electric Field

- **Previous Homework:**
  - **Read Chapter 22**
  - **Do Ch. 22 Questions 3, 5, 7, 9**
  - **Do Ch. 22 Problems 5, 19, 24, 34**
  - **Do WileyPlus assignment: Chapter 22**
- **Quiz tomorrow: Chapter 22**

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