

Examination 2 for PHYS 6450/7450 08th April 2026

Student Name: First

Last

Instructions: This test is worth 23 points which will be scaled to 18% of the final letter grade.

In this test the Boltzmann's constant, temperature, volume, the Helmholtz free energy, the specific heats at constant pressure and at constant volume, entropy, fugacity, the Fermi functions and the chemical potential are k , T , V , H , $C_P(T)$, $C_V(T)$, S , z and $f_v(z)$, μ respectively.

[1] Calculate the slope of the solid–liquid transition line for water near the triple point $T = 273.16$ K, given that the latent heat of melting is 3.35×10^9 erg/g, the density of the liquid phase is 1.00 g/cm³, and the density of the ice phase is 0.92 g/cm³. Estimate the melting temperature at $P = 100$ atm.

(a) Calculate dP/dT and hence dT/dP in K/atm? **[4 points]**

(b) What is ΔT ? **[1 point]**

[2] Consider two ideal Boltzmannian gases A and B, at pressures P_A and P_B and temperatures T_A and T_B and molecular masses m_A and m_B respectively, contained in two regions of space that communicate through a very narrow opening in the partitioning wall. The two gases effuse into each other such that $\phi = n \int_0^\infty v_z f(v_z) dv_z$, where ϕ is the number of molecules per unit time, passing through a small hole. The velocity v_z is along z axis, and $f(v_z)$ is the probability of the particles to have velocity v_z , n is the number density, where $f(v_z) = [m/(2\pi kT)]^{1/2} \exp(-mv_z^2/(2kT))$, where m is the mass of the particle.

(a) What is the relationship between the two quantities in ϕ_A and ϕ_B ? **[1 point]**

(b) Find the effusion velocity of particles A and B in terms of P , T and m . **[4 points]**

(c) In equilibrium what is this relationship between quantities that characterize these two gases, P_A , P_B , T_A , T_B , m_A and m_B ? **[2 points]**

[3] The Hamiltonian of a spin $\frac{1}{2}$ quantum system is given by $\hat{H} = -c\hat{B}$, where c is a positive constant of appropriate dimensions and \hat{B} is a (2×2) matrix with elements $b_{11} = -b_{22} = b_{12} + 1 = b_{21} + 1 = 1$.

(a) Write the density matrix $\hat{\rho}$. **[1 point]**

(b) Find $\langle \hat{H} \rangle$. **[1 point]**

(c) A unitary matrix \hat{M} transforms all matrices \hat{A} to $\hat{A}' = \hat{M}\hat{A}\hat{M}^{-1}$. Matrix elements of M are such that $m_{11} = m_{22} = m_{12} = -m_{21} = 1/\sqrt{2}$. Calculate \hat{H}' . **[2 points]**

(d) Calculate the energy eigenvalues of \hat{H}' . **[2 points]**

(e) Calculate the corresponding orthonormal eigenvectors. **[2 points]**

(f) Hence find the density matrix $\hat{\rho}'$. **[2 points]**

(g) Find $\langle \hat{H}' \rangle$. **[1 point]**