Student Name: First   Last

Instructions: This test is worth 20 points which will be scaled to 18% of the final letter grade.

In this test the Bolztmann's constant, temperature, volume, the Helmholtz free energy, the specific heats at constant pressure and at constant volume, entropy, fugacity, the Fermi functions and the chemical potential are k, T, V, H, $C_P(T)$, $C_v(T)$, S, z and $f_v(z)$, μ respectively.

[1] Two systems, A and B, of identical composition, are brought together and allowed to exchange both energy and particles, keeping volumes V_A and V_B constant.
(a) Write a simple expression for dE_A and dN_A? [**2 points**]
(b) Write an expression for dS_A and dS_B? [**2 points**]
(c) Write an expression for the change in total entropy dS_{tot}. [**1point**]
(d) Show that the minimum value of the quantity (dE_A/dN_A) is given by μ and T. Find it. [**2 points**]

[2] Consider a thermodynamic system of N identical distinguishable particles with only two allowed energy eigenvalues 0 and $\varepsilon > 0$.
(a) Find the canonical partition function for a single particle $Q_1(V, T)$. [**1 point**]
(b) Hence, find $Q_N (V, T)$. [**1 point**]
(c) From answer to (b) find the Helmholtz free energy (A), the entropy (S), the total internal energy (U), the pressure (P) and the chemical potential (μ). [**5 points**]

[3] A lattice gas consists of a lattice of N distinguishable sites. Each of which can be empty, in which case its energy is zero or occupied by one particle in which case its energy is $\varepsilon > 0$. Each particle has a magnetic moment of magnitude σ, which in the presence of an applied magnetic field H, can adopt two orientations (parallel or antiparallel to the field). The Hamiltonian is therefore given by $H = n(\varepsilon - \sigma Hs)$, with $n = 0, 1$ and $s = -1, 1$.
(a) Find the one particle canonical partition function for this system. [**2 points**]
(b) Evaluate the N particle canonical partition function. [**1 point**]
(c) Evaluate the average energy of the system. [**1 point**]
(d) Evaluate the magnetization of the system. [**2 points**]