Examination II for PHYS 6220/7220, Fall 2015

1. A particle of mass m and angular momentum ℓ moves in a central force field and has an equation of the orbit given by $r = a(1 + \cos(\theta))$, where a is a positive constant of appropriate dimensions.
(a) Find the form of the potential V(r). The answer should only contain the constants a, ℓ and m. (**2 points**)
(b) A particle of mass M approaches this center of force from far away with initial speed V_0 and impact parameter b. In what follows completely ignore the influence of the first particle of mass m on M. Derive and hence draw the shape of the effective potential, $V_e(r)$, that the mass M experiences. Justify with precise mathematical reasoning the shape of $V_e(r)$. Mark any critical points (r_c) that may occur on this graph giving expressions for these points r_c and the corresponding $V_e(r_c)$. (**4 points**)
(c) From the answer in part (b) determine the critical value of the impact parameter of M above which the particle fails to reach the origin? Quantitative answers in parts (b) and (c) should only contain the constants a, b, ℓ, m, M and V_0. (**2 points**)

2. A right handed Cartesian frame S is rotated about its fixed origin to obtain another frame S'. The initial set of right handed unit vectors along the X, Y and Z axes of S are $\mathbf{e_1}$, $\mathbf{e_2}$ and $\mathbf{e_3}$, respectively Those along the new axes, X', Y' and Z', of the frame S' are $\mathbf{e_1'}$, $\mathbf{e_2'}$ and $\mathbf{e_3'}$, respectively. It is known that $(\mathbf{e_1})^T = (1, 0, 0)$ and $(\mathbf{e_3})^T = (0, 0, 1)$. Similarly it is known that $(\mathbf{e_1'})^T = (1/5)(4, 1, \sqrt{8})$, $(\mathbf{e_2'})^T = (1/5)(1, 4, -\sqrt{8})$ and $(\mathbf{e_3'})^T = (1/5)(-\sqrt{8}, \sqrt{8}, 3)$. Express all angles in radians in the interval $[0, 2\pi]$.

(a) Find the matrix corresponding to this rotation. Do not convert any matrix elements to decimals. (**5 points**)
(b) Check that it is indeed a rotation matrix. (**1 point**)
(c) Find the angle of rotation Φ. (**1 point**)
(d) Find the unit vector along the axis of rotation **n**. (**2 points**)
(e) If this rotation was carried out not about a single axis and angle but by three Euler angle rotations then find the three Euler angles θ, ψ, and ϕ. Assume θ is an acute angle. (**3 points**)