Examination 1 for PHYS 6450/7450, 19th February 2025
First Last First Name

Student Name:

Instructions: 1) This test is worth a total of 27 points which will be scaled to a weight of 18% of the final letter grade.

(1) The Helmholtz free energy A(N,V, T) of a thermodynamic system is an extensive property of some of its arguments. Therefore, it can be written in the form,

$$A(N, V, T) = f(N, V, T)\left(\frac{\partial A}{\partial V}\right)_{N,T} + g(N, V, T)\left(\frac{\partial A}{\partial N}\right)_{V,T}.$$

Derive this form and hence the expressions for the functions f and g. [**6 points**]

Questions (2) and (3) refer to the same physical system described here. Consider a thermodynamic system of N identical distinguishable particles with only two allowed energy eigenvalues 0 and $\mathcal{E} > 0$. The Boltzmann constant is k.

(2) (a) Find the canonical partition function for a single particle $Q_1(V, T)$. [**1 point**]
(b) Hence, find $Q_N(V, T)$. [**1 point**]
(c) From answer to (b) find the Helmholtz free energy (A), the entropy (S), the total internal energy (U), the pressure (P) and the chemical potential (μ). [**5 points**]

(3) In a microcanonical ensemble find the total number of ways (Γ) to distribute a total of N particles in these two energy states. Assume there are n_1 particles in state with energy $\mathcal{E}$.
(a) Write an expression for (Γ) in terms of N and n_1. [**3 points**]
(b) Find the entropy (S) in terms of the answer to part (a). [**1 point**]
(c) Eliminate factorials and n_1 in terms of an appropriate thermodynamic variable. Hence express S in purely thermodynamic variables. [**2 points**]
(d) From the answer to (c) find the temperature (T), the pressure (P), the chemical potential (μ) the total internal energy (U), and the Helmholtz free energy (A). [**5 points**]
(e) What is the total number of particles in the ground state at temperature T? [**1 point**]
(f) Consider the case where the total energy of the system at temperature T is $U = fN\mathcal{E}$, where $0 < f < 1$. There are unexpected values for T. What range of f will yield such values? What are these unexpected values? [**2 points**]